Ch 19 C - Kinematics

Example: Given $f'(x) = 2x - 1$ and $f(0) = 3$. Find $f(x)$

$$f(x) = \int f'(x)\,dx$$

$$= \int (2x - 1)\,dx$$

$$f(x) = x^2 - x + C$$

$$3 = (0)^2 - (0) + C$$

$$3 = C$$

$$f(x) = x^2 - x + 3$$

Example: Given $f''(x) = 2x + 1$, $f'(1) = 3$ and $f(2) = 7$

Find $f(x)$

$$f'(x) = \int f''(x)\,dx$$

$$= \int (2x+1)\,dx$$

$$f'(x) = x^2 + x + C$$

$$3 = (1)^2 + (1) + C$$

$$1 = C$$

$$f'(x) = x^2 + x + 1$$

$$f(x) = \int f'(x)\,dx$$

$$= \int (x^2 + x + 1)\,dx$$

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + k$$

$$7 = \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2 + k$$

$$7 = \frac{8}{3} + 4 + k$$

$$3 = \frac{8}{3} + k$$

$$9 = 8 + 3k$$

$$1 = 3k$$

$$\frac{1}{3} = k$$

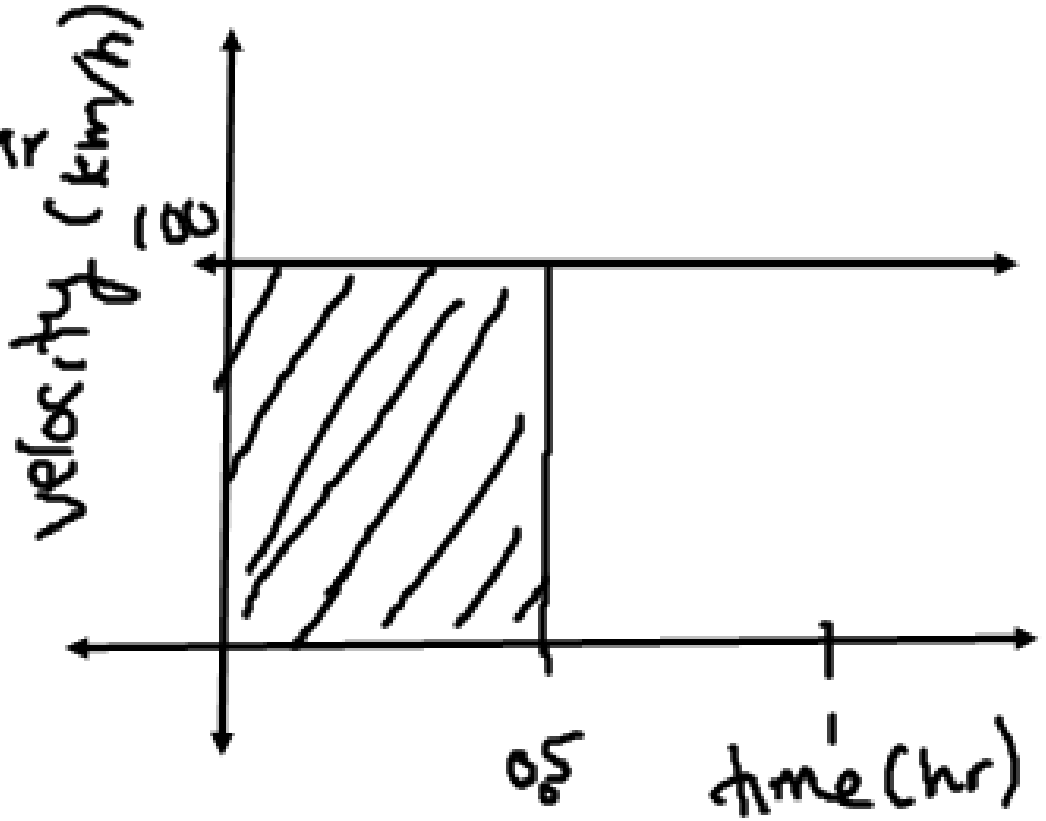
$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{1}{3}$$

Suppose a car travels at a constant positive velocity of 100 km/h for 30 minutes. How far has it travelled?

Distance = speed x time

$= \frac{100\ km}{hr} \times 0.5\ hr$

$= 50\ km$



Distance is the Area under a velocity-time graph.

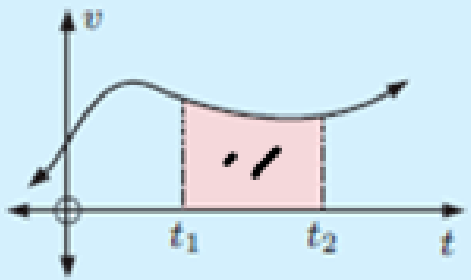
$A = l \times w$

$= (0.5)(100)$

$= 50$

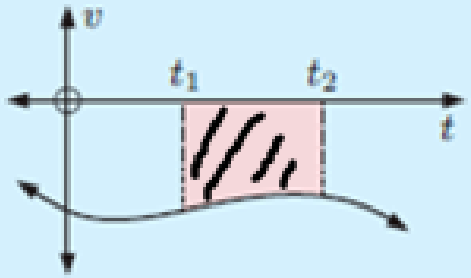
For a velocity-time function $v(t)$ where $v(t) \geqslant 0$ on the interval $t_1 \leqslant t \leqslant t_2$,

$$\textbf{distance travelled} = \int_{t_1}^{t_2} v(t)\,dt.$$



For a velocity-time function $v(t)$ where $v(t) \leqslant 0$ on the interval $t_1 \leqslant t \leqslant t_2$,

$$\textbf{distance travelled} = -\int_{t_1}^{t_2} v(t)\,dt.$$

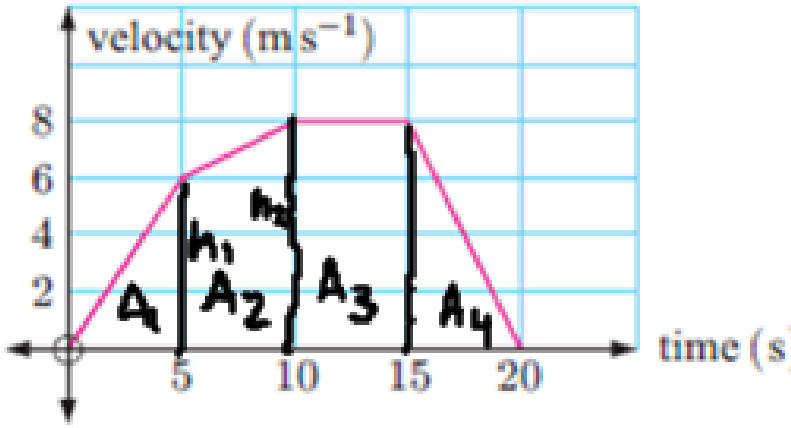


$$d = \int_{t_1}^{t_2} (y_{up} - y_{low})\,dt$$

$$= \int_{t_1}^{t_2} (0 - v(t))\,dt$$

EXERCISE 19C.1

1 A runner has the velocity-time graph shown. Find the total distance travelled by the runner.



Total distance Travelled

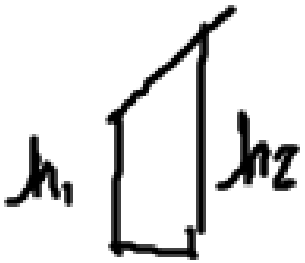
$= A_1 + A_2 + A_3 + A_4$

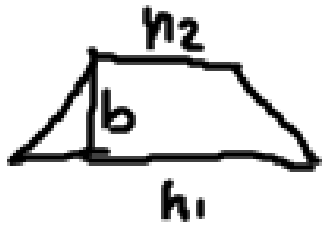
$= \frac{1}{2}bh + \frac{1}{2}(h_1+h_2)b + lw + \frac{1}{2}bh$

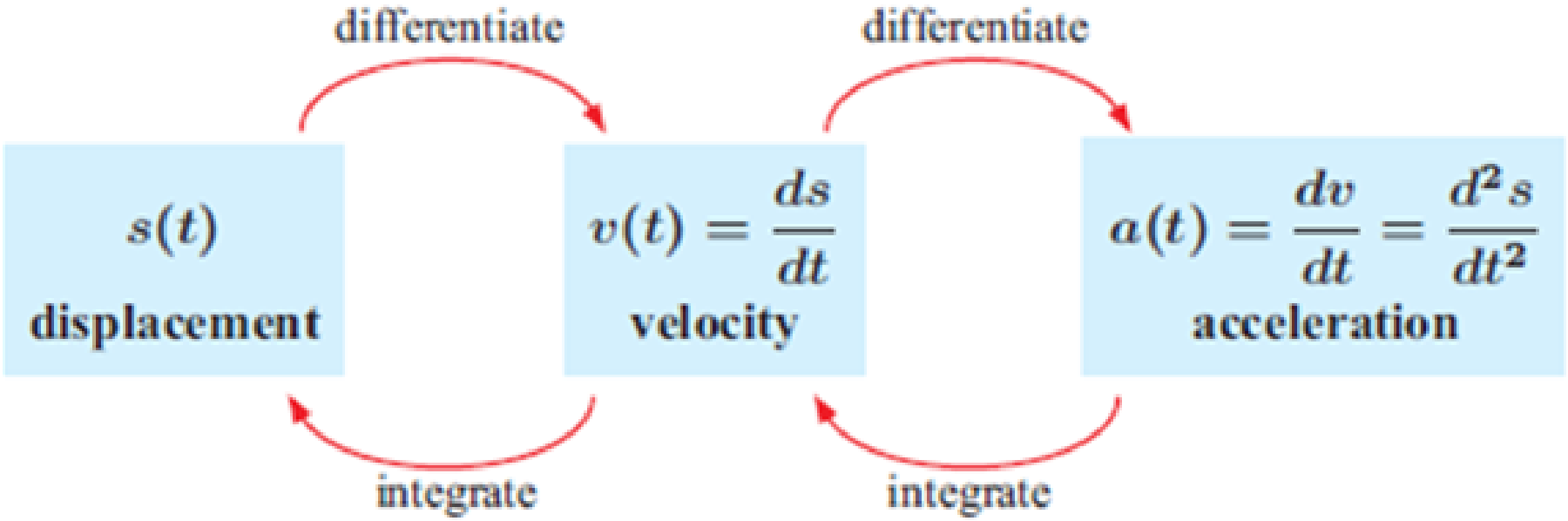
$= \frac{1}{2}(5)(6) + \frac{1}{2}(6+8)5 + (5)(8) + \frac{1}{2}(5)(8)$

$= 15 + 35 + 40 + 20$

$= 110$






differentiate
differentiate
$s(t)$
displacement
$v(t) = \frac{ds}{dt}$
velocity
$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
acceleration
integrate
integrate

A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6cm/\sec$ and its initial displacement is $s(0) = 9cm$. Find its position function.

$$v(t) = \int a(t)\,dt$$

$$= \int (6t + 4)\,dt$$

$$v(t) = 3t^2 + 4t + C$$

$$-6 = 3(0)^2 + 4(0) + C$$

$$-6 = C$$

$$v(t) = 3t^2 + 4t - 6$$

$$s(t) = \int v(t)\,dt$$

$$= \int (3t^2 + 4t - 6)\,dt$$

$$s(t) = t^3 + 2t^2 - 6t + K$$

$$9 = K$$

$$s(t) = t^3 + 2t^2 - 6t + 9$$

EXERCISE 19C.2

1 A particle has velocity function $v(t) = 1 - 2t$ cm s^{-1} as it moves in a straight line. The particle is initially 2 cm to the right of O. $s(0) = 2$

a Write a formula for the displacement function $s(t)$.

b Find the total distance travelled in the first second of motion.

c Find the displacement of the particle at the end of one second.

A) $s(t) = \int v(t)\,dt$

$= \int (1-2t)\,dt$

$s(t) = t - t^2 + C$

$2 = 0 - 0 + C$

$2 = C$

$s(t) = -t^2 + t + 2$

B) $v(t) = 1 - 2t$

$0 = 1 - 2t$

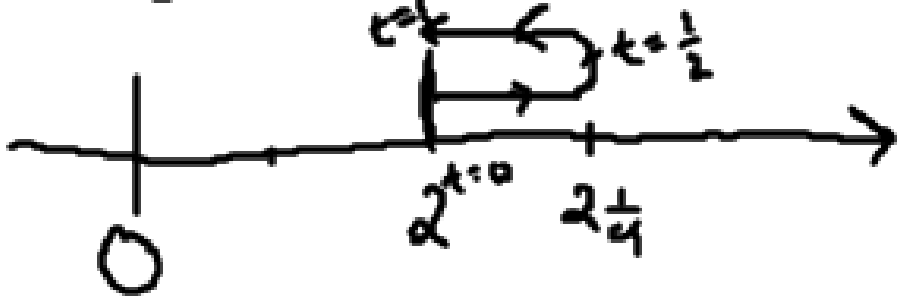
$t = \frac{1}{2}$

motion diagram

$s(0) = 2$

$s(\frac{1}{2}) = -(\frac{1}{2})^2 + \frac{1}{2} + 2 = 2\frac{1}{4}$

$s(1) = -(1)^2 + 1 + 2 = 2$



Total distance travelled =

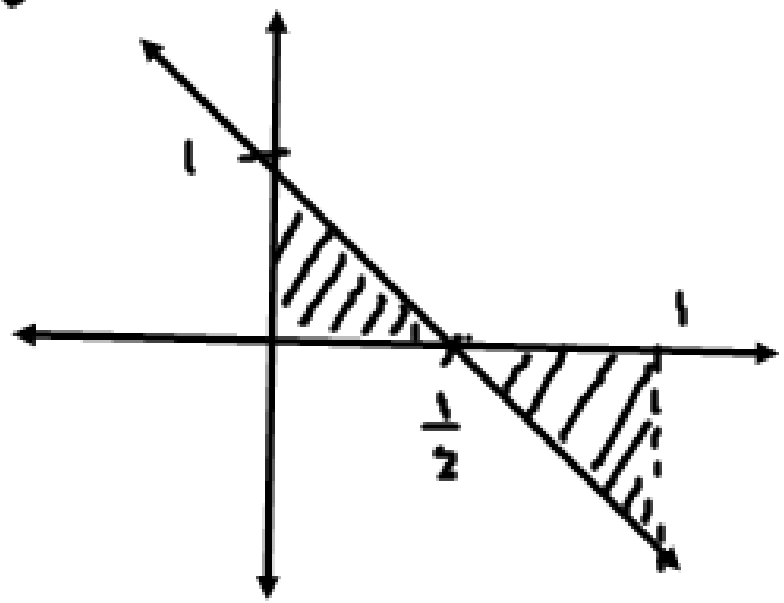
$(2\frac{1}{4} - 2) + (2\frac{1}{4} - 2)$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

B) or

graph $v(t) = 1-2t$



$$\text{Total D} = \int_0^{\frac{1}{2}} (1-2t)\,dt + \int_{\frac{1}{2}}^{1} (0-(1-2t))\,dt$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

C) Displacement $= S(1) - S(0)$

$= (2) - (2)$

$= 0$

Example: A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$

A) Find the displacement of the particle during the time period $1 \le t \le 4$ $\quad s(4) - s(1)$

$$s(t) = \int_1^4 v(t)\,dt$$

$$= \int_1^4 (t^2 - t - 6)\,dt$$

$$= \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t \Big]_1^4$$

displacement

$$= \left(\frac{1}{3}(4)^3 - \frac{1}{2}(4)^2 - 6(4)\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$$

$$= -\frac{9}{2}$$

$$= -4.5$$ ← means the particle moved 4.5 meters to the left!!

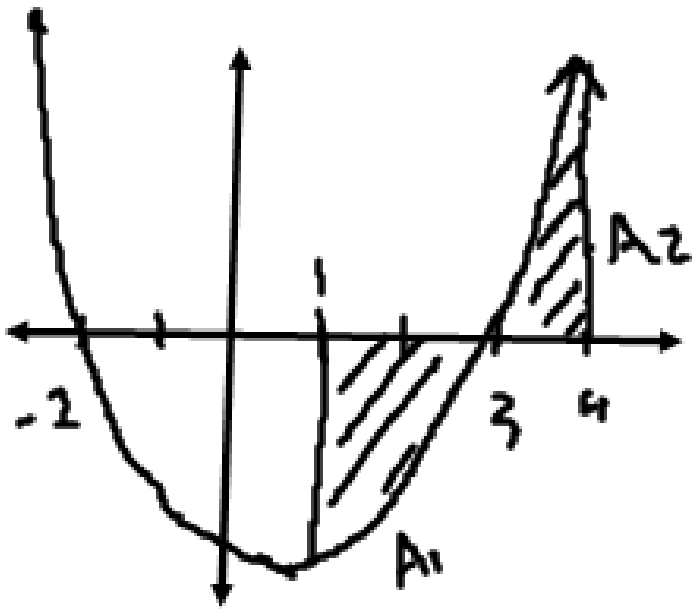
B) Find the distance travelled during this time period

$v(t) = t^2 - t - 6$

$0 = (t-3)(t+2)$

$t = 3 \quad t = -2$

$t = 1, \; t = 4$



Distance Travelled

$= A_1 + A_2$

$= \int_1^3 (t^2 - t - 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$

$= \frac{22}{3} + \frac{17}{6}$

$= \frac{61}{6} \quad \sim 10.1\overline{6}$

Hw pg 485 #2
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