

# 5.3

## The Tangent Function

### Focus on...

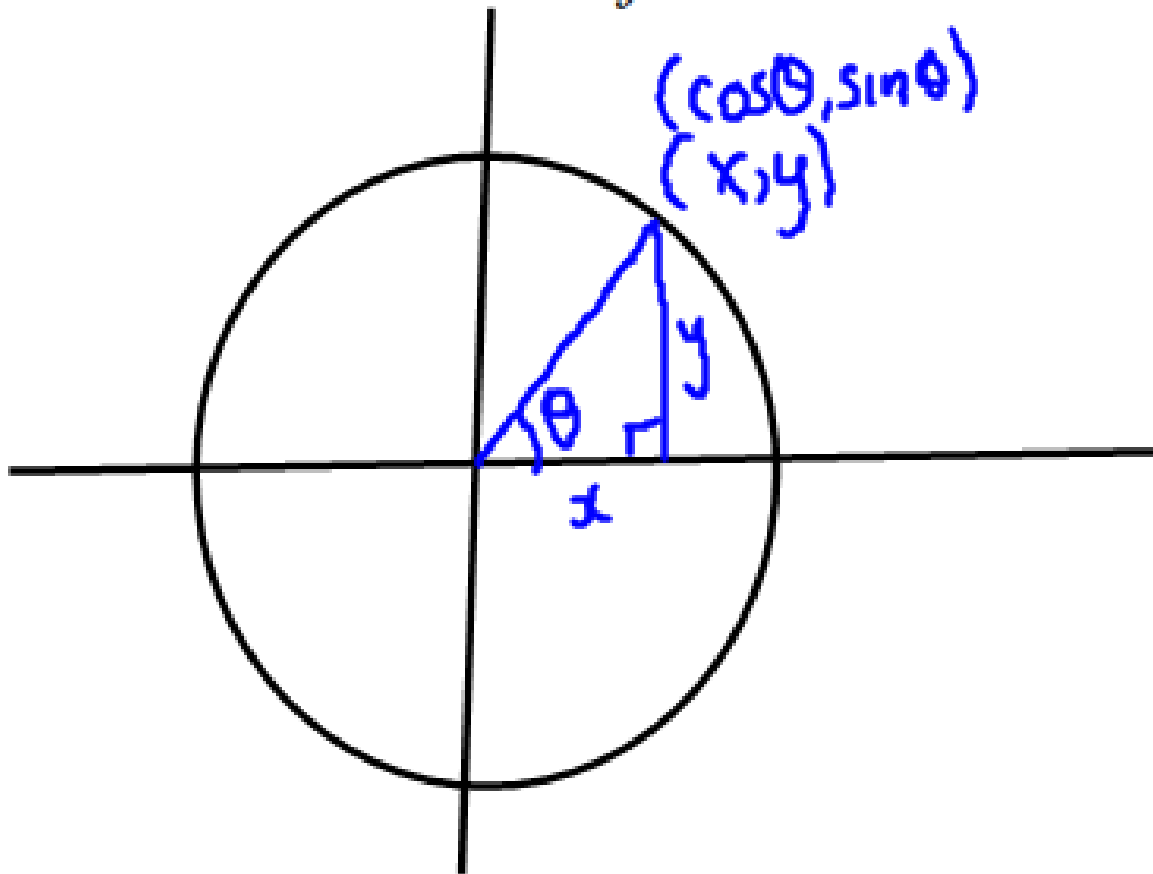
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- sketching the graph of  $y = \tan x$
- determining the amplitude, domain, range, and period of  $y = \tan x$
- determining the asymptotes and  $x$ -intercepts for the graph of  $y = \tan x$
- solving a problem by analysing the graph of the tangent function

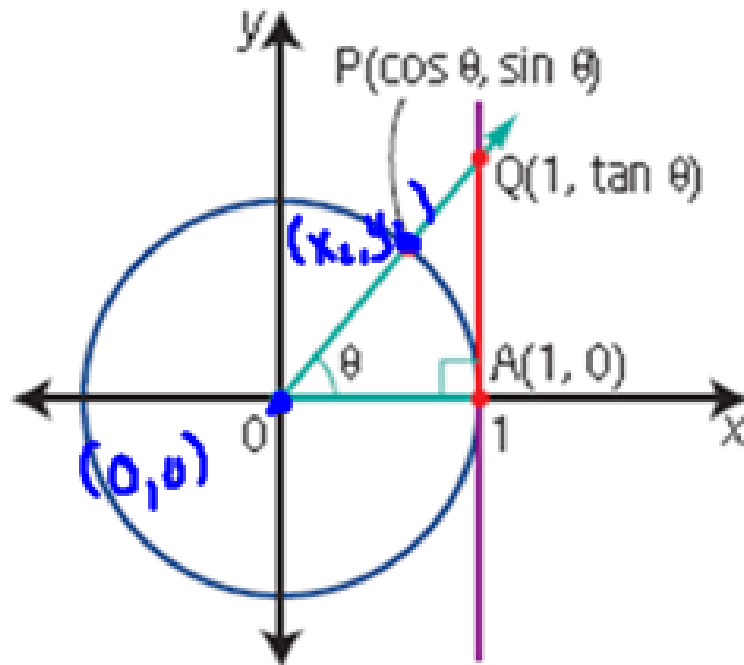
# Tangent

A tangent line is a line that touches a curve, or a graph of a function, at a single point.

Recall:  $\tan \theta = \frac{Opp}{Adj} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$



What is the slope of the terminal arm of angle  $\theta$  in standard position?  
 Determine the equation of the terminal arm.



How could you show that the coordinates of Q are  $(1, \tan \theta)$ ?

$$\textcircled{2} m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\sin \theta - 0}{\cos \theta - 0} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\textcircled{1} y = mx + b$$

$\uparrow$  slope       $\leftarrow$  y-int

$$y = mx$$

$$y = (\tan \theta)x \quad \text{when } x = 1$$

$$y = (\tan \theta)(1)$$

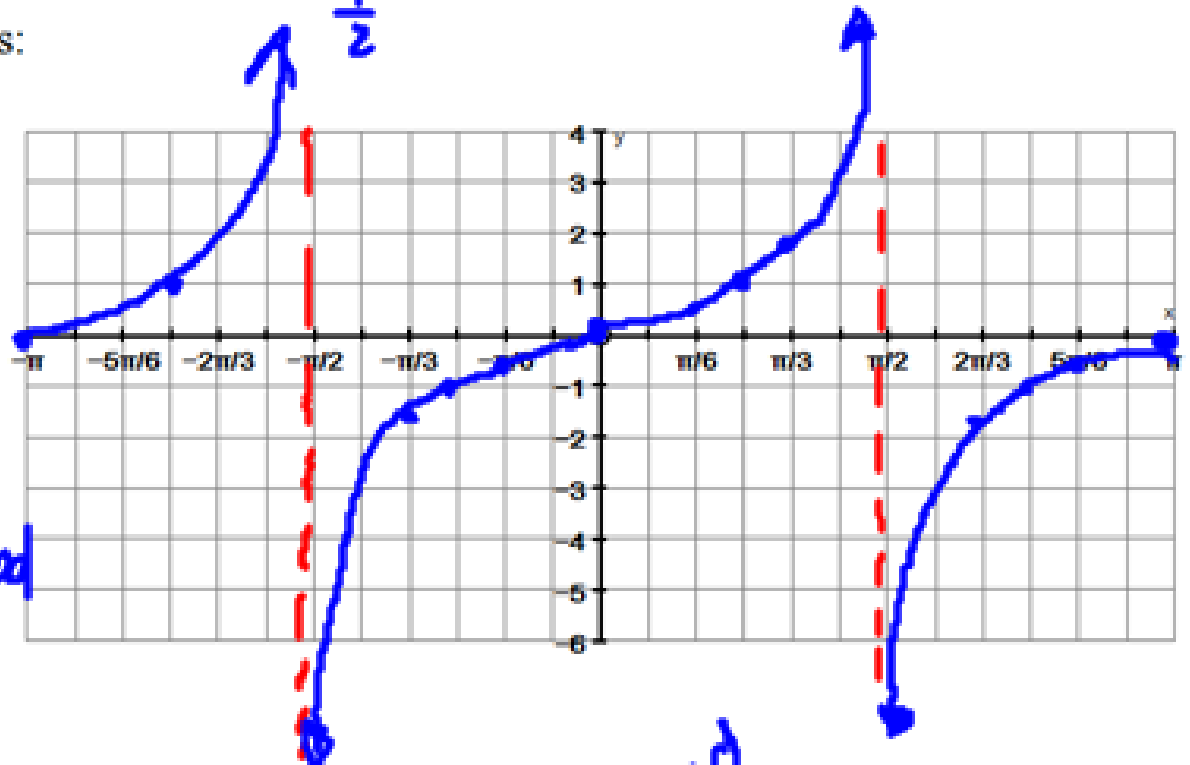
$$y = \tan \theta$$

# The Graph of $y = \tan x$

$$\tan \frac{\pi}{3} = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Complete the following table of values:

$\theta$	$\tan \theta$	$\theta$	$\tan \theta$
0	0	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{3}$
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\frac{5\pi}{4}$	1
$\frac{\pi}{4}$	1	$\frac{4\pi}{3}$	$-\sqrt{3}$
$\frac{\pi}{3}$	$\sqrt{3}$	$\frac{3\pi}{2}$	Undefined
$\frac{\pi}{2}$	Undefined	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{3}$
$\frac{2\pi}{3}$	$-\sqrt{3}$	$\frac{7\pi}{4}$	-1
$\frac{3\pi}{4}$	-1	$\frac{11\pi}{6}$	$-\frac{\sqrt{3}}{3}$
$\frac{5\pi}{6}$	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$	$2\pi$	0
$\pi$	0		



Period:  $\pi$

VA:  $x = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$

← 1 period

inflection  
pt.

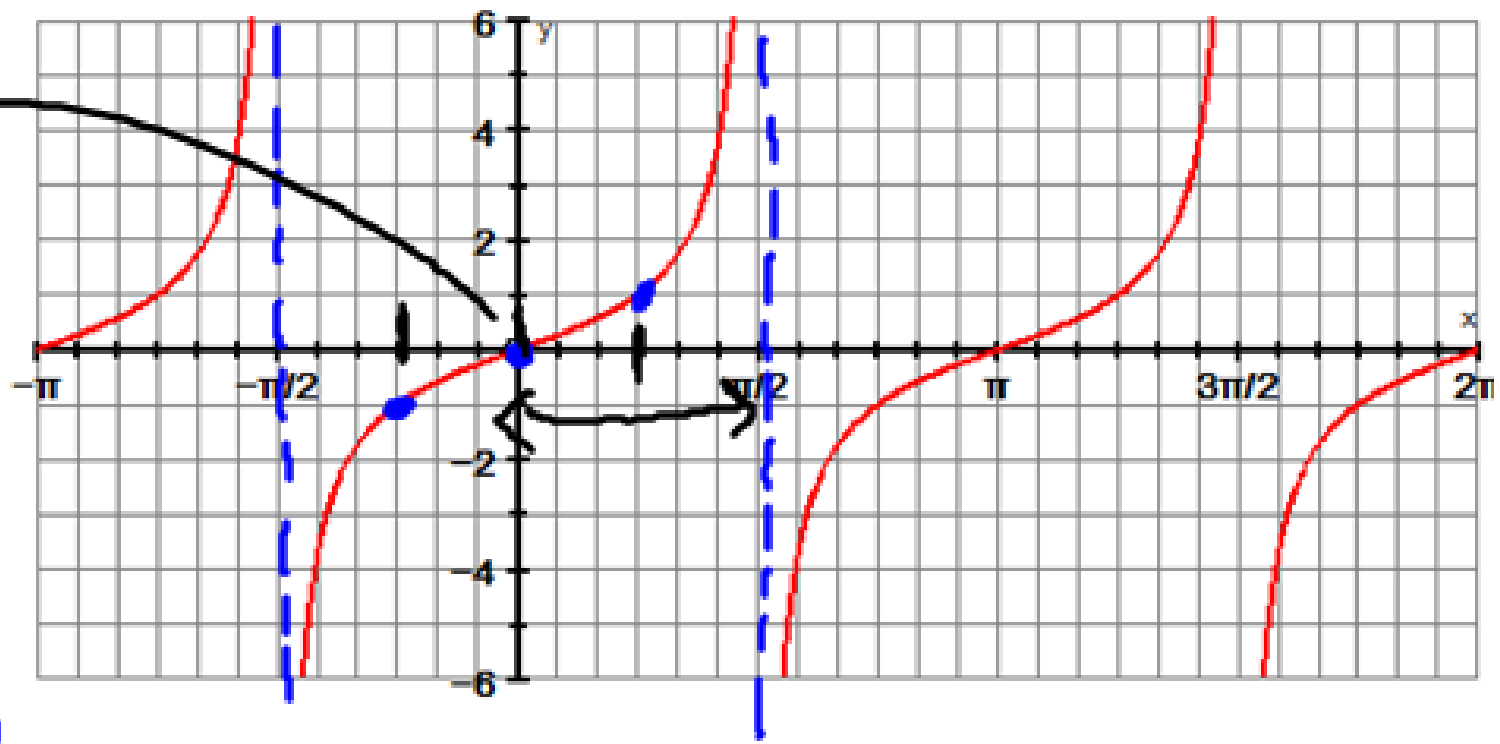
Amplitude: None

Period:  $\pi$

Range:  $\{y \mid y \in \mathbb{R}\}$

Vertical Asymptotes:  $x = \frac{\pi}{2} + \pi k, k \in \mathbb{I}$

Domain:  $\{x \mid x \neq \frac{\pi}{2} + \pi k, k \in \mathbb{I}, x \in \mathbb{R}\}$



everything but the VA

- 5 key pts  $(-\frac{\pi}{2}, \text{und})$  VA  
 $(-\frac{\pi}{2}, -1)$   
 $(0, 0)$   
 $(\frac{\pi}{2}, 1)$   
 $(\frac{\pi}{2}, \text{und})$  VA

$$y = 2 \tan\left(x - \frac{\pi}{2}\right)$$

List transformations

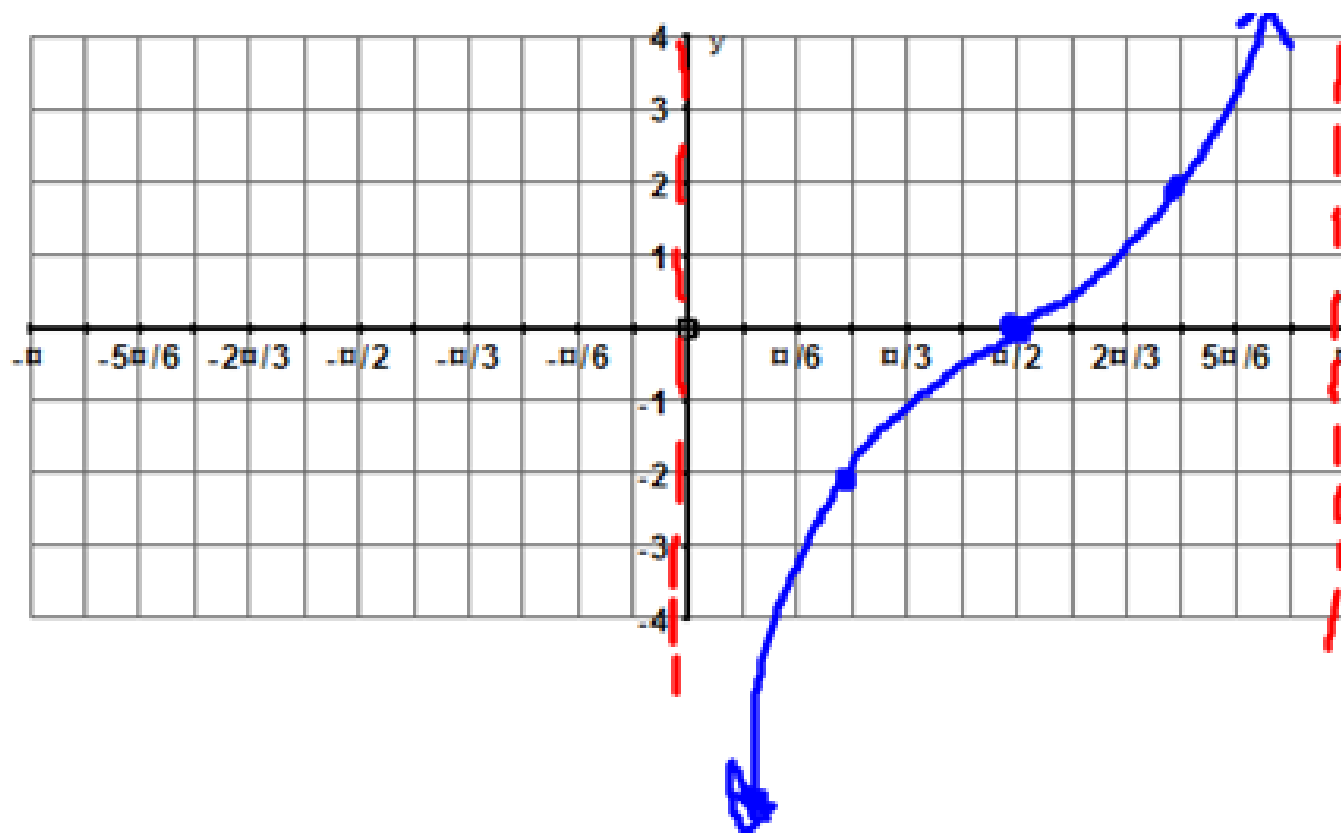
$$VSZ \quad HT + \frac{\pi}{2}$$

Period:  $\pi$

$$\text{Period} = HS \cdot \pi$$

$$\text{Period} = HS \cdot 180^\circ$$

$$\text{Inflection pt } (HT, VT) \\ \left(\frac{\pi}{2}, 0\right)$$



$$VA: x = 0 + \pi k, k \in \mathbb{I}$$

$$\text{Domain: } \{x \mid x \neq 0 + \pi k, k \in \mathbb{I}, x \in \mathbb{R}\}$$

$$\text{Range: } \{y \mid y \in \mathbb{R}\}$$

$$y = \tan\left(2\left(x + \frac{\pi}{4}\right)\right)$$

List transformations

$$HS \frac{1}{2} \quad HT -\frac{\pi}{4}$$

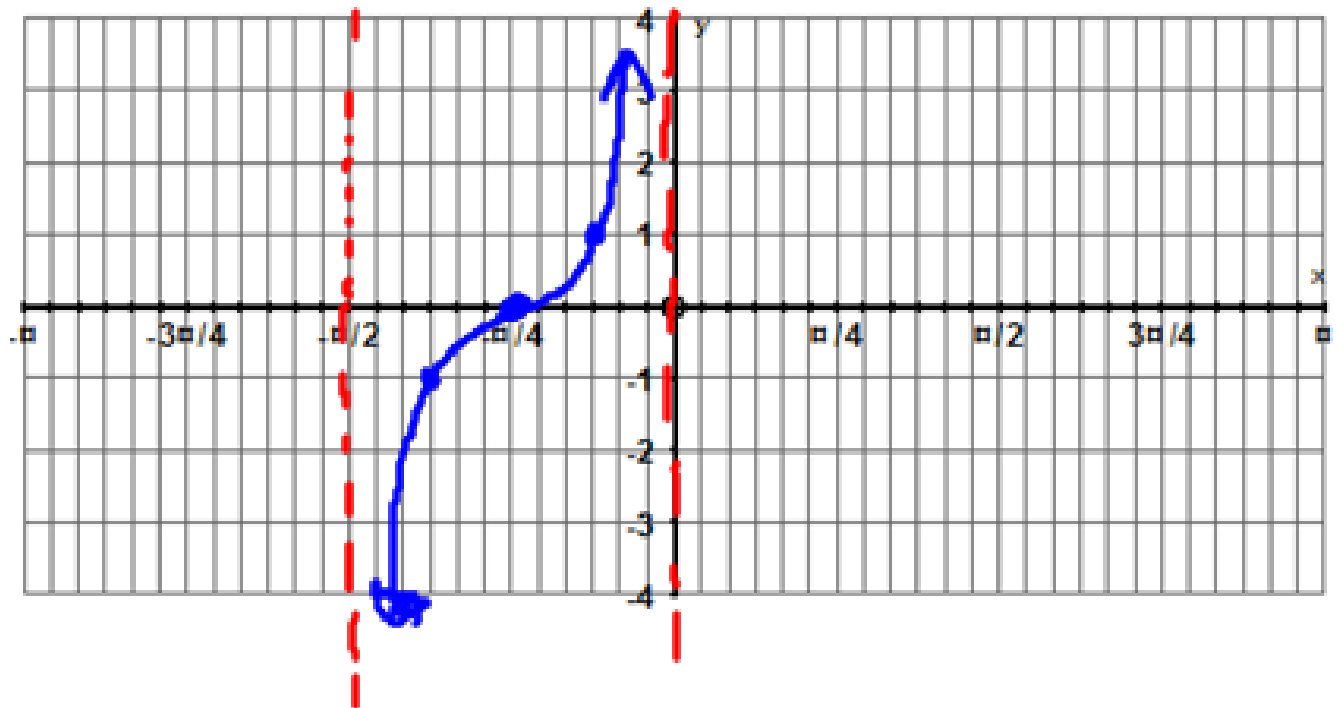
$$\text{Period} = \frac{1}{2} \cdot \pi = \frac{\pi}{2}$$

$$\text{inf. pt } \left(-\frac{\pi}{4}, 0\right)$$

$$VA: x = 0 + \frac{\pi}{2}k, k \in \mathbb{I}$$

$$\text{Domain: } \left\{x \mid x \neq 0 + \frac{\pi}{2}k, k \in \mathbb{I}, x \in \mathbb{R}\right\}$$

$$\text{Range: } \{y \mid y \in \mathbb{R}\}$$



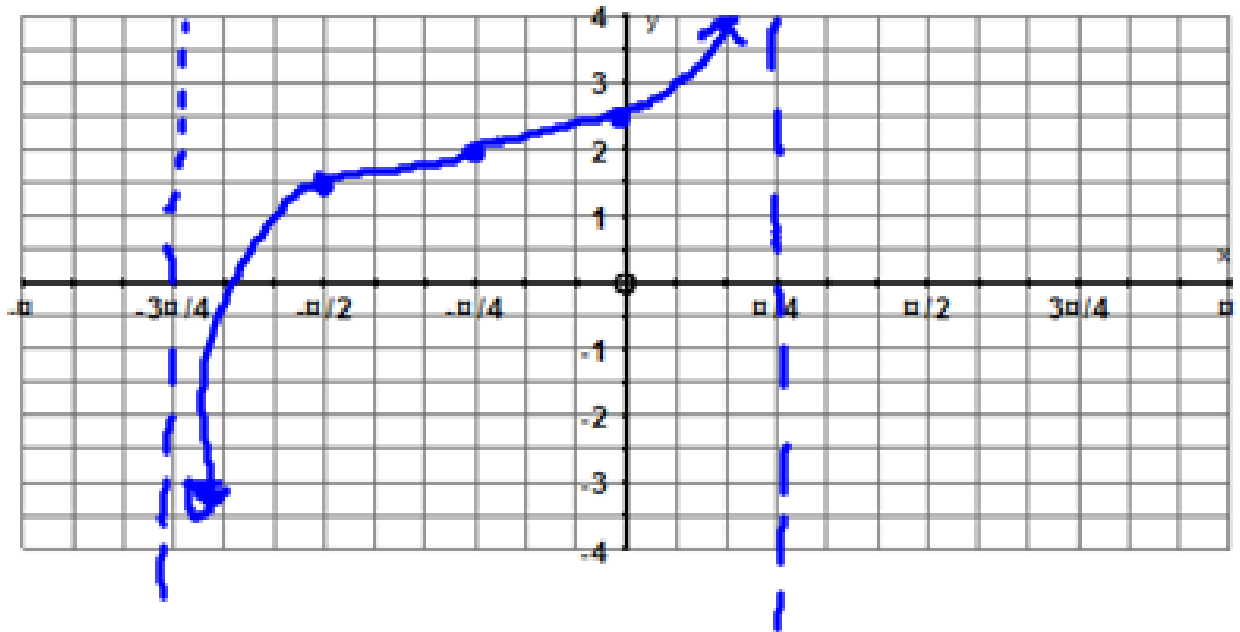
$$y = \frac{1}{2} \tan\left(x + \frac{\pi}{4}\right) + 2$$

List transformations

$$VS \frac{1}{2}$$

$$HT - \frac{\pi}{4}$$

$$VT + 2$$



Period:  $\pi$

$$\text{inf pt } \left(-\frac{\pi}{4}, 2\right)$$

$$VA: x = \frac{\pi}{4} + \pi k, k \in \mathbb{Z}$$

Domain:  $\left\{x \mid x \neq \frac{\pi}{4} + \pi k, k \in \mathbb{Z}, x \in \mathbb{R}\right\}$

Range:  $\{y \mid y \in \mathbb{R}\}$