Introduction to Euler's constant and Natural logs (NOT IN THE TEXTBOOK...but an outcome just the same:)

RF07.03 Determine, without technology, the exact value of a logarithm, such as $\log_2 8$ and $\ln e$.

RF08.04 Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$ and $\ln 10$.

Say "Oiler"

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Ex: you deposit \$1000 at 6.26% per annum (year), compounding semi-annually. How much will you have after 2 years?

$P = 1000$

$r = 0.0626$

$n = 2$

$t = 2$

$$A = 1000\left(1+\frac{0.0626}{2}\right)^{2\cdot 2}$$

$$A = 1000(1.0313)^4$$

$$= 1000(1.131201757)$$

$$= 1131.2017$$

$$= \$1131.20$$

Example: ~~deposit~~ borrow \$1 for one year at 100% compounded:

Annually: $A = 1\left(1+\frac{1}{1}\right)^{1\cdot 1} = 1(2)^1 = 2$

Monthly: $A = 1\left(1+\frac{1}{12}\right)^{12} = (1.08333)^{12} = \$2.61303529$

Daily: $A = 1\left(1+\frac{1}{365}\right)^{365} = (1.002739726)^{365} = 2.714567482$

Hourly: $A = 1\left(1+\frac{1}{8760}\right)^{8760} = (1.000114155)^{8760} = 2.718126691$

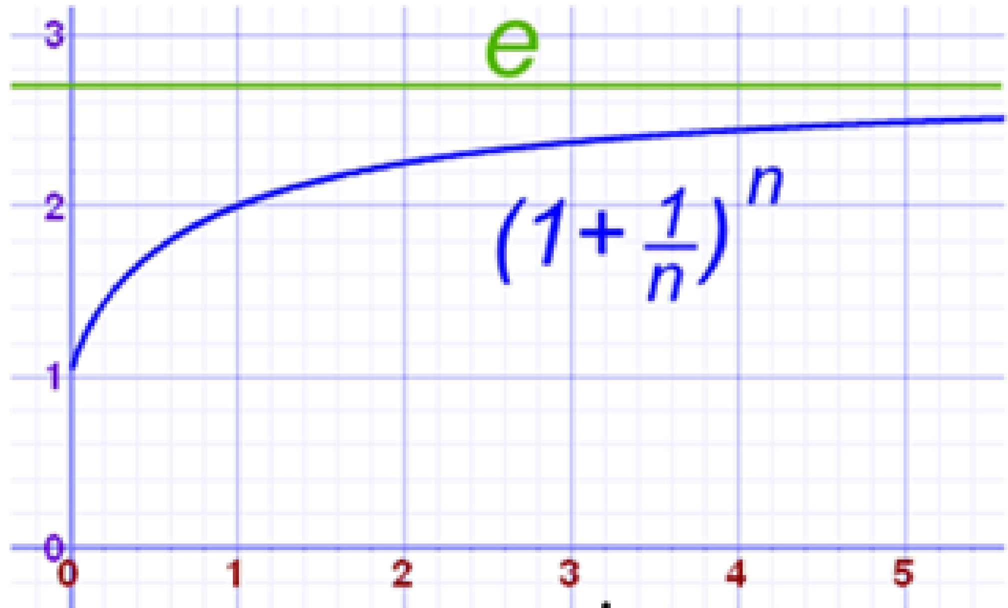
Secondly: $A = \left(1+\frac{1}{31536000}\right)^{31536000} = 2.718282473$

As the number of compounding periods increases the value of the investment approaches a certain number:

e = 2.71828182846 Euler's number ← Irrational π

$\sqrt{2}$

n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

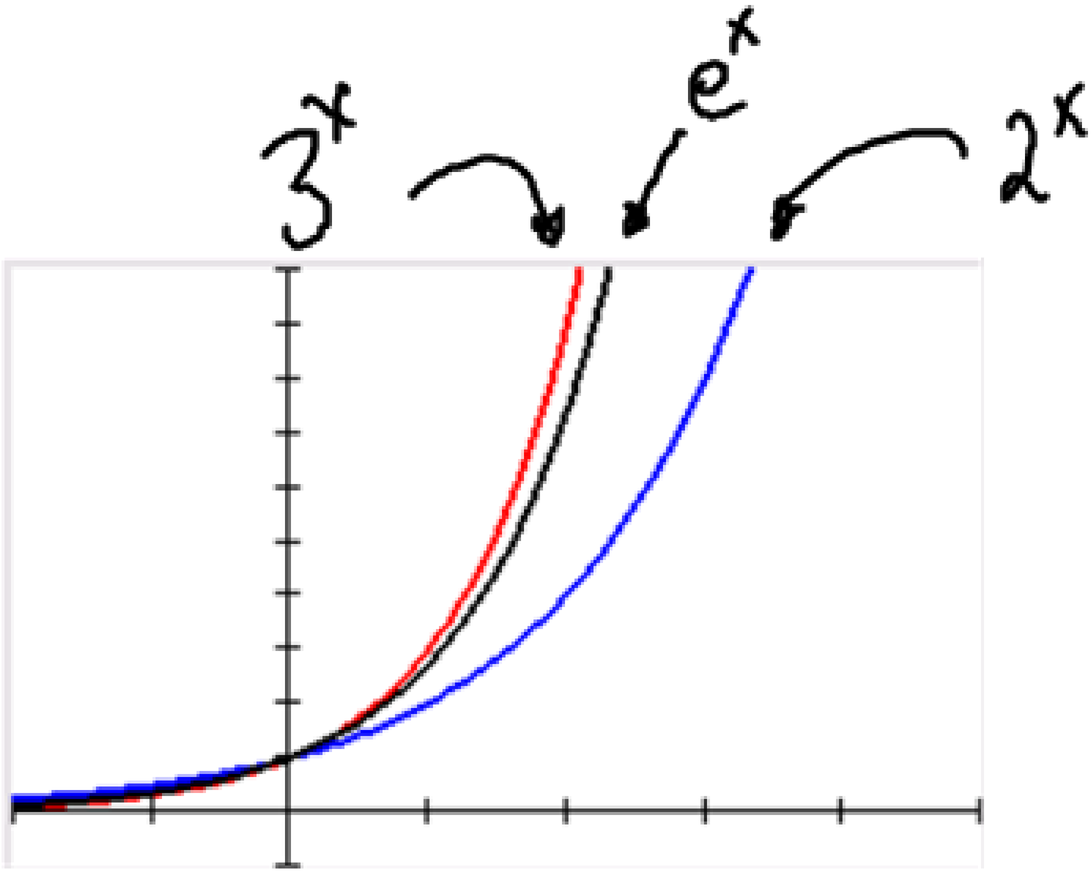


Don't copy

1 Use the $\boxed{e^x}$ key of your calculator to find the approximate value of e to as many digits as are possible.

2 Sketch, on the same set of axes, the graphs of $y = 2^x$, $y = e^x$ and $y = 3^x$. Comment on any observations.

3 Sketch, on the same set of axes, the graphs of $y = e^x$ and $y = e^{-x}$. What is the geometric connection between these two graphs?



Laws of Exponents

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Laws of Exponents with e

$$e^m \cdot e^n = e^{m+n}$$

$$(e^m)^n = e^{m \cdot n}$$

Example: Simplify the following:

A) $\left(e^3\right)^{3x-2}$

e^{9x-6}

B) $\left(e^{1.4}\right)\left(e^{0.7}\right)$

$e^{1.4+0.7} = e^{2.1}$

Solve the following equation $3^x = 5$ for the EXACT value of x using:

$\hookrightarrow \quad x = \log_3 5$

$\hookrightarrow \quad \log_{12}(3^x) = \log_{12} 5$

$x \cdot \log_{12} 3 = \log_{12} 5$

$x = \dfrac{\log_{12} 5}{\log_{12} 3}$

C) base e

$\log_e(3^x) = \log_e 5$

$x \cdot \log_e 3 = \log_e 5$

$x = \dfrac{\log_e 5}{\log_e 3}$

$x = \dfrac{\ln 5}{\ln 3}$

$\log_e x = \ln x$ natural logarithm

say "lawn"

Natural Logarithms:

$e^y = x \quad \rightarrow$ switch to log form: $\log_e x = y$

When the base is e, write as a natural logarithm:

$$y = \log_e x \text{ is the same as } y = \ln x$$

1. Evaluate:

(a) $\ln 1 = 0$

(b) $\ln e = 1$

(c) $\ln 15$
$= 2.708050201$

D) $\ln e^2 = 2$
$\downarrow$
$2 \cdot \ln e$
$2 \cdot (1)$

Find the exact value for x:

$$e^{x-1} = 7$$

$\log_e 7 = x-1$

$\ln 7 = x-1$

$x = 1 + \ln 7$

$$e^{2x} - 5e^x + 6 = 0$$

$(e^x)^2 - 5(e^x) + 6 = 0$

let $m = e^x$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2$ $\quad$ $m = 3$

$e^x = 2$ $\quad$ $e^x = 3$

$x = \log_e 2$ $\quad$ $x = \ln 3$

$x = \ln 2$

$$\frac{5e^{2x}}{5} = \frac{0.815}{5}$$

$$e^{2x} = 0.163$$

$$2x = \ln 0.163$$

$$x = \frac{1}{2} \cdot \ln(0.163)$$

Example: Solve for x

A) $\left(e^{2}\right)^{3x-2} = e^{5x+12}$

$e^{6x-4} = e^{5x+12}$

$6x-4 = 5x+12$

$6x-5x = 12+4$

$x = 16$

B) $3\left(5-2e^{x-4}\right)-2 = 10$

$\frac{3(5-2e^{x-4})}{3} = \frac{12}{3}$

$5-2e^{x-4} = 4$

$-2e^{x-4} = -1$

$e^{x-4} = \frac{1}{2}$

$\ln\left(\frac{1}{2}\right) = x-4$

$x = 4+\ln\left(\frac{1}{2}\right)$

C) $2e^{2x} - 7e^{x} + 12 = 9$

$2(e^x)^2 - 7(e^x) + 3 = 0$

let $m = e^x$

$2m^2 - 7m + 3 = 0$

$(2m - 1)(m - 3) = 0$

$m = \frac{1}{2}$ $\quad$ $m = 3$

$e^x = \frac{1}{2}$ $\quad$ $e^x = 3$

$x = \ln\left(\frac{1}{2}\right)$ $\quad$ $x = \ln 3$

laws of natural logs

simplify:

A) $\ln 7 + \ln x^2 - 3\ln 2$

$\ln 7 + \ln x^2 - \ln(2^3)$

$\ln(7 \cdot x^2) - \ln 8$

$\ln\left(\frac{7x^2}{8}\right)$

B) $4\ln y - (2\ln x - \ln y)$

$\ln(y^4) - (\ln(x^2) - \ln y)$

$\ln(y^4) - \left(\ln\left(\frac{x^2}{y}\right)\right)$

$\ln\left(\frac{y^4}{\frac{x^2}{y}}\right) = \ln\left(\frac{y^5}{x^2}\right)$

factor:

A) $(\ln x)^2 - 5\ln x + 6$ let $m = \ln x$

$m^2 - 5m + 6$

$(m-3)(m-2) \rightarrow (\ln x - 3)(\ln x - 2)$

B) $\ln^3 x - 4\ln x$ let $m = \ln x$ $\ln^3 x = (\ln x)^3$

$\neq \ln(x^3)$

$m^3 - 4m$

$m(m^2 - 4)$

$m(m+2)(m-2)$

$\ln x(\ln x + 2)(\ln x - 2)$