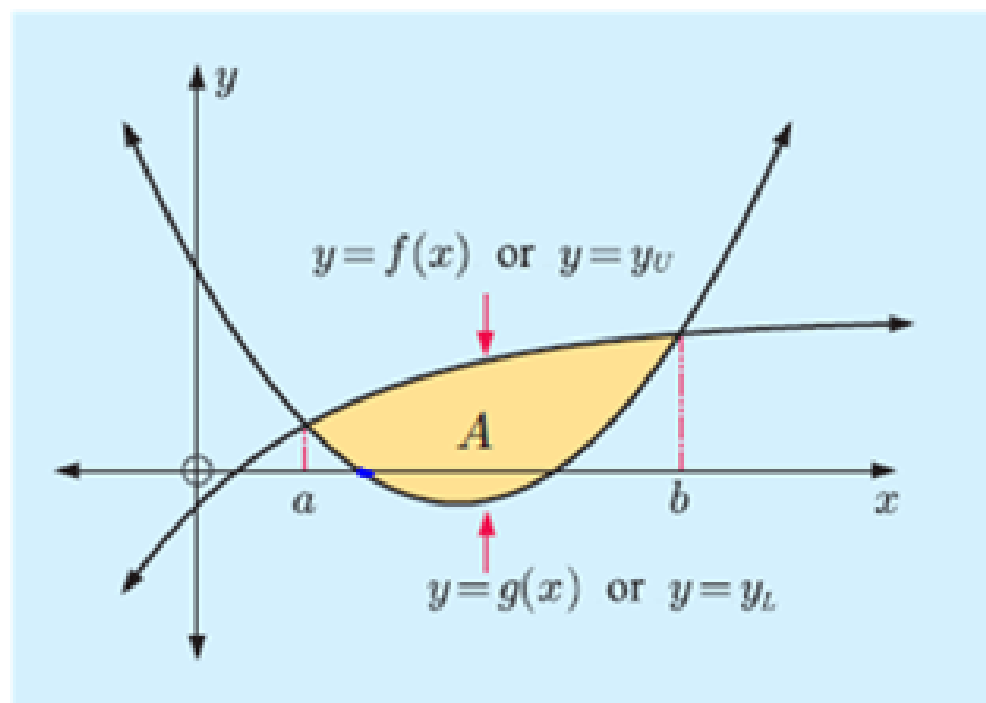
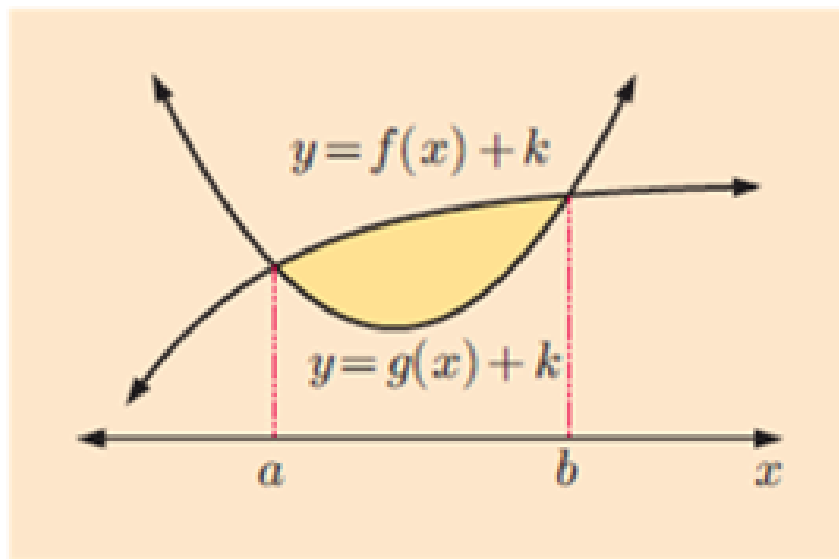


Ch 19B -Area Between TWO Functions Day 2

What if the curve is UNDER the x-axis??



Apply a vertical translation to each function, such that



$$A = \int_a^b (y_{\text{up}} - y_{\text{low}}) dx$$

$$A = \int_a^b [(f(x) + k) - (g(x) + k)] dx$$

$$= \int_a^b (f(x) - g(x)) dx$$

Example: Find the area bounded by the x-axis and $y = x^2 - 4x$

→ Quadratic

→ opens up

→ vertex: $x = \frac{-b}{2a}$

$$x = +2$$

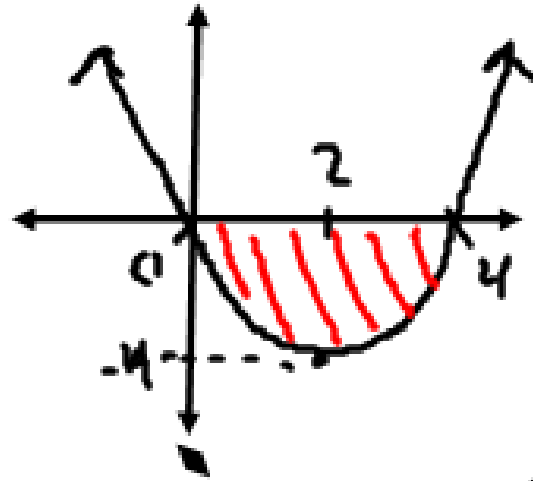
$$y = 4 - 8$$
$$= -4$$

$$(2, -4)$$

→ x-int (y=0)

$$0 = x^2 - 4x$$
$$= x(x - 4)$$

$$x = 0 \quad x = 4$$
$$(0, 0) \quad (4, 0)$$



$$A = \int_0^4 (y_{up} - y_{low}) dx$$
$$= \int_0^4 (0 - (x^2 - 4x)) dx$$
$$= \int_0^4 (-x^2 + 4x) dx$$
$$= \left[-\frac{1}{3}x^3 + 4\left(\frac{1}{2}x^2\right) \right]_0^4$$
$$= \left(-\frac{1}{3}(4)^3 + 2(4)^2 \right) - (0)$$
$$= -\frac{64}{3} + \frac{96}{3} = \frac{32}{3}$$

evaluate $y = \int_0^4 (x^2 - 4x) dx$

$$= -10.67$$

evaluating the integral is
diff than finding the area

$$= \left(\frac{1}{3}x^3 - 2x^2 \right)_0^4$$

Example:

A) Find the total area of the regions contained by

$$y = x^3 + 2x^2 - x - 2 \text{ and the } x\text{-axis.}$$

-cubic



$$A_T = A_1 + A_2$$

-y-int (0, -2)

-x-int (y=0)

$$0 = \underbrace{x^3 + 2x^2}_{\text{group 1}} - \underbrace{x - 2}_{\text{group 2}}$$

Factor by grouping

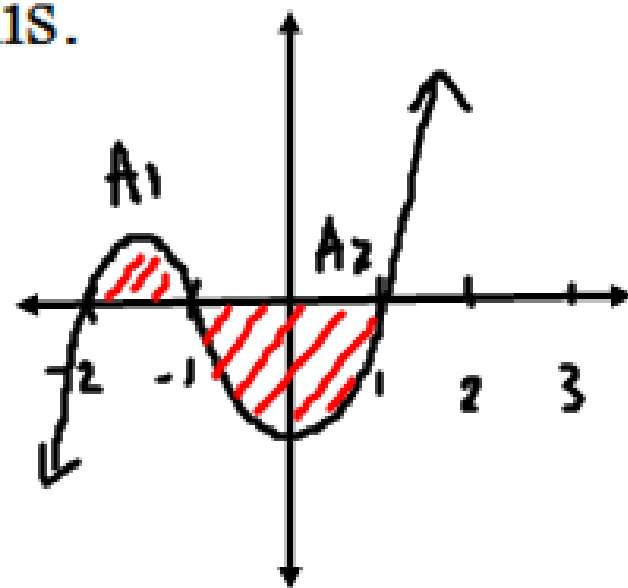
$$0 = x^2(x+2) - 1(x+2)$$

$$= (x+2)(x^2-1)$$

$$0 = (x+2)(x+1)(x-1)$$

$x = -2 \quad x = -1 \quad x = 1$

$$(-2, 0) \quad (-1, 0) \quad (1, 0)$$



$$A = A_1 + A_2$$

$$= \int_{-2}^{-1} [(x^3 + 2x^2 - x - 2) - 0] dx + \int_{-1}^1 [0 - (x^3 + 2x^2 - x - 2)] dx$$

$$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2x \right]_{-2}^{-1} + \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_{-1}^1$$

$$= \left[\left(\frac{1}{4}(-1)^4 + \frac{2}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) \right) - \left(\frac{1}{4}(-2)^4 + \frac{2}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 2(-2) \right) \right] +$$
$$\left[\left(-\frac{1}{4} - \frac{2}{3} + \frac{1}{2} + 2 \right) - \left(-\frac{1}{4}(-1)^4 - \frac{2}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1) \right) \right]$$

$$= \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$$

B) Find $\int_{-2}^1 (x^3 + 2x^2 - x - 2) dx$

$\int_{-2}^1 (x^3 + 2x^2 - x - 2) dx$
-2.25 ← $\frac{5}{12} - \left(\frac{8}{3}\right)$

HW: ch 19B# 5,6,7,9