

Finding Angles Given Their Trigonometric Ratios

← radians

A) $\sin \theta = 0.654$,

$$0 \leq \theta < 2\pi$$

↻ 1 full rotation

1.5157 QI + II



NOT MULTIPLICATION

$$\sin^{-1}(\sin \theta) = \sin^{-1}(0.654)$$

$$\theta = \sin^{-1}(0.654)$$

$$\theta_1 = 0.7128 \text{ rad QI}$$

$$\theta_2 = \pi - 0.7128$$

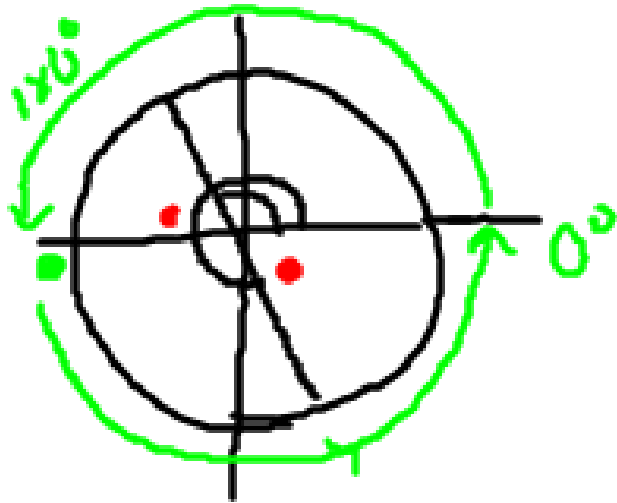
$$= 2.4287$$

Interval:

$$\theta = \begin{cases} 0.71 \\ 2.43 \end{cases}$$

$$\text{B) } \tan \theta = -1.235,$$

QII + IV



$$\tan^{-1}(\tan \theta) = \tan^{-1}(-1.235)$$

$$\theta = \tan^{-1}(-1.235)$$

• $\theta_1 = -51.00^\circ$ QIV
 $\theta_1 = 360^\circ + (-51.00^\circ)$
 $= 309.00^\circ$

$$-180^\circ \leq \theta < 180^\circ$$

↑ degrees

$$\theta_2 = 180^\circ - 51.00^\circ$$
$$= 129.00^\circ$$

$$\theta = \begin{cases} -51.00^\circ \\ 129.00^\circ \end{cases}$$

$$\text{C) } \csc \theta = \frac{2}{\sqrt{3}}$$

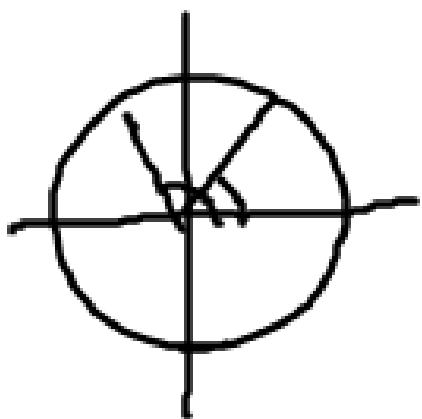
$$-2\pi \leq \theta < 2\pi$$

$$\frac{1}{\sin \theta} = \frac{2}{\sqrt{3}}$$

$$\frac{2 \sin \theta}{2} = \frac{\sqrt{3}}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

Q I & II



unit circle
value.
Exact
answers

$$\theta = \left\{ \begin{array}{l} \frac{\pi}{3} \\ \frac{2\pi}{3} \end{array} \right\} + 2\pi k, \quad k \in \mathbb{I}$$

general solution

Interval:

$$\theta = \left\{ \begin{array}{l} \frac{\pi}{3}, \frac{2\pi}{3} \\ -\frac{5\pi}{3}, -\frac{4\pi}{3} \end{array} \right\}$$

full rotations

5. For each point, sketch two coterminal angles in standard position whose terminal arm contains the point. Give one positive and one negative angle, in radians, where neither angle exceeds one full rotation.

a) (3, 5)

b) (-2, -1)

c) (-3, 2)

d) (5, -2)

$$\tan \theta = \frac{O}{A}$$

Positive rotation

$$\begin{aligned} \theta_2 &= 2\pi - 0.38 \\ &= 5.90 \text{ rad.} \end{aligned}$$

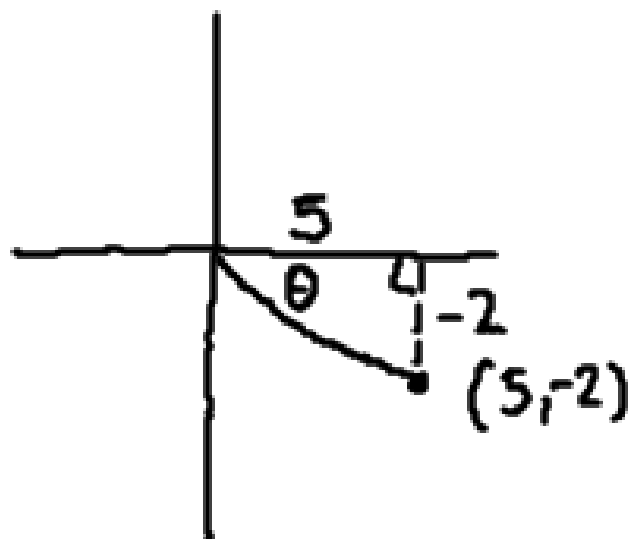
Q.IV

$$\tan \theta = \frac{-2}{5}$$

$$\theta = \tan^{-1}\left(-\frac{2}{5}\right)$$

$$\theta = -0.38 \text{ radians}$$

↑
negative
rotation



pg 202 No calculator. DO NOT SOLVE FOR THE ANGLE

12. Determine the exact values of the other five trigonometric ratios under the given conditions.

a) $\sin \theta = \frac{3}{5}, \frac{\pi}{2} < \theta < \pi$

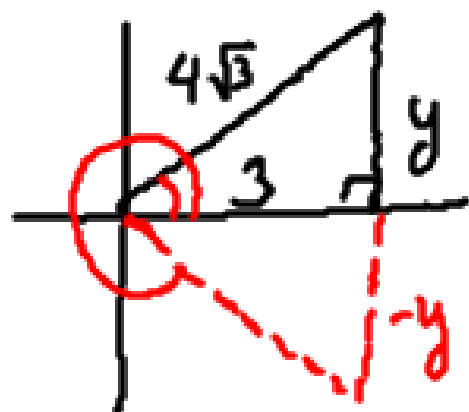
b) $\cos \theta = \frac{-2\sqrt{2}}{3}, -\pi \leq \theta \leq \frac{3\pi}{2}$

c) $\tan \theta = \frac{2}{3}, -360^\circ < \theta < 180^\circ$

d) $\sec \theta = \frac{4\sqrt{3}}{3}, -180^\circ \leq \theta \leq 180^\circ$

$$\frac{1}{\cos \theta} = \frac{4\sqrt{3}}{3}$$

$$\cos \theta = \frac{3}{4\sqrt{3}} = \frac{A}{H}$$



$$a^2 + b^2 = c^2$$

$$(3)^2 + y^2 = (4\sqrt{3})^2$$

$$y^2 = 48 - 9$$

$$y^2 = 39$$

$$y = \pm \sqrt{39}$$

$$\tan \theta = \frac{O}{A} = \frac{\pm \sqrt{39}}{3}$$

$$\cot \theta = \frac{A}{O} = \frac{3}{\pm \sqrt{39}} \left(\frac{\sqrt{39}}{\sqrt{39}} \right)$$

$$\cot \theta = \frac{\pm 3\sqrt{39}}{39} = \frac{\pm \sqrt{39}}{13}$$

$$\sin \theta = \frac{\pm \sqrt{39}}{4\sqrt{3}}$$

$$\sin \theta = \frac{\pm \sqrt{13}}{4}$$

$$\cos \theta = \frac{3}{4\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$$

$$\cos \theta = \frac{\sqrt{3}}{4}$$

$$\csc \theta = \frac{H}{O} = \frac{4\sqrt{3}}{\pm \sqrt{39}}$$

$$= \frac{\pm 4}{\sqrt{13}}$$

$$\csc \theta = \frac{\pm 4\sqrt{13}}{13}$$

NO CALCULATOR

15. a) Determine the positive value of $\sin(\cos^{-1} 0.6)$. Use your knowledge of the unit circle to explain why the answer is a rational number.

~~b~~ Without calculating, what is the positive value of $\cos(\sin^{-1} 0.6)$? Explain.

$$\sin(\underbrace{\cos^{-1} 0.6}_{\text{angle}})$$

$$\text{let } \theta = \cos^{-1} 0.6$$

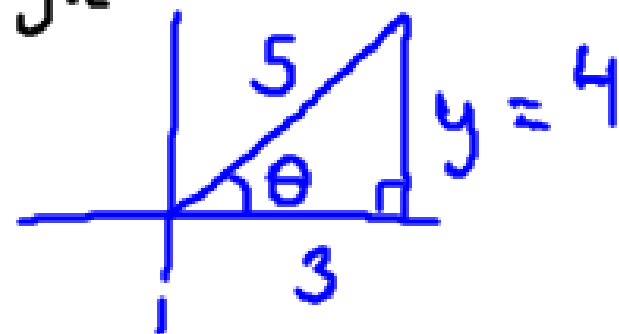
ratio of side lengths

$$\cos \theta = \cos(\cos^{-1} 0.6)$$

$$\cos \theta = 0.6 = \frac{3}{5} \left(\frac{A}{H} \right)$$

$$\sin \theta = \frac{O}{H} \left. \begin{array}{l} \text{ratio of} \\ \text{side lengths} \end{array} \right\}$$

angle



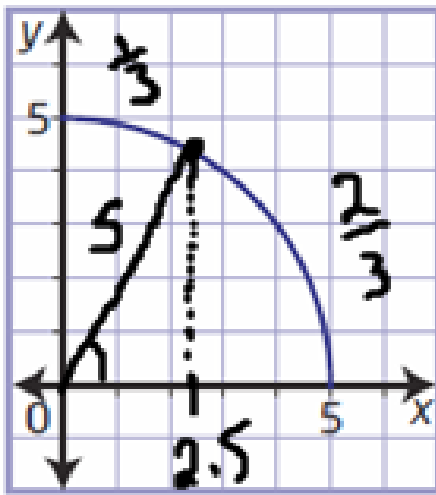
Pythagorean Triple 3-4-5

$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{4}{5}$$

$$\sin(\cos^{-1}(0.6)) = 0.8$$

21. The diagram shows a quarter-circle of radius 5 units. Consider the point on the curve where $x = 2.5$. Show that this point is one-third the distance between $(0, 5)$ and $(5, 0)$ on the arc of the circle.



$$\cos \theta = \frac{A}{r}$$

$$\cos \theta = \frac{2.5}{5}$$

HW: pg 202 #10-14, 17, 18, 19, C4

