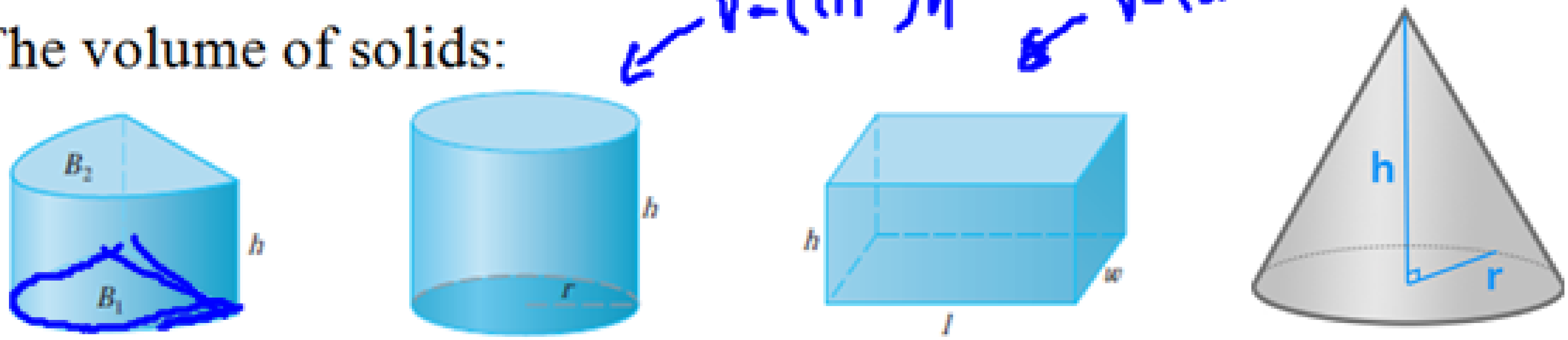
Ch 19 D – Solids of Revolution

The volume of solids:

$V=(\pi r^2)h$ $\quad$ $V=(lw)h$

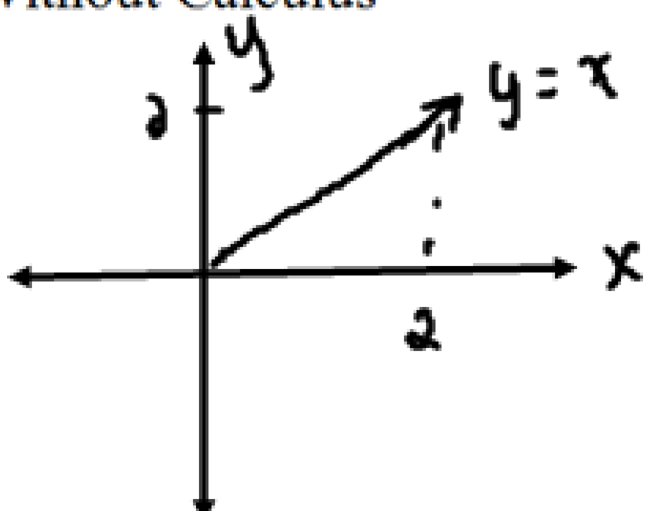


$V=\frac{1}{3}\pi r^2 \cdot h$

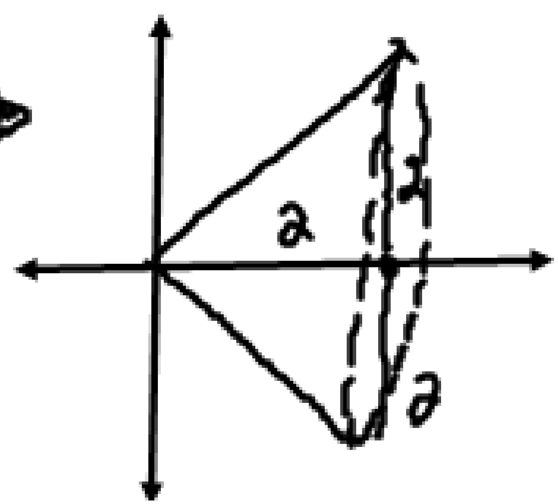
Volume= Area of base x height

Take $y=x$ and revolve it $360°$ about the x-axis to find the trapped VOLUME from $x=0$ to $x=2$

Without Calculus



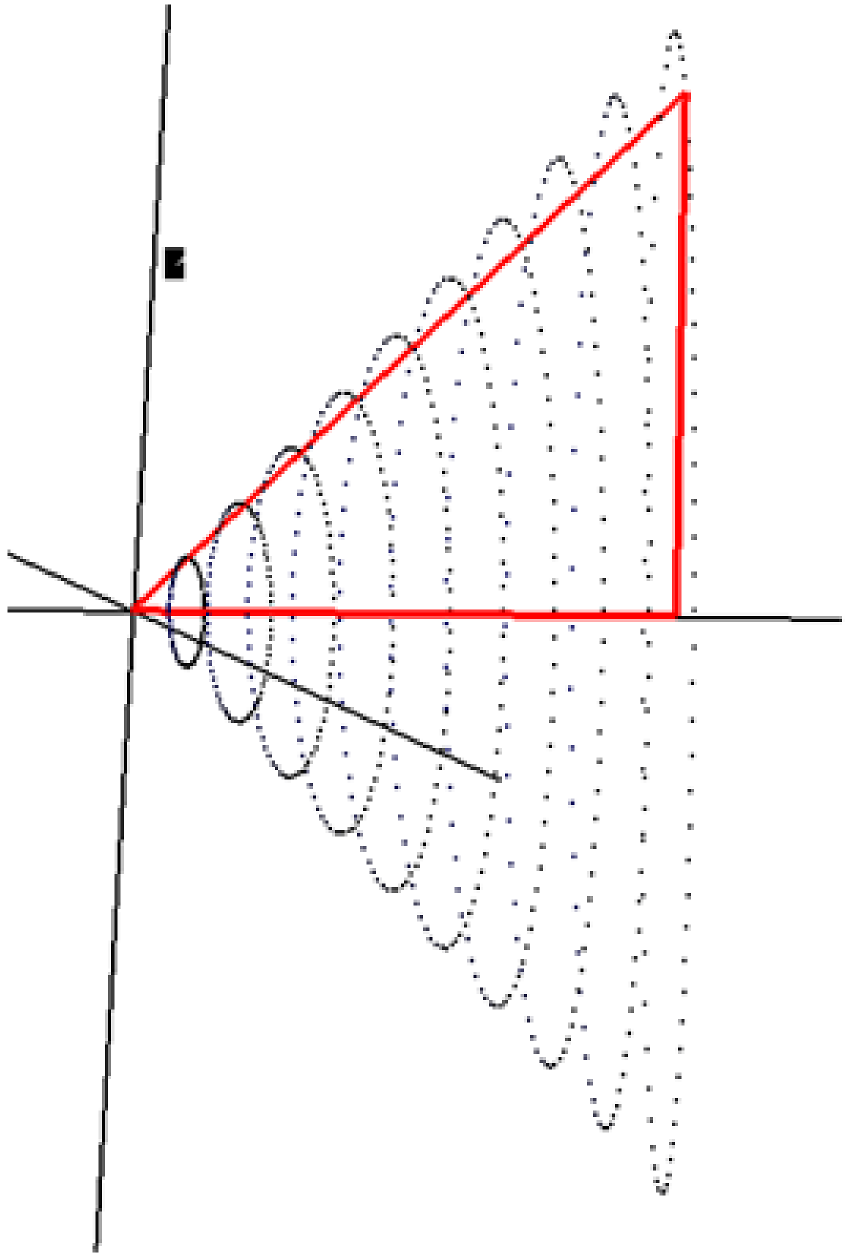
$\Rightarrow$



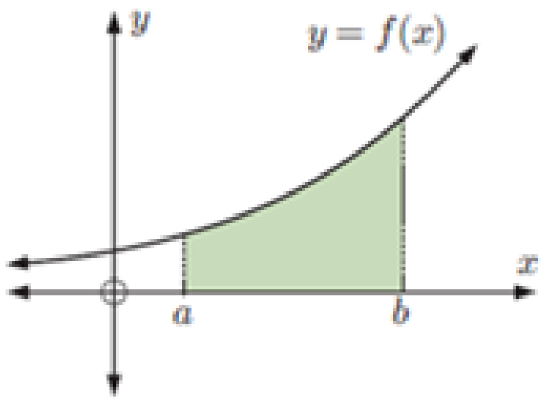
cone

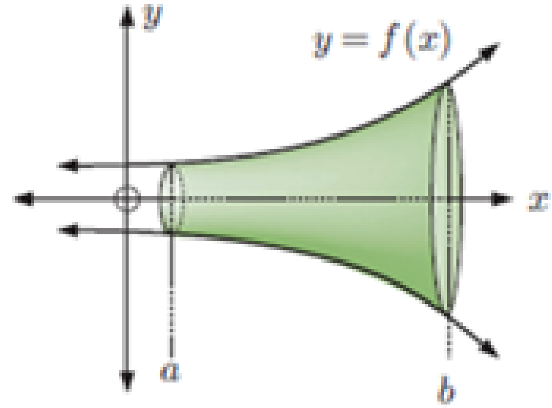
$$V=\frac{1}{3}\pi(2)^2(2)$$

$$=\frac{8\pi}{3}\ u^3$$



Given any function $y = f(x)$ that is rotated or revolved about the x-axis by $360°$ or 2π

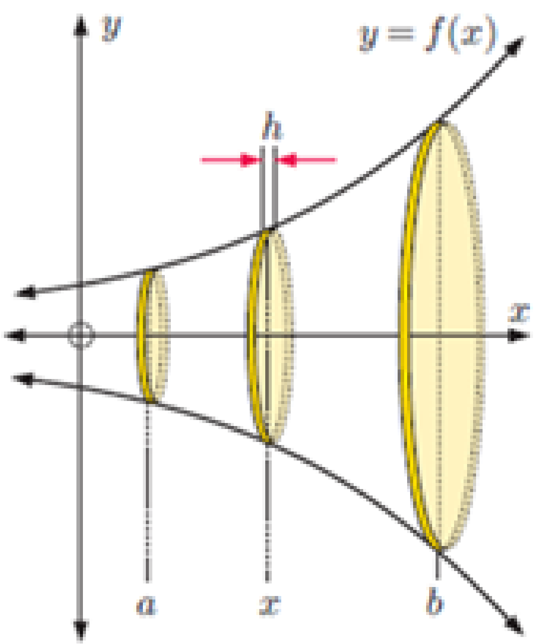




$\pi r^2 \cdot h$

There are an infinite number of cylindrical discs

$V_{cylinder} = A_{circle} \times h$



$V_{disc\ at\ x=a}: V = \pi(f(a))^2 \cdot h$

$V_{x=b} : V = \pi(f(b))^2 \cdot h$

$V_{x=x} : V = \pi(f(x))^2 \cdot h$

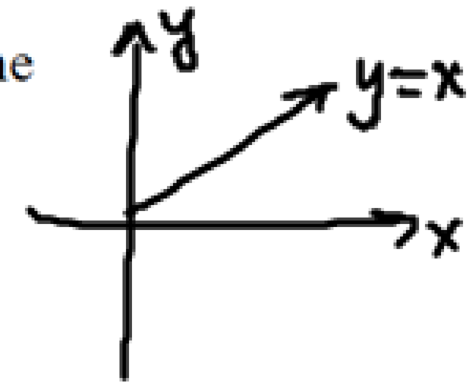
$$V = \sum_{x=a}^{b} \pi (f(x))^2 h$$

as h gets smaller

$$V = \lim_{h \to 0} \sum_{x=a}^{b} \pi (f(x))^2 h$$

$$V = \int_a^b \pi (f(x))^2 \, dx$$

Take $y = x$ and revolve it 360° about the x-axis to find the trapped VOLUME from $x = 0$ to $x = 2$



With Calculus

$$V = \int_0^2 \pi (x)^2 \, dx$$

$$= \pi \int_0^2 x^2 \, dx$$

$$= \pi \left[\frac{1}{3} x^3 \right]_0^2$$

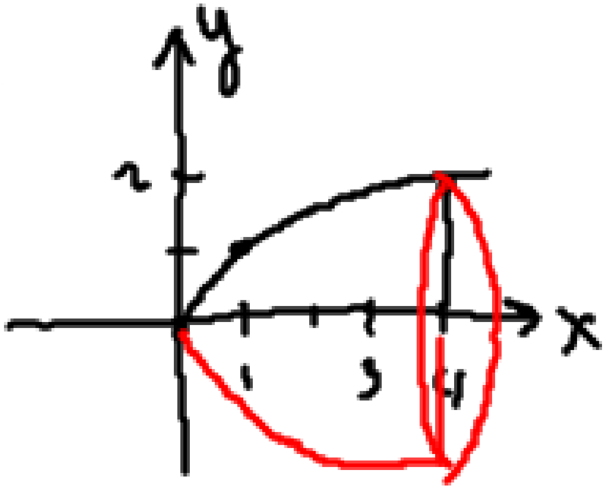
$$= \pi \left[\frac{2^3}{3} - \frac{0^3}{3} \right]$$

$$= \pi \left(\frac{8}{3} \right)$$

$$V = \frac{8\pi}{3} \quad u^3$$

Example:

Find the volume of the solid formed when $y = \sqrt{x}$ from $x = 0$ to $x = 4$



$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$= \int_0^4 \pi x \, dx$$

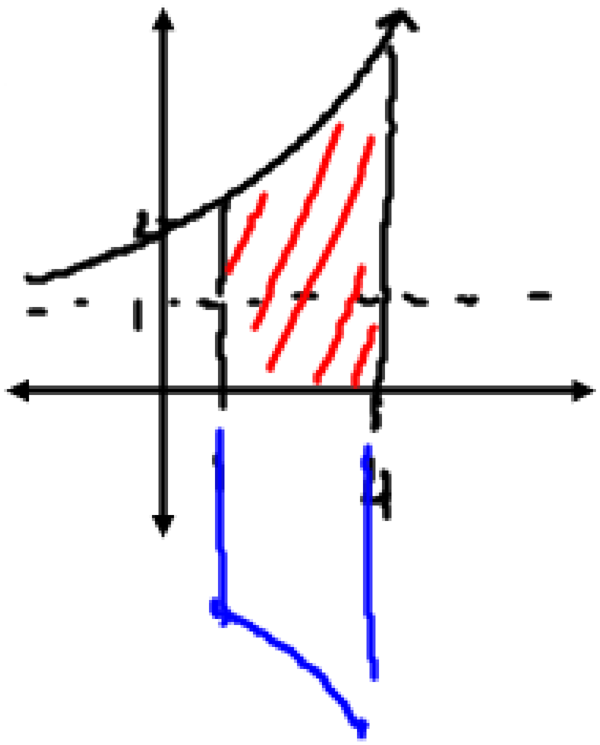
$$= \pi \left(\frac{1}{2}x^2\right)_0^4$$

$$= \pi \left(\frac{16}{2} - \frac{0}{2}\right)$$

$$= 8\pi$$

Example:

Find the volume of the solid formed when $y = e^x + 1$ from $x = 1$ to $x = 4$



$$V = \int_1^4 \pi (e^x + 1)^2 \, dx$$

vertical translation of 1 unit

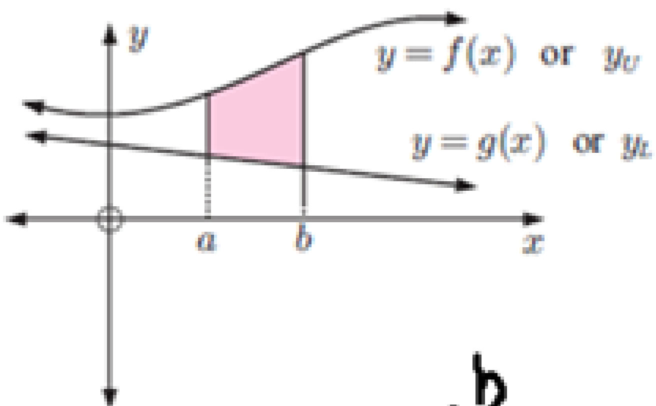
$$= \int_1^4 \pi (e^{2x} + 2e^x + 1) \, dx$$

$$= \pi \left[\frac{1}{2} e^{2x} + 2e^x + x \right]_1^4$$

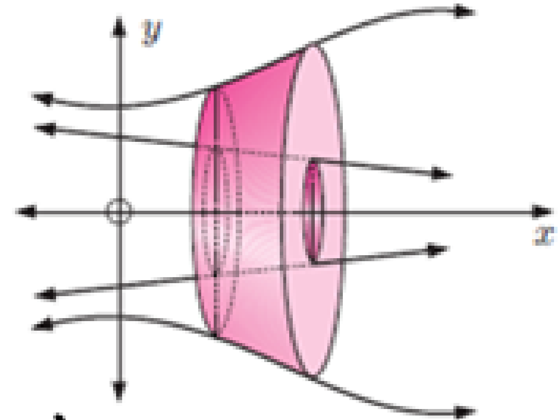
$$= \pi \left[\left(\frac{e^8}{2} + 2e^4 + 4 \right) - \left(\frac{e^2}{2} + 2e + 1 \right) \right]$$

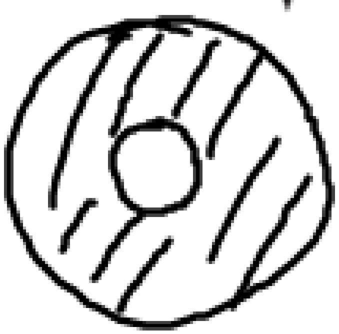
If the region bounded by an upper function and lower function is revolved about the x-axis, then:

Rotating



about the x-axis gives





$$V = \int_a^b \left(\pi (f(x))^2 - \pi (g(x))^2\right) dx$$

$$V = \pi \int_a^b \left((f(x))^2 - (g(x))^2\right) dx$$

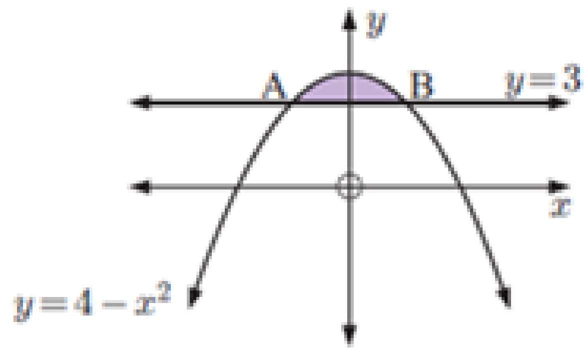
y upper $\quad$ y lower

EXERCISE 19D.2

1 The shaded region between $y = 4 - x^2$ and $y = 3$ is revolved about the x-axis.

a What are the coordinates of A and B?

b Find the volume of revolution.



A) POI

$4 - x^2 = 3$

$4 - 3 = x^2$

$1 = x^2$

$x = \pm 1$

$A(-1, 3)$

$B(1, 3)$

expand

$$V = \int_{-1}^{1} \pi \left[(4 - x^2)^2 - (3)^2 \right] dx$$

$y_{up} = 4 - x^2$

$y_{low} = 3$

$$V = \pi \int_{-1}^{1} \left[16 - 8x^2 + x^4 - 9 \right] dx$$

$$V = \pi \int_{-1}^{1} (x^4 - 8x^2 + 7) dx$$

$$V = \pi \left[\frac{1}{5}x^5 - \frac{8}{3}x^3 + 7x \right]_{-1}^{1}$$

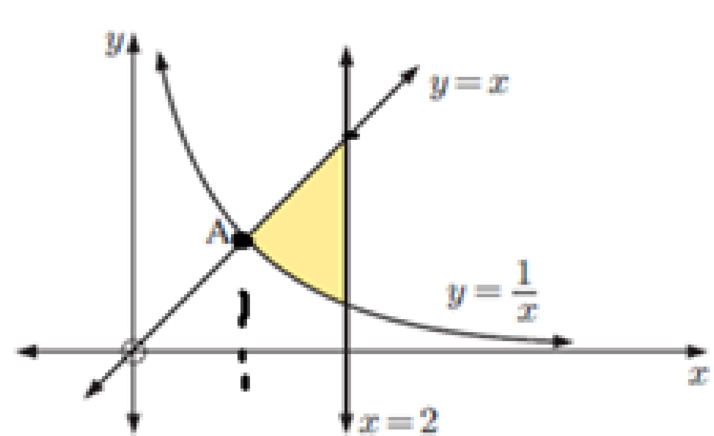
$\vdots$

$$V = \frac{136\pi}{15}$$

3 The shaded region between $y = x$, $y = \frac{1}{x}$, and $x = 2$ is revolved about the x-axis.

a Find the coordinates of A.

b Find the volume of revolution.



A) POI

$$x = \frac{1}{x}$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x = 1,\ y = 1$$

A(1,1)

Yup: $y = x$

Ylow: $y = \frac{1}{x}$

$$V = \int_1^2 \pi\left[(x)^2 - \left(\frac{1}{x}\right)^2\right]dx$$

$$= \int_1^2 \pi\left(x^2 - x^{-2}\right)dx$$

$$= \pi\left[\frac{1}{3}x^3 + x^{-1}\right]_1^2$$

$$= \pi\left(\frac{x^3}{3} + \frac{1}{x}\right)_1^2$$

$$= \frac{11\pi}{6}$$

Hw pg 491 # 1dfh
2b
3c
8b
pg 494 # 2
4

EXERCISE 19D.1

1 Find the volume of the solid formed when the following are revolved through 2π about the x-axis:

a $y = 2x$ for $0 \leqslant x \leqslant 3$

b $y = \sqrt{x}$ for $0 \leqslant x \leqslant 4$

c $y = x^3$ for $1 \leqslant x \leqslant 2$

d $y = x^{\frac{3}{2}}$ for $1 \leqslant x \leqslant 4$

e $y = x^2$ for $2 \leqslant x \leqslant 4$

f $y = \sqrt{25 - x^2}$ for $0 \leqslant x \leqslant 5$

g $y = \dfrac{1}{x-1}$ for $2 \leqslant x \leqslant 3$

h $y = x + \dfrac{1}{x}$ for $1 \leqslant x \leqslant 3$

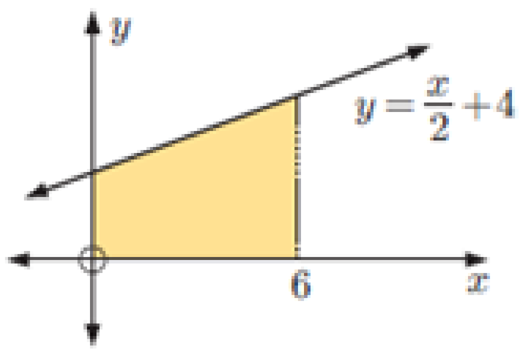
2 Use technology to find, correct to 3 significant figures, the volume of the solid of revolution formed when these functions are rotated through 360° about the x-axis:

a $y = \dfrac{x^3}{x^2+1}$ for $1 \leqslant x \leqslant 3$

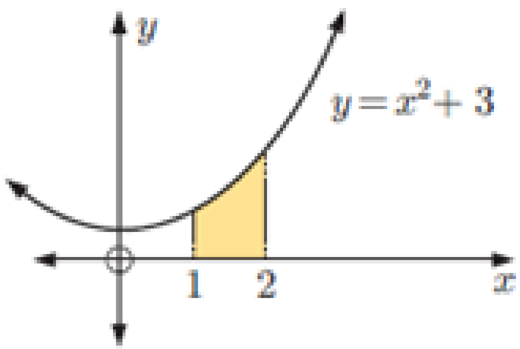
b $y = e^{\sin x}$ for $0 \leqslant x \leqslant 2$.

3 Find the volume of revolution when the shaded region is revolved through 2π about the x-axis.

a



b



c