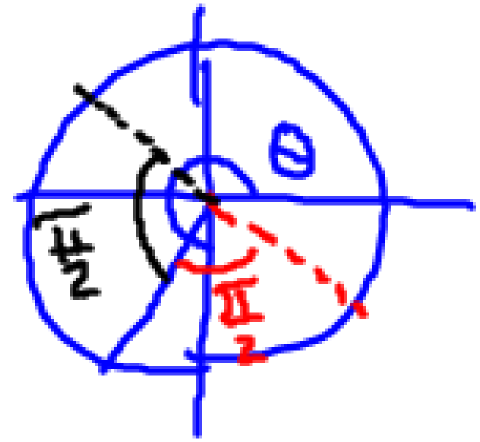
13. If $P(\theta) = \left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$, determine the following.

a) What does $P(\theta)$ represent? Explain using a diagram.

b) In which quadrant does θ terminate? ← QIII

c) Determine the coordinates of $P\left(\theta + \frac{\pi}{2}\right)$.

d) Determine the coordinates of $P\left(\theta - \frac{\pi}{2}\right)$.



$P\left(\theta + \frac{\pi}{2}\right) = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

$\frac{1}{4}$ turn in positive direction

QIV

$P\left(\theta - \frac{\pi}{2}\right) = \left(-\frac{2\sqrt{2}}{3}, +\frac{1}{3}\right)$

$\frac{1}{4}$ turn negative direction, QII

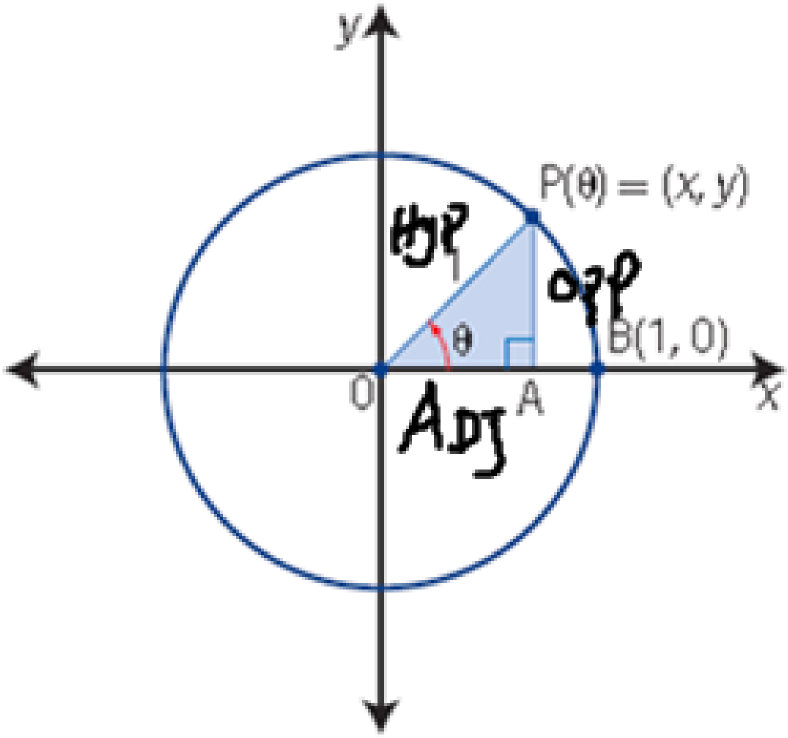
4.3

Trigonometric Ratios

Focus on...

- relating the trigonometric ratios to the coordinates of points on the unit circle
- determining exact and approximate values for trigonometric ratios
- identifying the measures of angles that generate specific trigonometric values
- solving problems using trigonometric ratios

Primary Trigonometric Ratios SOH CAH TOA



$$\cos\theta = \frac{ADJ}{HYP} = \frac{x}{1}$$

$$x = \cos\theta$$

$$\sin\theta = \frac{OPP}{HYP} = \frac{y}{1}$$

$$y = \sin\theta$$

$$\tan\theta = \frac{OPP}{ADJ} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

The coordinates of any point $P(\theta)$ at the intersection of the terminal arm of an angle, θ, and the unit circle are given by:

$$(\cos\theta, \sin\theta)$$

Reciprocal Trigonometric Ratios

cosecant $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{O}{H}} = \frac{H}{O}$

$1 \div \frac{O}{H}$

$1 \times \frac{H}{O}$

$\sin\theta = \frac{1}{\csc\theta}$

secant $\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A}$

$\cos\theta = \frac{1}{\sec\theta}$

cotangent $\cot\theta = \frac{1}{\tan\theta} = \frac{A}{O}$

$\tan\theta = \frac{1}{\cot\theta}$

$\cot = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$

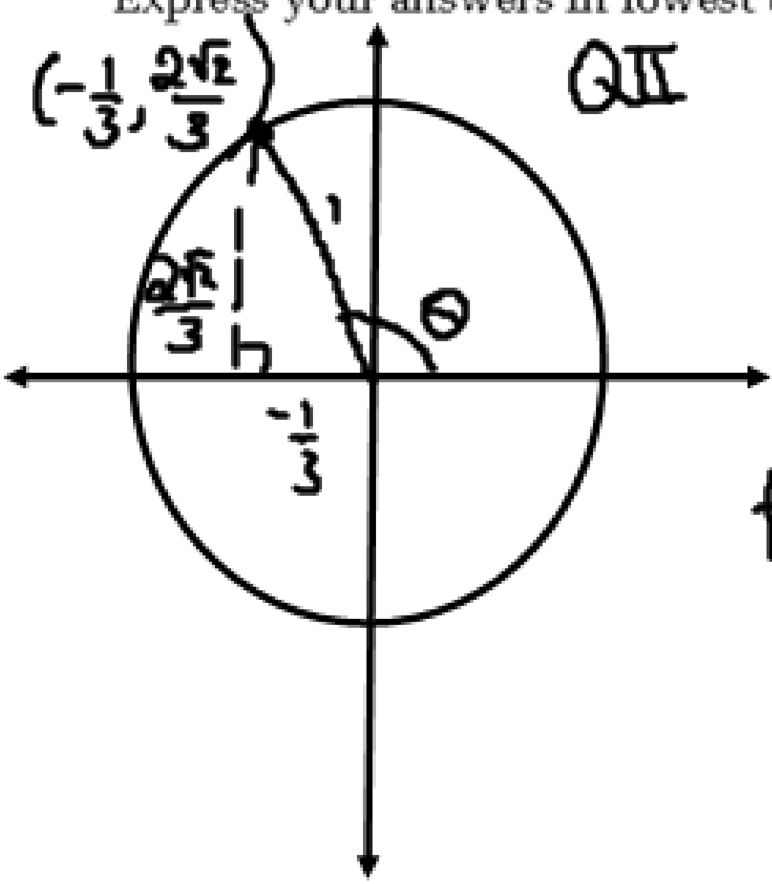
Pg. 194

Your Turn

The point $B\left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

a) Draw a diagram to model the situation.

b) Determine the values of the six trigonometric ratios for θ. Express your answers in lowest terms.



$$\sin\theta = \frac{O}{H} = \frac{\frac{2\sqrt{2}}{3}}{1} = \frac{2\sqrt{2}}{3}$$

$$\cos\theta = \frac{A}{H} = \frac{\frac{-1}{3}}{1} = \frac{-1}{3}$$

$$\tan\theta = \frac{O}{A} = \frac{\frac{2\sqrt{2}}{3}}{\frac{-1}{3}} = \frac{2\sqrt{2}}{3} \div \frac{-1}{3}$$

$$= \frac{2\sqrt{2}}{\cancel{3}} \times \frac{-\cancel{3}}{1} = -2\sqrt{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2\sqrt{2}}{3}}$$

$$= \frac{3}{2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{3\sqrt{2}}{4}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{-\frac{1}{3}}$$

$$= -3$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$= \frac{1}{-2\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

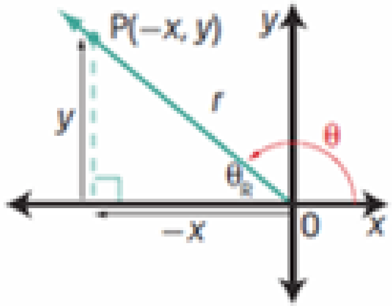
$$= \frac{-\sqrt{2}}{4}$$

Quadrant II

$90° < \theta < 180°$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{y}{-x}$

$\sin\theta > 0$ $\cos\theta < 0$ $\tan\theta < 0$



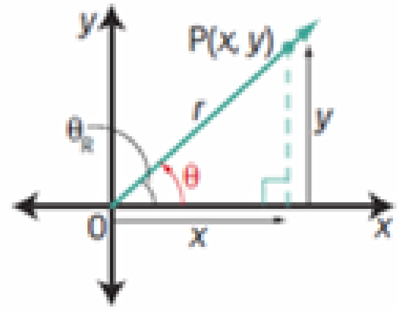
$\theta = 180° - \theta_R$

Quadrant I

$0° < \theta < 90°$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta > 0$ $\cos\theta > 0$ $\tan\theta > 0$



Why is r always positive?

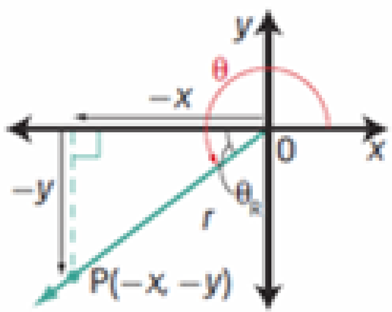
$\theta = \theta_R$

Quadrant III

$180° < \theta < 270°$

$\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{-x}{r}$ $\tan\theta = \frac{-y}{-x}$

$\sin\theta < 0$ $\cos\theta < 0$ $\tan\theta > 0$



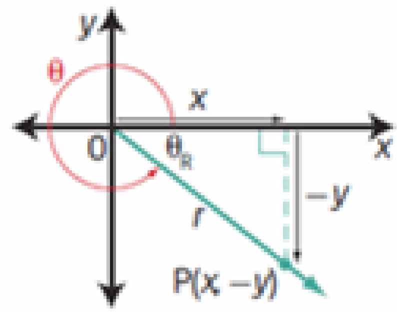
$\theta = 180° + \theta_R$

Quadrant IV

$270° < \theta < 360°$

$\sin\theta = \frac{-y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{-y}{x}$

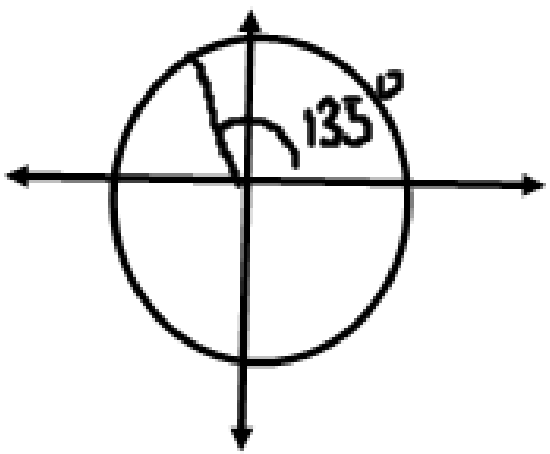
$\sin\theta < 0$ $\cos\theta > 0$ $\tan\theta < 0$



$\theta = 360° - \theta_R$

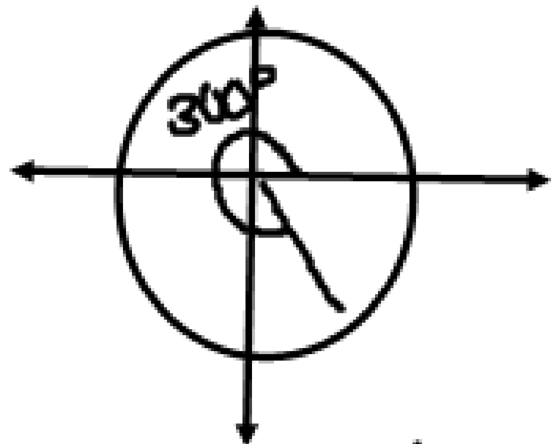
Examples: Draw diagrams to help you determine the exact value of each trigonometric ratio.

(a) $\sin 135°$ QII



$\sin 135° = \frac{\sqrt{2}}{2}$

(b) $\csc 300°$ QIV

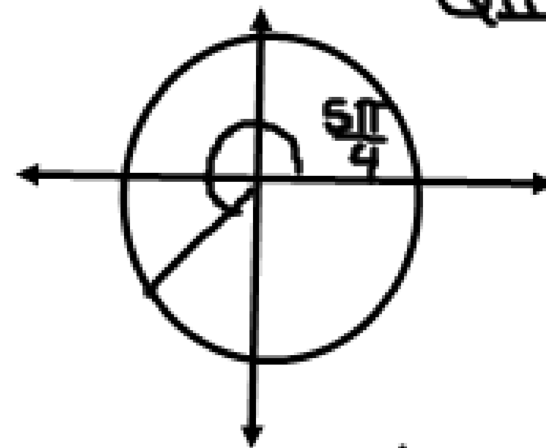


$\csc 300° = \frac{1}{\sin 300°}$

$= \frac{1}{\frac{-\sqrt{3}}{2}}$

$= \frac{-2}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{-2\sqrt{3}}{3}$

(c) $\sec \frac{5\pi}{4}$ QIII



$\sec \frac{5\pi}{4} = \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$= \frac{1}{\frac{-\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

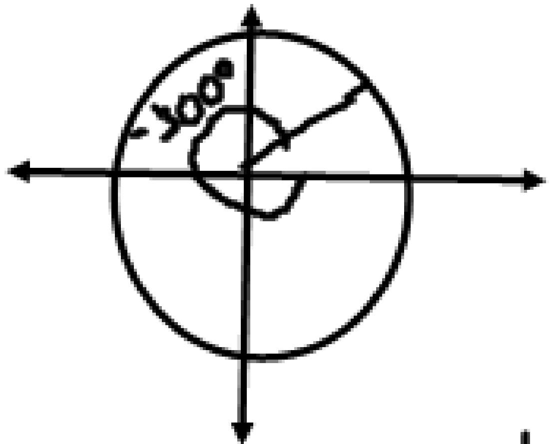
$= -\sqrt{2}$

(d) $\cot(-300°)$ QI

$-300°$ is coterminal with

$-300° + 360° = 60°$

$\cot(-300°) = \cot(60°)$



$\cot 60° = \dfrac{\cos 60°}{\sin 60°} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)$

$= \dfrac{\sqrt{3}}{3}$

(e) $\tan\frac{\pi}{2} = \dfrac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$

$= \dfrac{1}{0}$ oh no!! ☹

= undefined

HW pg 201
#1-3, moodle Quizzes
Study ch 11 ☺

Simplifying Trig EXPRESSIONS

$$\frac{1}{\sin\left(-\frac{2\pi}{3}\right)+\cos^2\left(\frac{11\pi}{6}\right)}$$

$$\frac{\cot(-60^\circ)\cos(300^\circ)}{\csc(-240^\circ)}$$

HW pg 201

1-3

5, 6,