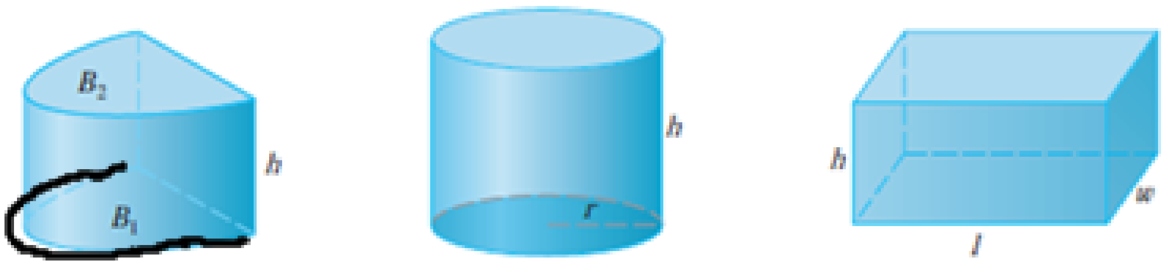
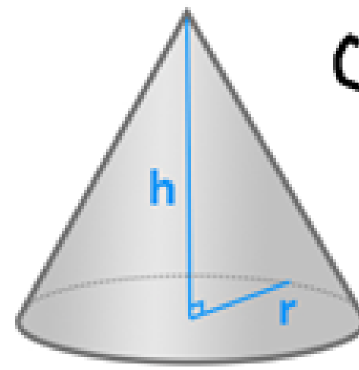
Ch 19 D – Solids of Revolution

$V = (\pi r^2)h$ $\quad V = (lw)h$

The volume of solids:





cone

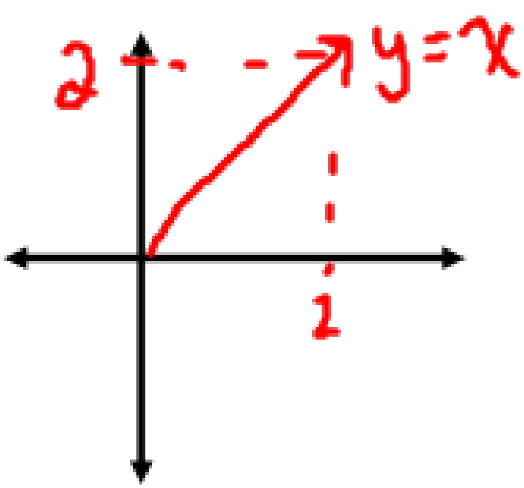
$$V = \frac{1}{3}\pi r^2 \cdot h$$

Volume= Area of base x height

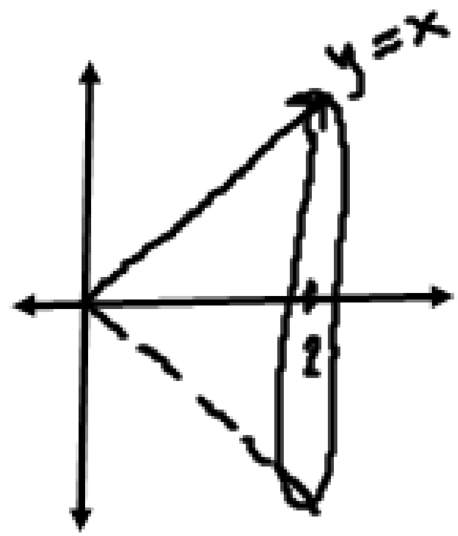
Take $y = x$ and revolve it $360°$ about the x-axis to find the trapped VOLUME from $x = 0$ to $x = 2$

a cone

Without Calculus



$\Rightarrow$

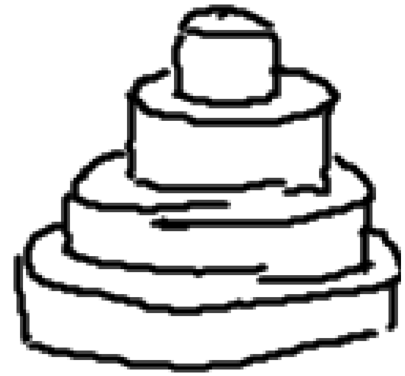


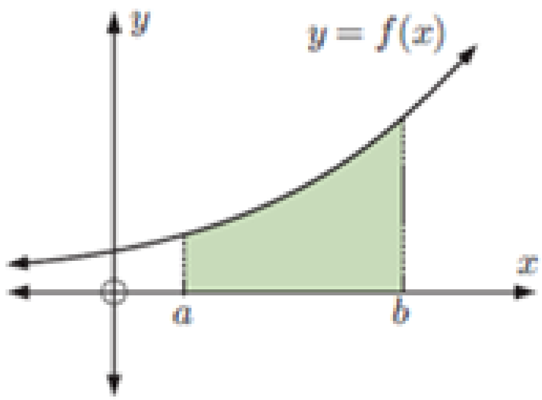
$$V = \frac{1}{3}\pi(2)^2(2)$$

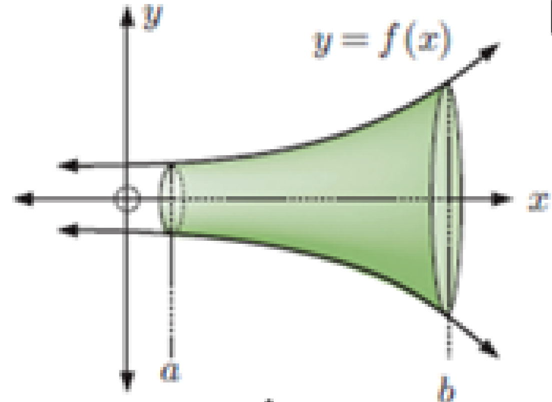
$$= \frac{1}{3}\pi(8)$$

$$= \frac{8\pi}{3} \text{ cubic units}$$

Given any function $y = f(x)$ that is rotated or revolved about the x-axis by $360°$ or 2π

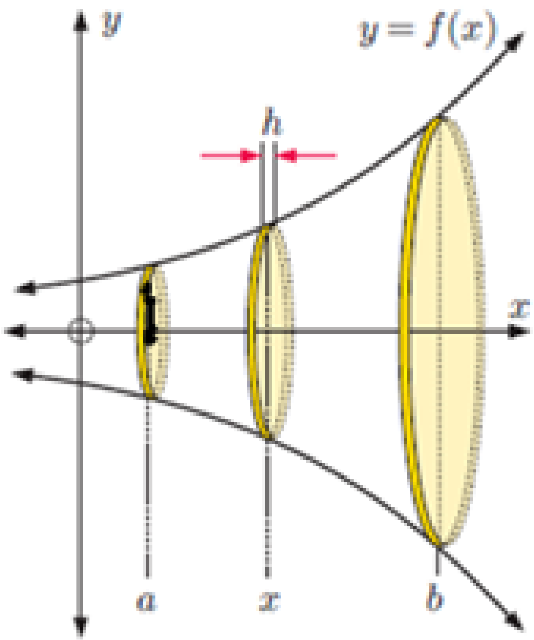






There are an infinite number of cylindrical discs

$V_{cylinder} = \pi r^2 h$



$V_{x=a} = \pi (f(a))^2 \cdot h$ radius of circle

$V_{x=b} = \pi (f(b))^2 \cdot h$

$V_x = \pi (f(x))^2 \cdot h$

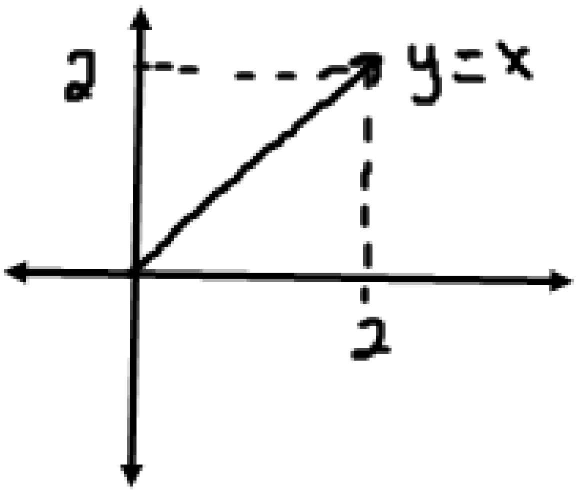
$$V_{TOTAL} = \sum_{x=a}^{b} \pi (f(x))^2 \cdot h$$

$$= \underbrace{\lim_{h \to 0} \sum_{x=a}^{b}} \pi (f(x))^2 \cdot h$$

$$= \int_a^b \pi (f(x))^2 \, dx$$

$$\left\{ \text{Take } y = x \text{ and revolve it } 360° \text{ about the x-axis to find the trapped VOLUME from } x = 0 \text{ to } x = 2 \right\}$$

With Calculus



$$V = \int_0^2 \pi (x)^2 \, dx$$

$$= \pi \int_0^2 x^2 \, dx$$

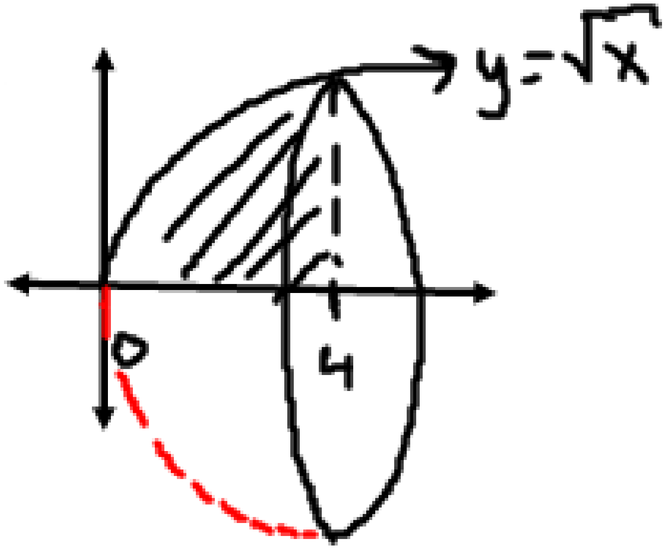
$$= \pi \left[\frac{1}{3} x^3 \right]_0^2$$

$$= \pi \left(\frac{1}{3}(2)^3 - \frac{1}{3}(0)^3 \right)$$

$$= \frac{8\pi}{3}$$

Example:

Find the volume of the solid formed when $y=\sqrt{x}$ from $x=0$ to $x=4$



$$V=\int_0^4 \pi(\sqrt{x})^2 dx$$

$$V=\pi\int_0^4 x\,dx$$

$$=\pi\left[\frac{1}{2}x^2\right]_0^4$$

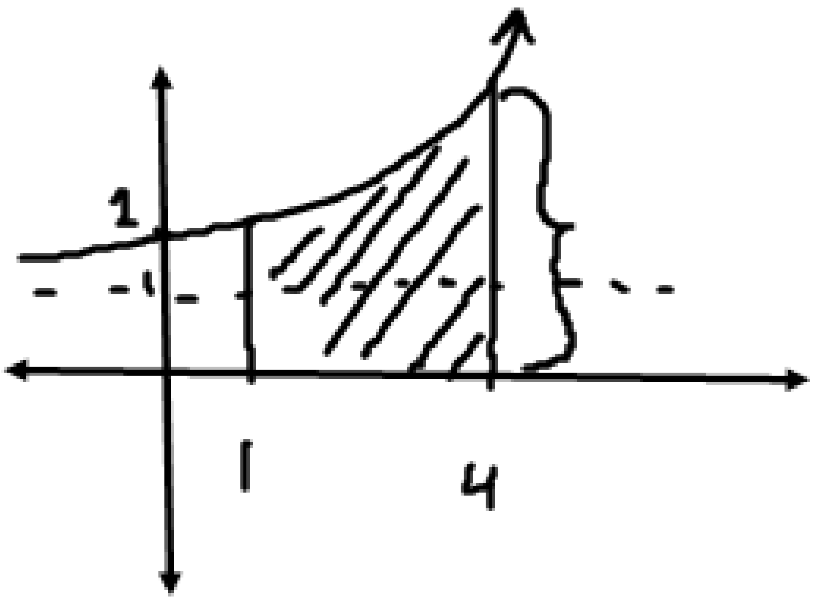
$$=\pi\left[\frac{1}{2}(4)^2-\frac{1}{2}(0)^2\right]$$

$$=8\pi$$

Example:

VT↑1

Find the volume of the solid formed when $y = e^x + 1$ from $x = 1$ to $x = 4$



$$V = \int_1^4 \pi (e^x + 1)^2 \, dx$$

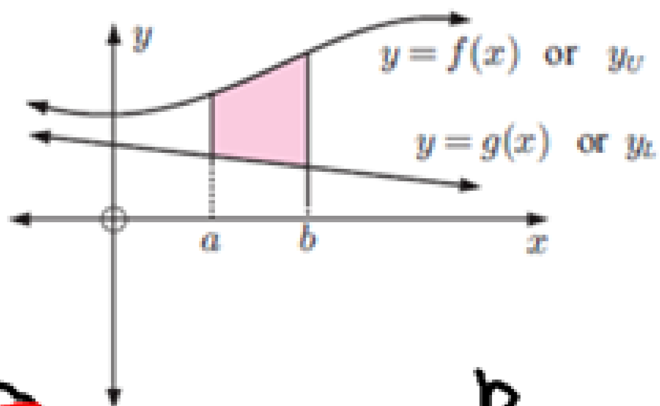
$$= \pi \int_1^4 (e^{2x} + 2e^x + 1) \, dx$$

$$= \pi \left[\frac{1}{2} e^{2x} + 2e^x + x \right]_1^4$$

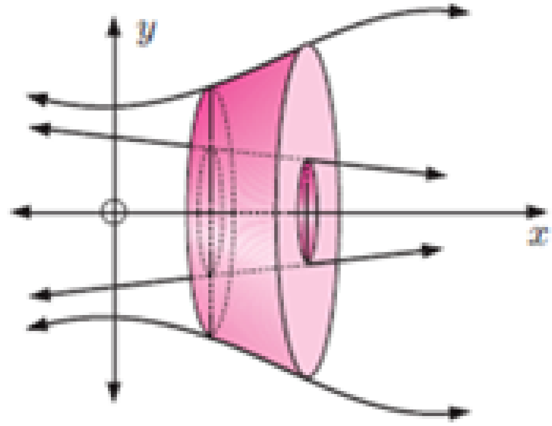
$$V = \pi \left[\left(\frac{1}{2} e^8 + 2e^4 + 4 \right) - \left(\frac{1}{2} e^2 + 2e + 1 \right) \right]$$

If the region bounded by an upper function and lower function is revolved about the x-axis, then:

Rotating



about the x-axis gives



$$V = \int_a^b \pi (f(x))^2\,dx - \int_a^b \pi (g(x))^2\,dx$$

$$= \int_a^b \pi\left[(f(x))^2 - (g(x))^2\right] dx$$

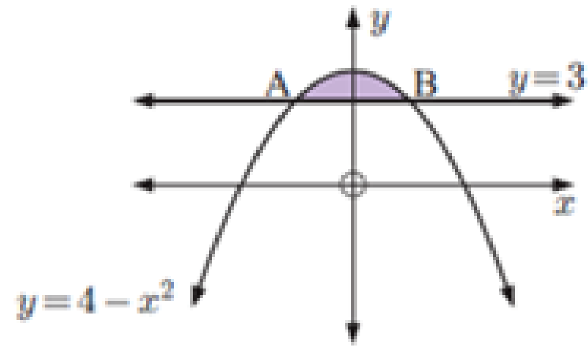
up low

EXERCISE 19D.2

1 The shaded region between $y = 4 - x^2$ and $y = 3$ is revolved about the x-axis.

a What are the coordinates of A and B?

b Find the volume of revolution.



A) POI

$4 - x^2 = 3$

$1 = x^2$

$\pm 1 = x$

$(-1,3)$ and $(1,3)$

$$V = \int_{-1}^{1} \pi\left[(4-x^2)^2 - (3)^2\right]dx$$

$$= \pi \int_{-1}^{1} (16 - 8x^2 + x^4 - 9)\,dx$$

$$= \pi \int_{-1}^{1} (x^4 - 8x^2 + 7)\,dx$$

$$= \pi \left[\frac{1}{5}x^5 - 8\left(\frac{1}{3}x^3\right) + 7x\right]_{-1}^{1}$$

$\vdots$

$$V = \frac{136\pi}{15}$$

$$\int_0^4 (x)\,dx = \left[\frac{1}{2}x^2 + C\right]_0^4$$

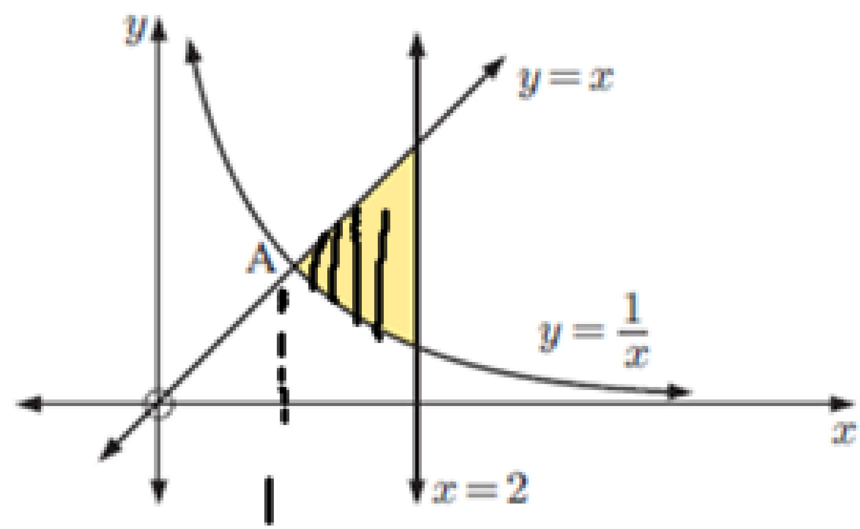
$$= \left[\frac{1}{2}(4)^2 + C\right] - \left(\frac{1}{2}(0)^2 + C\right]$$

up low

3 The shaded region between $y = x$, $y = \frac{1}{x}$, and $x = 2$ is revolved about the x-axis.

a Find the coordinates of A.

b Find the volume of revolution.



A) POI

$x = \frac{1}{x}$

$x^2 = 1$

$x = \pm 1$

$(1,1)$

$$V = \int_1^2 \pi\left((x)^2 - \left(\frac{1}{x}\right)^2\right)dx$$

$$= \pi \int_1^2 (x^2 - x^{-2})\,dx$$

$$= \pi\left[\frac{1}{3}x^2 + x^{-1}\right]_1^2$$

$\vdots$

$$= \frac{11\pi}{6}$$

HW pg 491 # 1dfh
2b
3c
8b
pg 494 # 2, 4