5.4

Equations and Graphs of Trigonometric Functions

Focus on...

- using the graphs of trigonometric functions to solve equations
- analysing a trigonometric function to solve a problem
- determining a trigonometric function that models a problem
- using a model of a trigonometric function for a real-world situation

Solving Trigonometric Equations

Solve both graphically and algebraically the equation $2\sin x + 1 = 1$ for $-2\pi \le x \le 2\pi$.

$y = 2\sin x + 1$

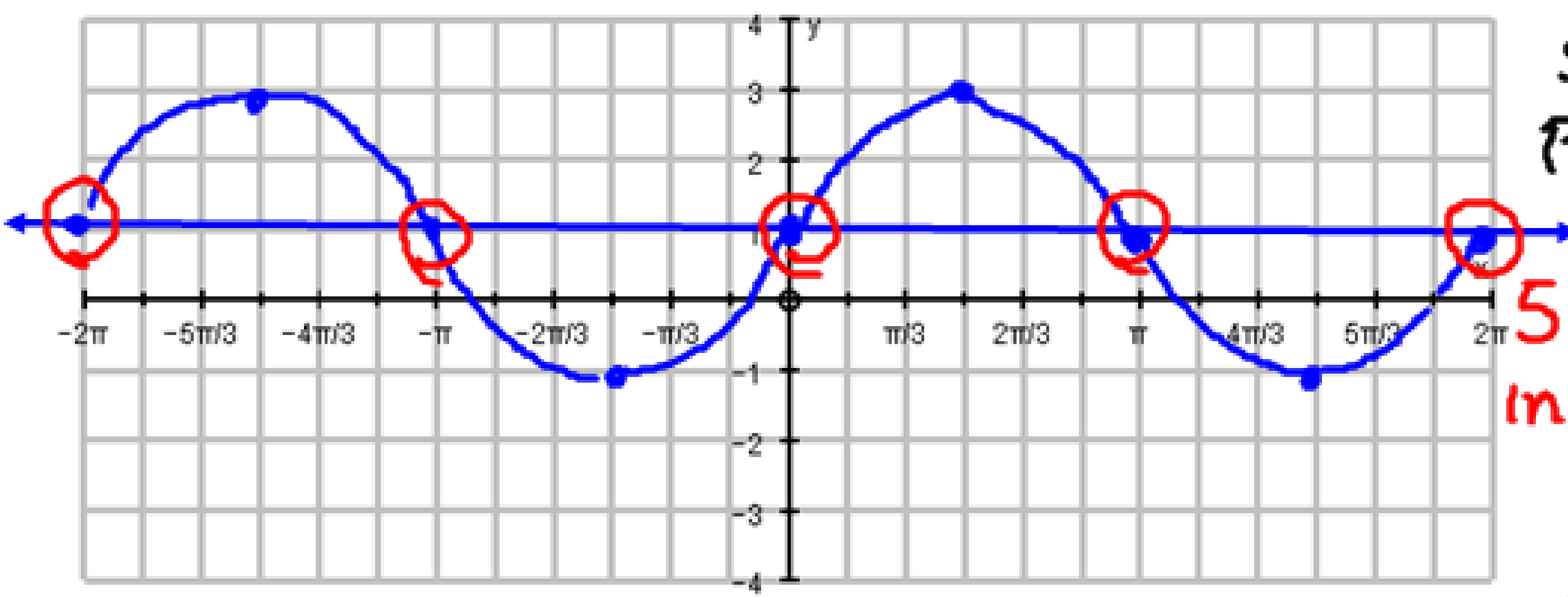
VS 2 VT 1

Amp: 2

SA: $y = 1$

Period: 2π

Graph the ~~line~~ curve $y = 2\sin x + 1$. Find the points where $y = 1$.



5 Points of intersection

answer: $x = -2\pi, -\pi, 0, \pi, 2\pi$ ← graph shows us

ALL sol'ns: $x = \left\{ \begin{matrix} 0 \\ \pi \end{matrix} \right. + 2\pi k, k \in I$

Can also solve algebraically: ✓ radian

$2\sin x + 1 = 1 \qquad -2\pi \le x \le 2\pi$

$2\sin x = 0$ axis value

$\sin x = 0$ unit circle value

interval

$x = \sin^{-1}(0)$

$x = \begin{cases} 0, 2\pi, -2\pi \\ \pi, -\pi \end{cases}$

$\underbrace{x = \begin{cases} 0 \\ \pi \end{cases} + 2\pi k, k \in I}_{\text{general sol'n}}$

Examples:

1. Solve graphically: $2\cos(2x+30^\circ) = -2$, general solution.

Verify by solving algebraically.

$$y = 2\cos(2x+30^\circ)$$
$$= 2\cos(2(x+15^\circ))$$

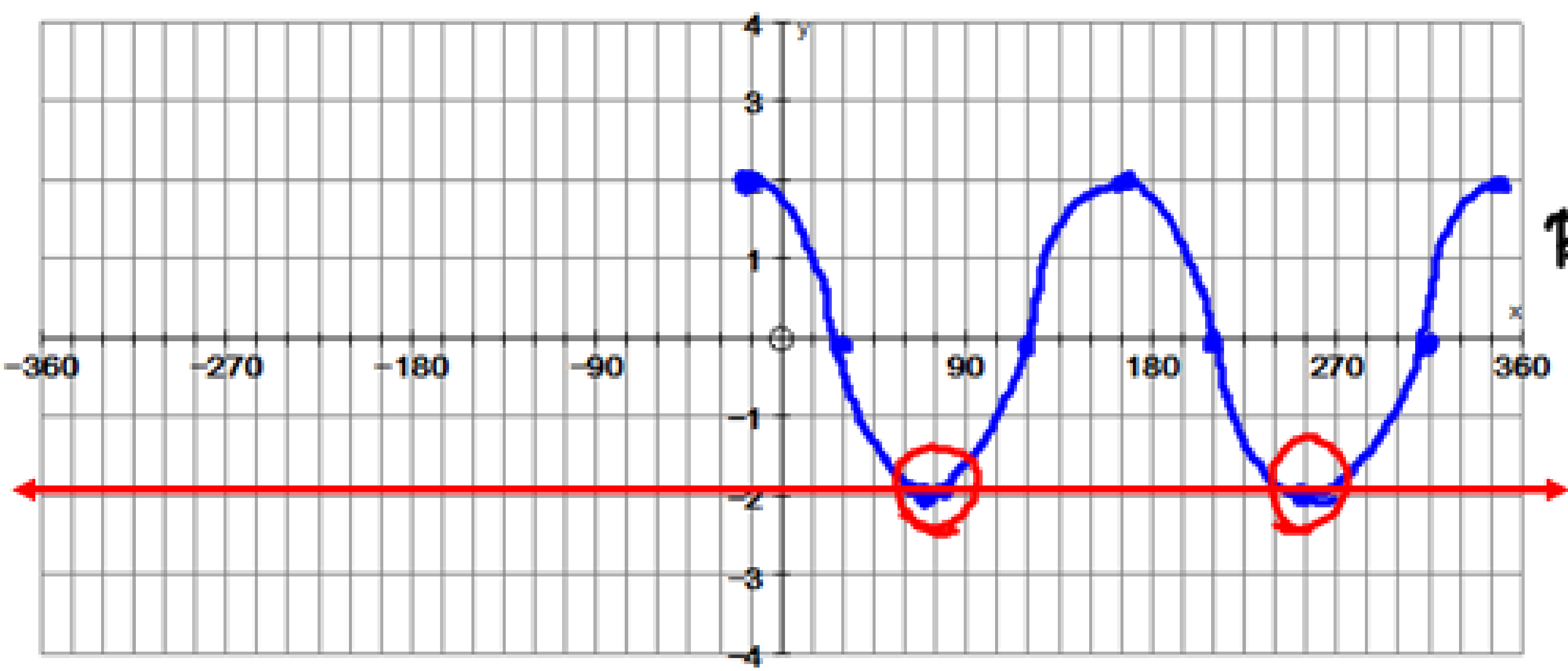
VS 2 VT0

HS $\frac{1}{2}$ HT -15°

Amp 2

SA: $y=0$

Period 180°



$$x = 75^\circ + 180^\circ k, \; k \in I$$

$$2\cos(2x+30^\circ) = -2$$

these brackets DO NOT mean multiply

let $m = 2x + 30^\circ$

$$\frac{2\cos m}{2} = \frac{-2}{2}$$

$\cos m = -1$ ← unit circle value

$m = 180^\circ + 360^\circ k, k \in I$

$2x + 30^\circ = 180^\circ + 360^\circ k$

$2x = 180^\circ - 30^\circ + 360^\circ k$

HT does not affect Period

$$\frac{2x}{2} = \frac{150^\circ}{2} + \frac{360^\circ k}{2}$$

$x = 75^\circ + 180^\circ k, k \in I$

C) $\sin(\frac{1}{2}x + 20°) = \frac{-\sqrt{2}}{2}$ (all values in degrees)

unit circle value

let $\theta = \frac{1}{2}x + 20°$

$\sin\theta = -\frac{\sqrt{2}}{2}$ QIII + IV

$$\theta = \begin{cases} 225° \\ 315° \end{cases} + 360°k,\ k \in I$$

$$\frac{1}{2}x + 20° = \begin{cases} 225° \\ 315° \end{cases} + 360°k.$$

$$\frac{1}{2}x = \begin{cases} 205° \\ 295° \end{cases} + 360°k$$

$$x = \begin{cases} 410° \\ 590° \end{cases} + 720°k,\ k \in I$$

D) $6\cos\left[2\left(x-\frac{\pi}{4}\right)\right]-1=2$ $\quad x\in(-\pi,4\pi]$

multiply by $\frac{1}{2}$

let $\theta = 2\left(x-\frac{\pi}{4}\right)$

$6\cos\theta - 1 = 2$

$\frac{6\cos\theta}{6} = \frac{3}{6}$

$\cos\theta = \frac{1}{2}$ unit circle

$\theta = \begin{cases} \frac{\pi}{3} \\ \frac{5\pi}{3} \end{cases} + 2\pi k, k\in\mathbb{Z}$

$\left(\frac{1}{2}\right)2\left(x-\frac{\pi}{4}\right) = \begin{cases} \frac{\pi}{3}\left(\frac{1}{2}\right) \\ \frac{5\pi}{3}\left(\frac{1}{2}\right) \end{cases} + 2\pi k\left(\frac{1}{2}\right)$

$x-\frac{\pi}{4} = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} + \pi k$

* Get a common denominator

$x-\frac{3\pi}{12} = \begin{cases} \frac{2\pi}{12} \\ \frac{10\pi}{12} \end{cases} + \pi k$

$x = \begin{cases} \frac{2\pi}{12}+\frac{3\pi}{12} \\ \frac{10\pi}{12}+\frac{3\pi}{12} \end{cases} + \pi k$

$$x = \begin{cases} \dfrac{2\pi}{12} + \dfrac{3\pi}{12} \\ \dfrac{10\pi}{12} + \dfrac{3\pi}{12} \end{cases} + \pi k$$

interval $(-\pi, 4\pi]$

$$\left(-\frac{12\pi}{12}, \frac{48\pi}{12}\right]$$

$$\underbrace{x = \begin{cases} \dfrac{5\pi}{12} \\ \dfrac{13\pi}{12} \end{cases} + \frac{12\pi}{12}k,\ k \in I}_{\text{General SOL'N}}$$

$$x = \begin{cases} \dfrac{5\pi}{12}, \dfrac{17\pi}{12}, \dfrac{29\pi}{12}, \dfrac{41\pi}{12}, \dfrac{-7\pi}{12} \\ \dfrac{13\pi}{12}, \dfrac{25\pi}{12}, \dfrac{37\pi}{12}, \dfrac{\pi}{12}, \dfrac{-11\pi}{12} \end{cases}$$

F) $2\cos^4 x - 3\cos^2 x + 1 = 0$ $\qquad \underset{-\frac{4\pi}{4}}{-\pi} \le x \le \underset{\frac{12\pi}{4}}{3\pi}$

let $m = \cos^2 x$

$2m^2 - 3m + 1 = 0$

$(2m - 1)(m - 1) = 0$

$2m - 1 = 0 \qquad m - 1 = 0$

$m = \frac{1}{2} \qquad m = 1$

$\cos^2 x = \frac{1}{2} \qquad \cos^2 x = 1$

$\cos x = \pm\frac{1}{\sqrt{2}} \qquad \cos x = \pm 1$

$\cos x = \frac{\sqrt{2}}{2} \qquad \cos x = -\frac{\sqrt{2}}{2}$

$x = \begin{cases} \frac{\pi}{4} \\ \frac{7\pi}{4} \end{cases} + 2\pi k,\ k \in I \quad \frac{8\pi}{4} \qquad x = \begin{cases} \frac{3\pi}{4} \\ \frac{5\pi}{4} \end{cases} + 2\pi k,$

$\cos x = 1 \qquad \cos x = -1$

$x = \{0 + 2\pi k, \qquad x = \{\pi + 2\pi k,$

Interval:

$\{\frac{\pi}{4}, \frac{9\pi}{4}, \frac{7\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}$

$0, 2\pi, \pi, 3\pi, -\pi$