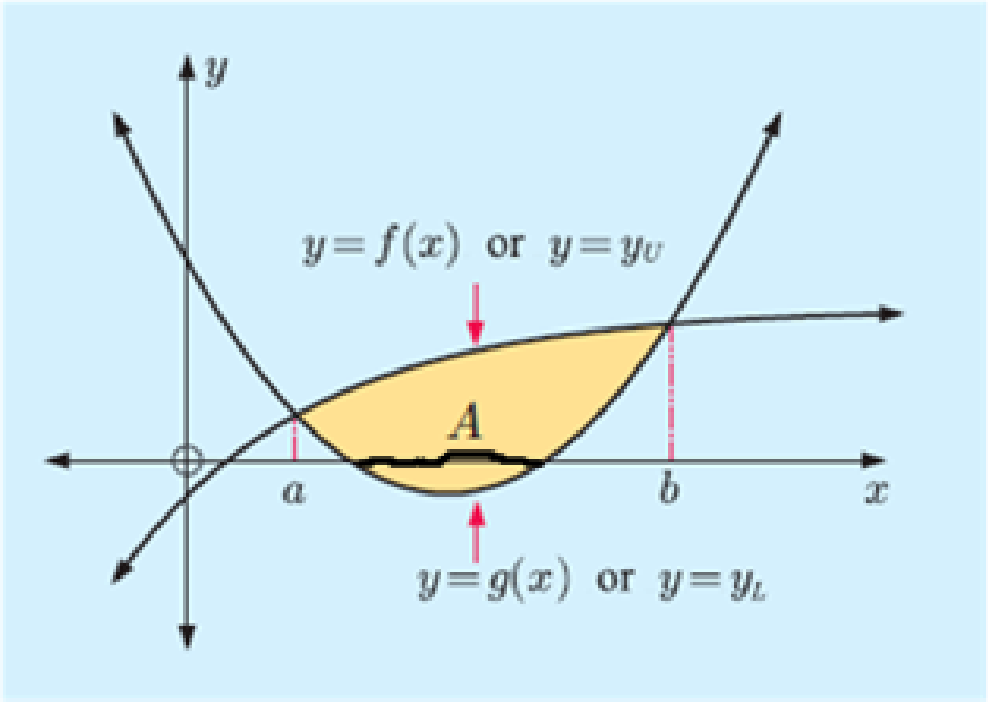
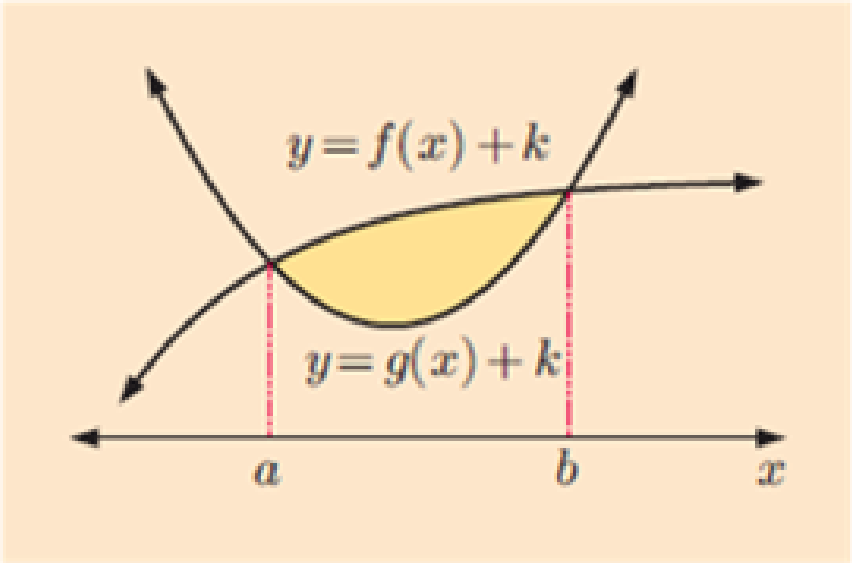
Ch 19B -Area Between TWO Functions Day 2

What if the curve in UNDER the x-axis??



Apply a vertical translation to each function, such that



$$A = \int_a^b (y_{upper} - y_{lower})\,dx$$

$$A = \int_a^b ((f(x)+k) - (g(x)+k))\,dx$$

$$= \int_a^b (f(x) - g(x))\,dx$$

→ When we are looking at the area between two functions, it doesn't matter which portion is below the x-axis.

Example: Find the area bounded by the x-axis and $y = x^2 - 4x$

→ Quadratic

→ opens up

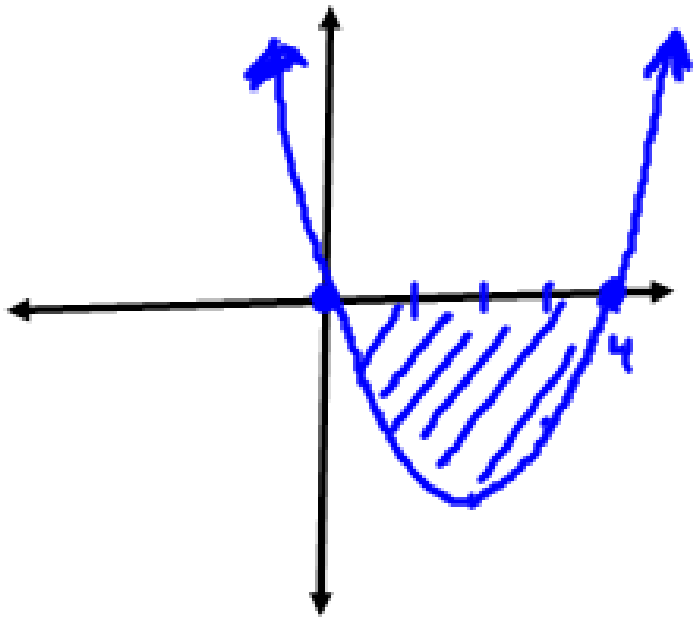
→ x-int ($y=0$)

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0 \quad x = 4$

$(0,0) \quad (4,0)$



$$A = \int_0^4 (y_{up} - y_{low})\,dx$$

$$= \int_0^4 (0 - (x^2 - 4x))\,dx$$

$$= \int_0^4 (-x^2 + 4x)\,dx$$

$$= -\frac{1}{3}x^3 + \frac{4}{2}x^2 \Big]_0^4$$

$$= \left(-\frac{1}{3}(4)^3 + 2(4)^2\right) - (0)$$

$$= -\frac{64}{3} + 32$$

$$= \frac{32}{3}$$

$\int_0^4 (x^2 - 4x)\,dx$

-10.66666667

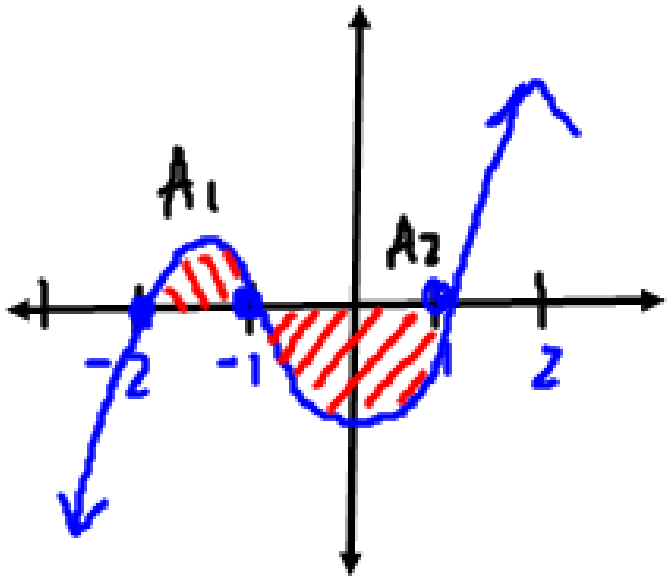
Example:

A)Find the total area of the regions contained by $y = x^3 + 2x^2 - x - 2$ and the x-axis.

→ cubic

→ odd function

start low, end high



$0 = x^3 + 2x^2 - x - 2$

factor by grouping

$0 = x^2(x+2) - 1(x+2)$

$= (x+2)(x^2-1)$

$= (x+2)(x+1)(x-1)$

$(-2, 0)$ $(-1, 0)$ $(1, 0)$

$$A_T = A_1 + A_2$$

$$= \int_{-2}^{-1} \left((x^3 + 2x^2 - x - 2) - 0\right) dx + \int_{-1}^{1} \left(0 - (x^3 + 2x^2 - x - 2)\right) dx$$

$$= \left(\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 - 2x\right)\Big]_{-2}^{-1} + \left[\left(-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 + 2x\right)\right]_{-1}^{1}$$

$$= -\left[\left(\frac{1}{4}(-1)^4 + \frac{2}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1)\right) - \left(\frac{1}{4}(-2)^4 + \frac{2}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 2(-2)\right)\right] +$$

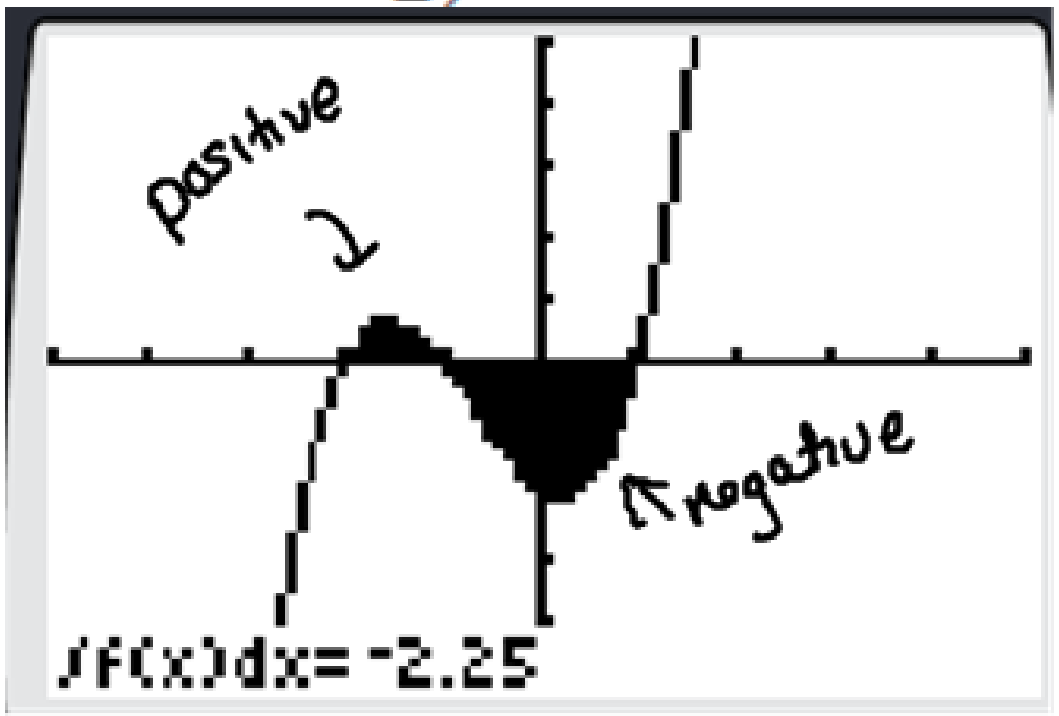
$$\left[\left(-\frac{1}{4}(1)^4 - \frac{2}{3}(1)^3 + \frac{1}{2}(1)^2 + 2(1)\right) - \left(\frac{1}{4}(-1)^4 - \frac{2}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1)\right)\right]$$

$$= \frac{5}{12} + \frac{8}{3}$$

$$= \frac{37}{12}$$

B) Find $\int_{-2}^{1} \left(x^3 + 2x^2 - x - 2\right) dx$ = -2.25

= $-\frac{9}{4}$



↑ not calc.

total Area ...

Net Area ...

HW: ch 19B# 5,6,7,9