2a. *[2 marks]*

Let $f(x) = \sin(e^x)$ for $0 \leq x \leq 1.5$. The following diagram shows the graph of f.

Find the x-intercept of the graph of f.

Let $\theta = e^x$ $\quad y = \sin\theta$

$0 = \sin\theta$

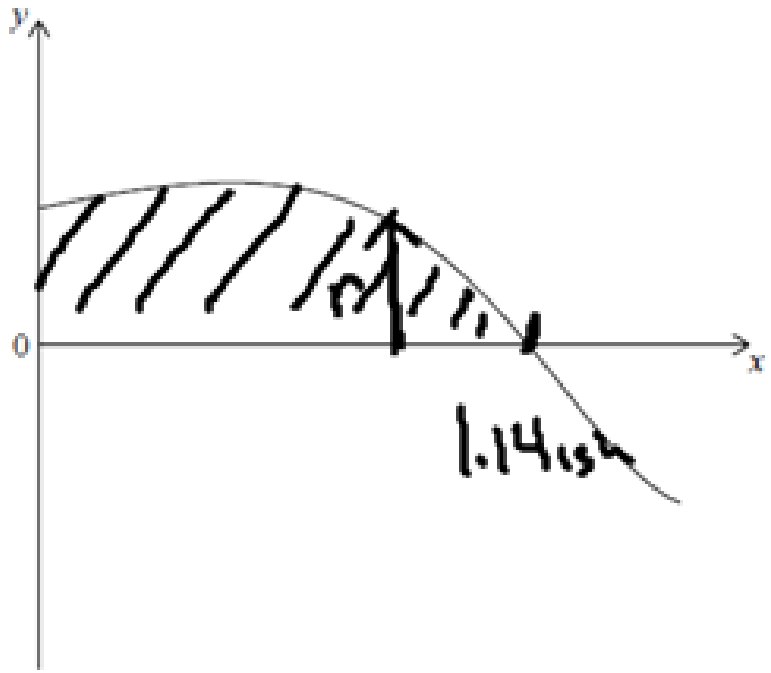
$\theta = \sin^{-1}(0)$

$\theta = \begin{cases} 0 \\ \pi \end{cases} + 2\pi k, k \in I$

$e^x = 0, e^x = \pi$

NO SOLN $\quad x = \ln(\pi)$

$x \approx 1.14$ ish



2b. *[3 marks]*

The region enclosed by the graph of f, the y-axis and the x-axis is rotated 360° about the x-axis.

Find the volume of the solid formed.

$$V = \int_a^b \pi \left(f(x)\right)^2 dx$$

$$V = \int_0^{\ln \pi} \pi \left[\sin(e^x)\right]^2 dx$$

$$= \pi \int_0^{\ln \pi} \left(\sin(e^x)\right)^2 dx$$

$$= \pi (0.7951)$$

$$\approx 2.50$$

$\int_0^{\ln(\pi)} ((\sin(e^x))^2$

.7951356882

Ans*π

2.497992437

5a. *[2 marks]*

Note: In this question, distance is in metres and time is in seconds.

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \leqslant t \leqslant 5$.

Write down the values of t when $a = 0$.

$3 \times 8 = 24$

$- \times - = 24$

$- + - = -14$

$0 = 3t^2 - 14t + 8$

$0 = \underline{3t^2 - 2t} - \underline{12t + 8}$

$= t(3t-2) - 4(3t-2)$

$0 = (3t-2)(t-4)$

$t = \frac{2}{3} \quad t = 4$

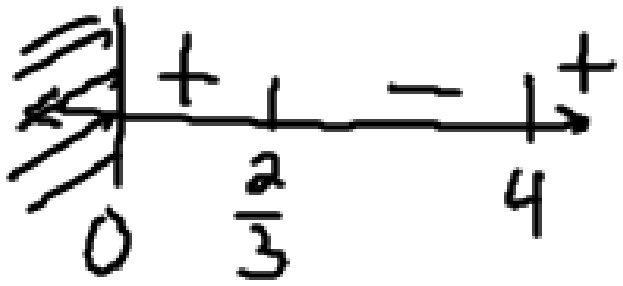
5b. *[2 marks]*

Hence or otherwise, find all possible values of t for which the velocity of P is decreasing.

$t = \frac{2}{3}$ $t = 4$

Velocity decreasing when acceleration is negative

Sign diagram for "a"



Velocity decreasing from $\frac{2}{3} < t < 4$

5c. *[6 marks]*

When $t = 0$, the velocity of P is $3\ \text{ms}^{-1}$.

Find an expression for the velocity of P at time t.

$$v(t) = \int a(t)\,dt$$

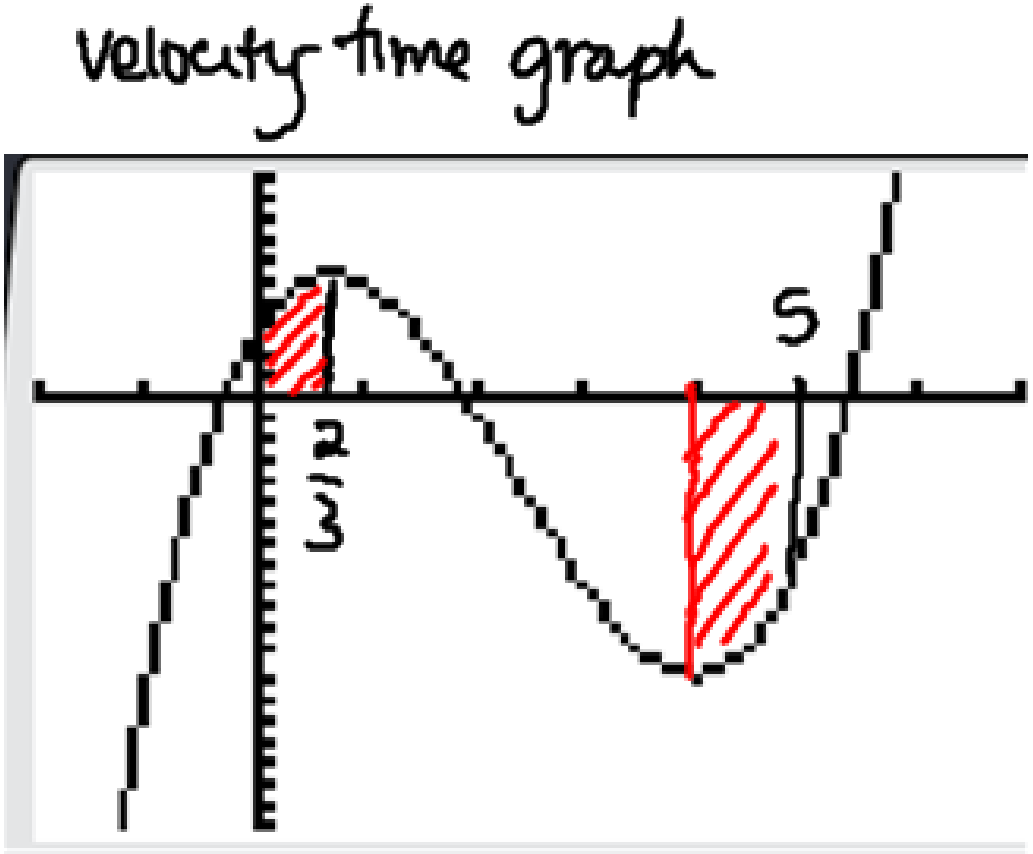
$$= \int (3t^2 - 14t + 8)\,dt$$

$$= 3\left(\frac{1}{3}t^3\right) - 14\left(\frac{1}{2}t^2\right) + 8t + C$$

$$v(t) = t^3 - 7t^2 + 8t + 3$$

5d. *[4 marks]*

Find the total distance travelled by P when its velocity is increasing.



$$\text{Total D} = A_1 + A_2$$

$$A_1 = \int_0^{\frac{2}{3}} (t^3 - 7t^2 + 8t + 3)\,dt$$

$$= 3.1358$$

$$A_2 = \int_4^5 (0 - (t^3 - 7t^2 + 8t + 3))\,dt$$

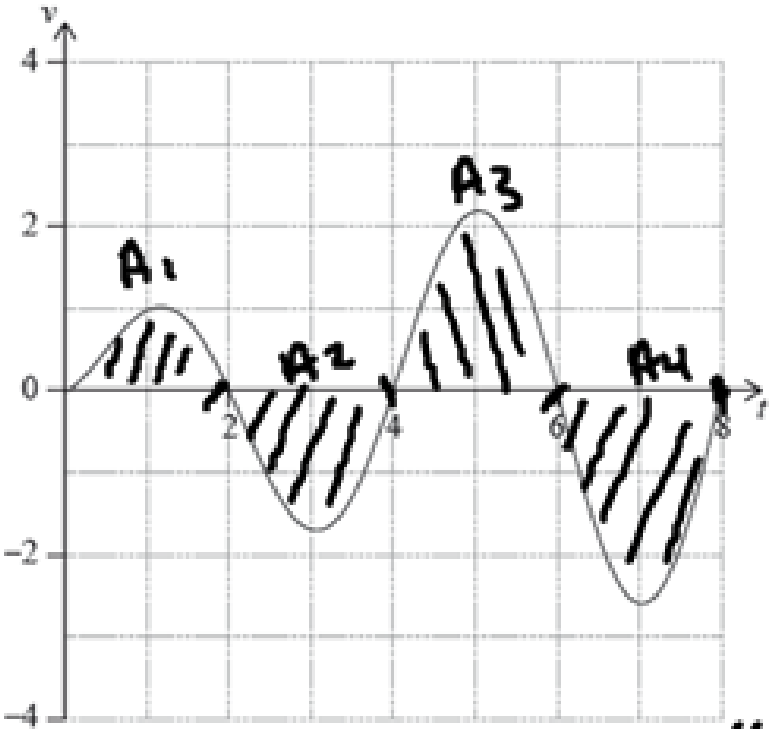
$$= 11.0833\overline{3}$$

$$\text{Total} = 3.1358 + 11.0833\overline{3}$$

$$= 14.21$$

7a. *[1 mark]*

A particle P moves along a straight line. Its velocity v_P m s^{-1} after t seconds is given by $v_P = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \leqslant t \leqslant 8$. The following diagram shows the graph of v_P.



Write down the first value of t at which P changes direction.

Velocity goes from positive to to negative ∴ $t = 2$

7b. *[2 marks]*

Find the **total** distance travelled by P, for $0 \leqslant t \leqslant 8$.

A Between 2 curves $= \int_a^b (y_{up} - y_{low})\,dx$

Total D $= A_1 + A_2 + A_3 + A_4$

$$D = \int_0^2 \left(\sqrt{t}\sin\left(\tfrac{\pi}{2}t\right)\right)dt + \int_2^4 \left(-\sqrt{t}\sin\left(\tfrac{\pi}{2}t\right)\right)dt + \int_4^6 \left(\sqrt{t}\sin\left(\tfrac{\pi}{2}t\right)\right)dt + \int_6^8 \left(-\sqrt{t}\sin\left(\tfrac{\pi}{2}t\right)\right)dt$$

$= 1.2370 + 2.1994 + 2.8443 + 3.3670$

$= 9.6478$ m

7c. [4 marks]

A second particle Q also moves along a straight line. Its velocity, v_Q m s^{-1} after t seconds is given by $v_Q = \sqrt{t}$ for $0 \leqslant t \leqslant 8$. After k seconds Q has travelled the same total distance as P.

Find k.

Total D of particle P is 9.65 m.

$$D_Q = \int_0^k \sqrt{t}\, dt$$

$$= \frac{2}{3} t^{\frac{3}{2}} \Big]_0^k$$

$$= \frac{2}{3} k^{3/2}$$

$$= 9.65$$

$$9.65 = \frac{2}{3} k^{3/2}$$

$$14.475 = k^{3/2} \rightarrow (a)^{2/3} = (b^{3/2})^{2/3}$$

$$k = (14.475)^{2/3}$$

$$= 5.93944$$

after $\sim$ 5.94 seconds the two particles have travelled the same distance.