5 a On the same set of axes, graph $y = e^x - 1$ and $y = 2 - 2e^{-x}$, showing axes intercepts and asymptotes.

b Find algebraically the points of intersection of $y = e^x - 1$ and $y = 2 - 2e^{-x}$.

c Find the area of the region enclosed by the two curves.

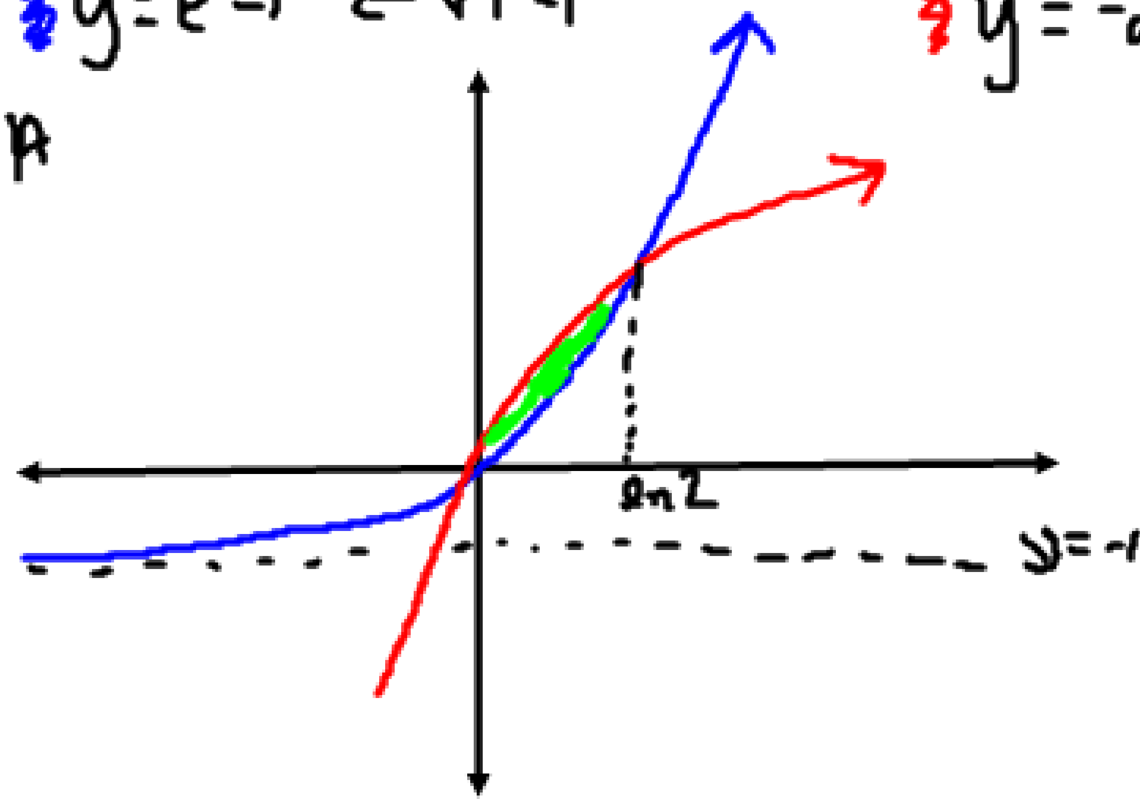
$y = e^x - 1$ ← VT −1

$y = -2e^{-x} + 2$

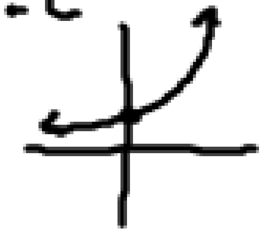
VT+2

VS2

A



$y = e^x$



$y = e^{-x}$ ← Ry

$y = -e^{-x}$ ← Ry

Rx

B)

$e^x - 1 = -2e^{-x} + 2$

$e^x + 2e^{-x} - 3 = 0$

$e^x + \frac{2}{e^x} - 3 = 0$

mult. each term by e^x

$(e^x)^2 + 2 - 3e^x = 0$

let $a = e^x$

$a^2 - 3a + 2 = 0$

$(a-2)(a-1) = 0$

$a = 2 \quad a = 1$

$e^x = 2 \qquad e^x = 1$

$x = \ln 2 \qquad x = 0$

c) $A = \int_0^{\ln 2} (y_{up} - y_{lw})\,dx$

$= \int_0^{\ln 2} ((2 - 2e^{-x}) - (e^x - 1))\,dx$

$= \int_0^{\ln 2} (3 - 2e^{-x} - e^x)\,dx$

$= 3x - 2(-e^{-x}) - (e^x)\Big]_0^{\ln 2}$

$\vdots$

$= 3\ln 2 - 2$

Ch 19 C - Kinematics

Example: Given $f'(x) = 2x - 1$ and $f(0) = 3$. Find $f(x)$

$$f(x) = \int f'(x)\,dx$$

$$= \int (2x-1)\,dx$$

$$f(x) = x^2 - x + C$$

$$3 = (0)^2 - (0) + C$$

$$3 = C \rightarrow f(x) = x^2 - x + 3$$

Example: Given $f''(x) = 2x + 1$, $f'(1) = 3$ and $f(2) = 7$

Find $f(x)$

$$f'(x) = \int f''(x)\,dx$$

$$= \int (2x+1)\,dx$$

$$f'(x) = x^2 + x + C$$

$$3 = (1)^2 + (1) + C$$

$$1 = C$$

$$f'(x) = x^2 + x + 1$$

$$f(x) = \int f'(x)\,dx$$

$$= \int (x^2 + x + 1)\,dx$$

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + d$$

$$7 = \frac{1}{3}(8) + \frac{1}{2}(4) + 2 + d$$

$$7 = \frac{8}{3} + 4 + d$$

$$3 = \frac{8}{3} + d$$

$$9 = 8 + 3d$$

$$1 = 3d$$

$$\frac{1}{3} = d$$

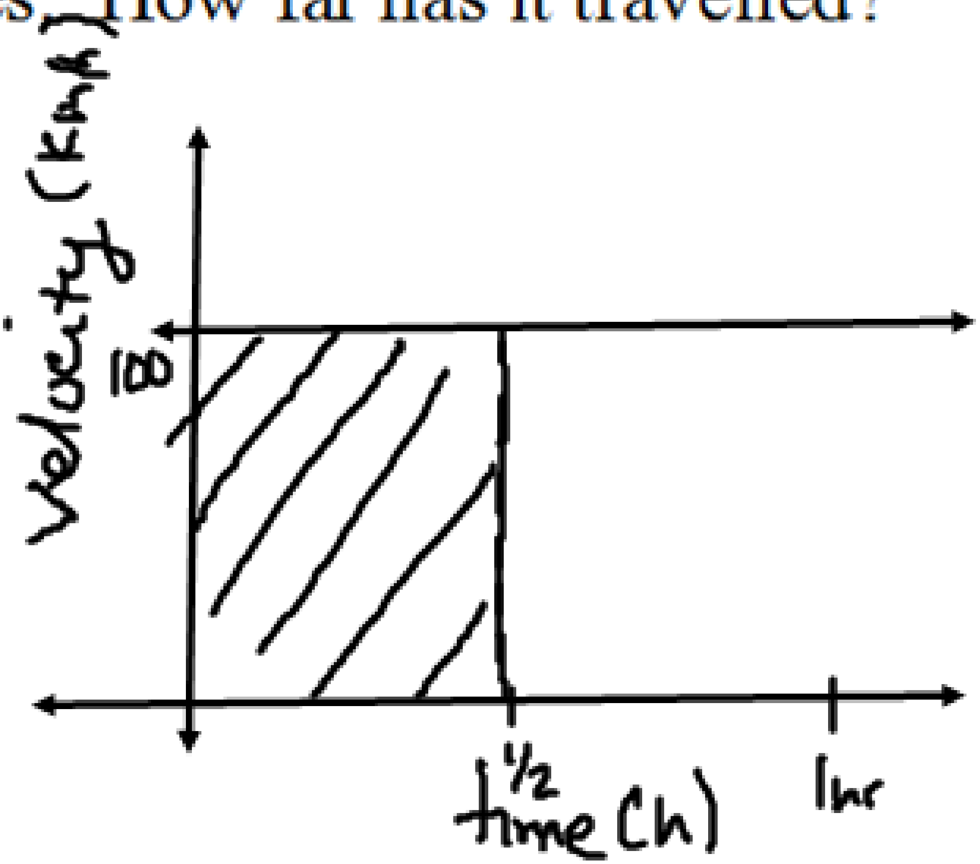
$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{1}{3}$$

Suppose a car travels at a constant positive velocity of 100 km/h for 30 minutes. How far has it travelled?

Distance = speed x time

$= \frac{100 km}{h} \times 0.5\ h$

$= 50 km$



A = l x w

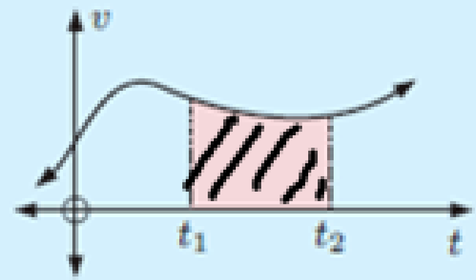
$= 100 \times \frac{1}{2}$

$= 50$

Distance is area of a velocity - time graph.

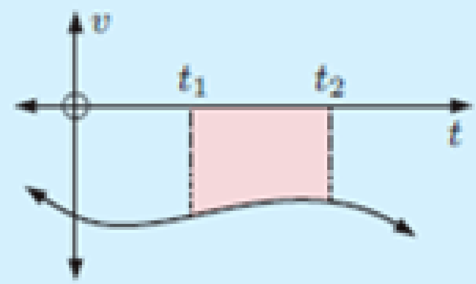
For a velocity-time function $v(t)$ where $v(t) \geqslant 0$ on the interval $t_1 \leqslant t \leqslant t_2$,

$$\textbf{distance travelled} = \int_{t_1}^{t_2} v(t)\, dt.$$



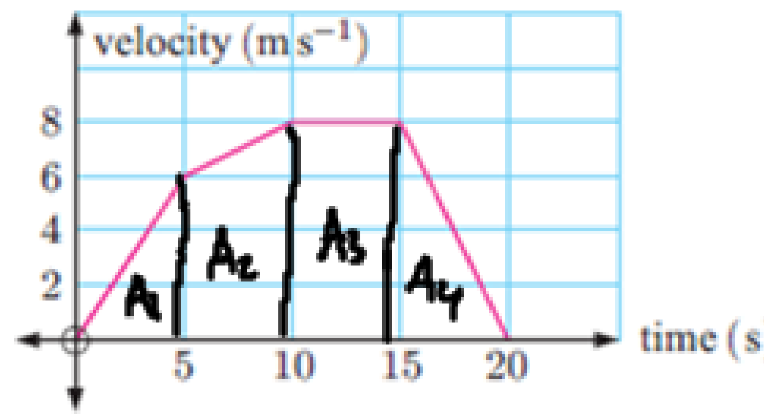
For a velocity-time function $v(t)$ where $v(t) \leqslant 0$ on the interval $t_1 \leqslant t \leqslant t_2$,

$$\textbf{distance travelled} = -\int_{t_1}^{t_2} v(t)\, dt.$$



EXERCISE 19C.1

1 A runner has the velocity-time graph shown. Find the total distance travelled by the runner.



Total Distance travelled =

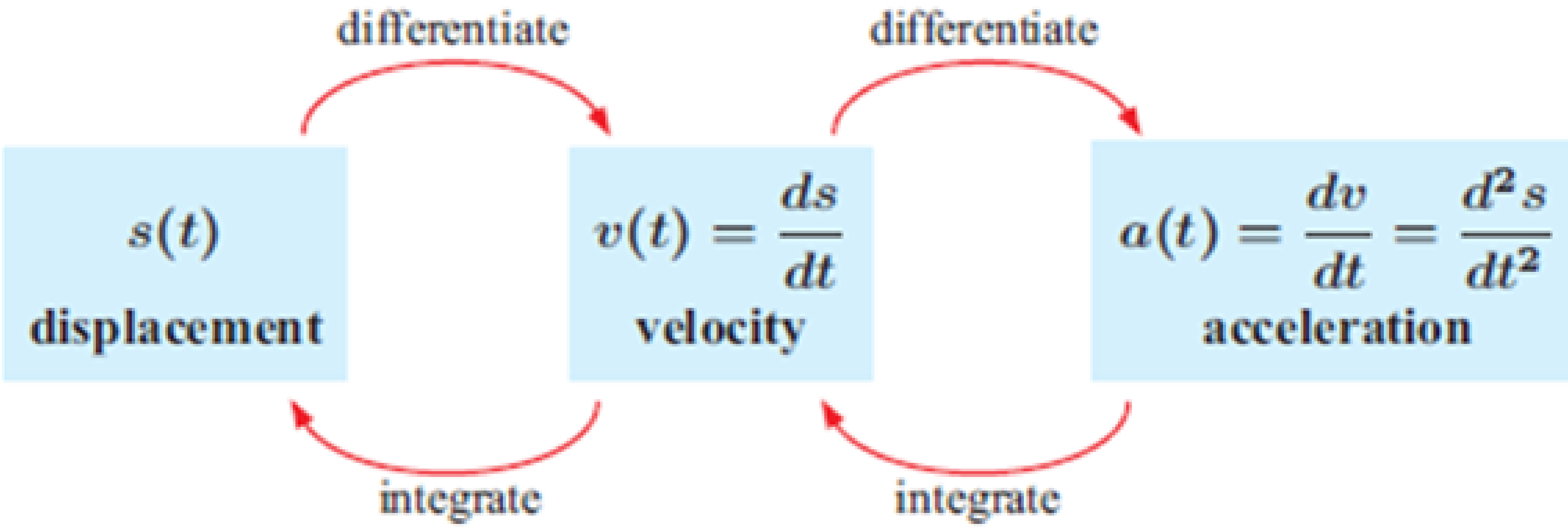
$= A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2}bh + \frac{1}{2}(h_1+h_2)b + l \times w + \frac{1}{2}bh$

$= \frac{1}{2}(5)(6) + \frac{1}{2}(6+8)(5) + (8)(5) + \frac{1}{2}(5)(8)$

$= 15 + 35 + 40 + 20$

$= 110\ m$


differentiate
differentiate
$s(t)$
displacement
$v(t) = \frac{ds}{dt}$
velocity
$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
acceleration
integrate
integrate

A particle moves in a straight line and has acceleration given by $a(t)=6t+4$. Its initial velocity is $v(0)=-6cm/\sec$ and its initial displacement is $s(0)=9cm$. Find its position function.

$$v(t)=\int a(t)\,dt$$
$$=\int (6t+4)\,dt$$
$$v(t)=3t^2+4t+C$$
$$-6=3(0)^2+4(0)+C$$
$$-6=C$$
$$v(t)=3t^2+4t-6$$

$$s(t)=\int v(t)\,dt$$
$$=\int (3t^2+4t-6)\,dt$$
$$s(t)=t^3+2t^2-6t+p$$
$$9=(0)^3+2(0)^2-6(0)+p$$
$$9=p$$
$$\boxed{s(t)=t^3+2t^2-6t+9}$$

EXERCISE 19C.2

1 A particle has velocity function $v(t) = 1 - 2t$ cm s^{-1} as it moves in a straight line. The particle is initially 2 cm to the right of O. $(S(0)=2)$

a Write a formula for the displacement function $s(t)$.

b Find the total distance travelled in the first second of motion.

c Find the displacement of the particle at the end of one second.

A) $S(t) = \int v(t)\,dt$

$= \int (1-2t)\,dt$

$S(t) = t - t^2 + C$

$2 = C$

$\boxed{S(t) = -t^2 + t + 2}$

B) find $v(t) = 0$

$1 - 2t = 0$

$t = \frac{1}{2}$

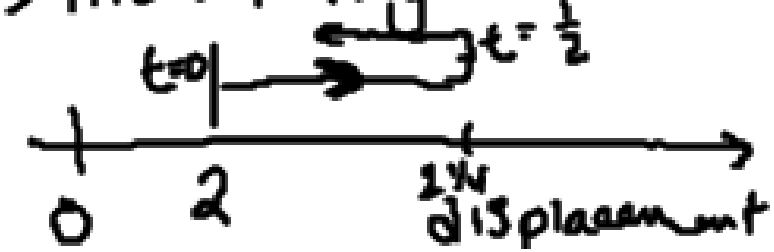
$S(0) = 2$

$S(\frac{1}{2}) = -(\frac{1}{2})^2 + (\frac{1}{2}) + 2$

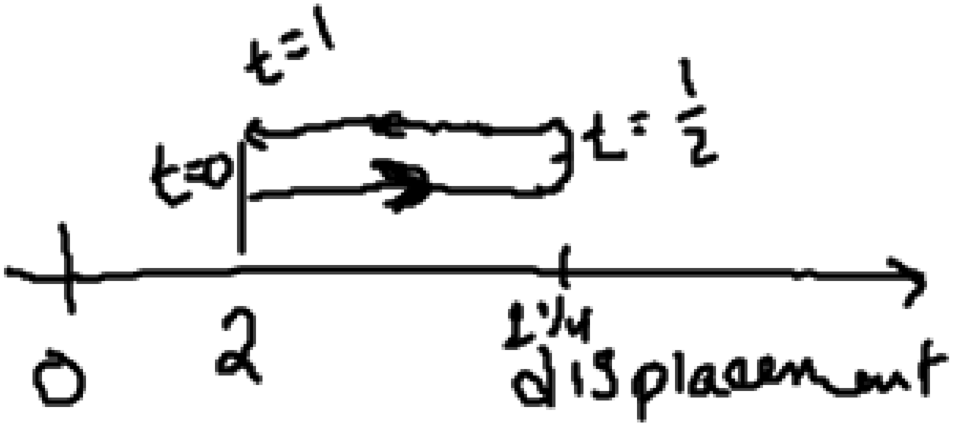
$= 2\frac{1}{4}$

$S(1) = -1 + 1 + 2 = 2$

→ motion diagram

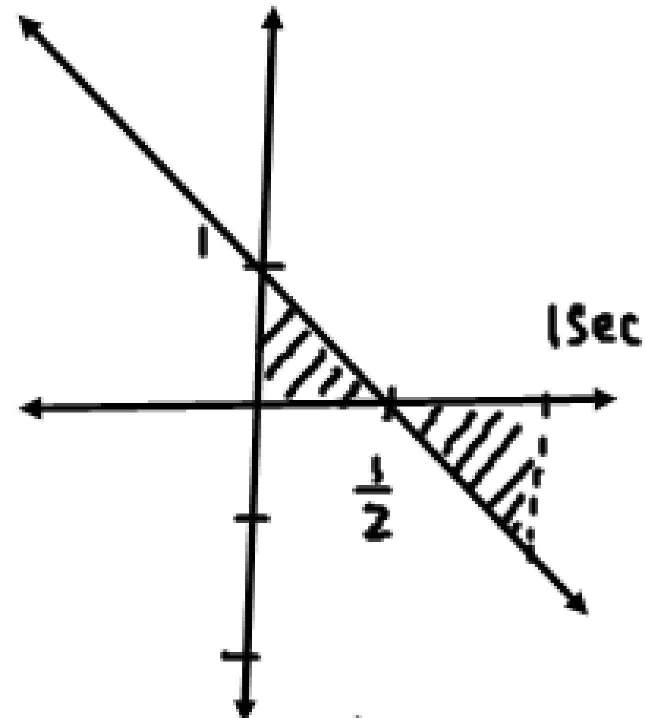


→ motion diagram,



total distance $= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$v(t) = 1-2t$



$$\text{Distance} = \int_0^{\frac{1}{2}} (1-2t)\,dt + \int_{\frac{1}{2}}^{1} (-1+2t)\,dt$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

Example: A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$

A) Find the displacement of the particle during the time period $1 \le t \le 4$

$s(t) = \int v(t)\,dt$

$= \int_1^4 (t^2 - t - 6)\,dt$

$= \frac{1}{3}t^3 - \frac{1}{2}t^2 - 6t \Big]_1^4$

Displacement: $\left(\frac{1}{3}(4)^3 - \frac{1}{2}(4)^2 - 6(4)\right) - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$

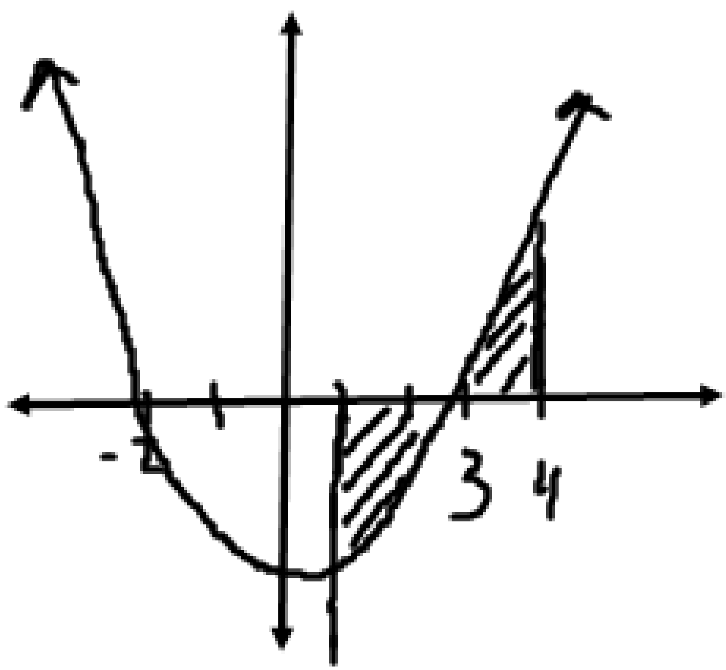
$= \frac{-9}{2}$ ← means the partical moved 4.5 m. to the left

B) Find the distance travelled during this time period

$v(t) = t^2 - t - 6$

$0 = (t-3)(t+2)$

$t = 3,\ t = -2$



$$\text{Distance} = \int_1^3 (-t^2 + t + 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$$

$$= \frac{22}{3} + \frac{17}{6} = \frac{61}{6}$$

HW: pg 485 #2

pg 487 # 2-5, 7