Chapter 19

Applications of integration

Syllabus reference: 6.5, 6.6

Contents:

Ch 19 A – Area under a curve… again

Use a graph of the integrand and areas to evaluate the integral:

$$\int_{-1}^{4} (2x+3)\,dx$$

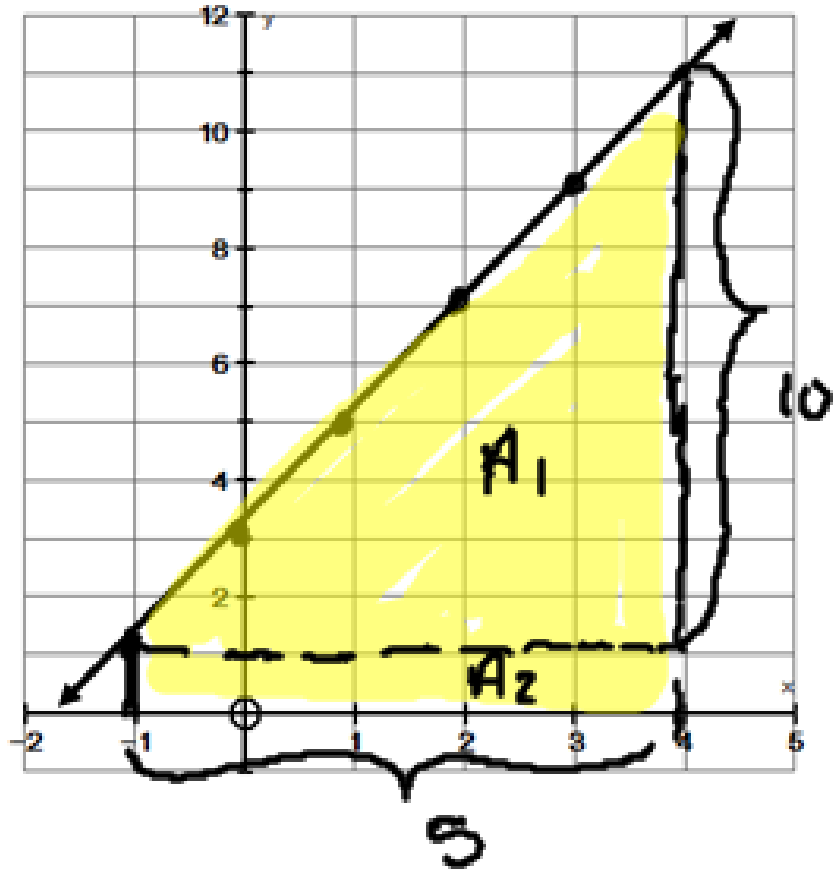
$A_{TOTAL} = A_1 + A_2$

$= \frac{1}{2} b \cdot h + l \cdot w$

$= \frac{1}{2}(5)(10) + (5)(1)$

$= 26 + 5$

$= 31$



$$A = \int_{-1}^{4} (2x+3)\,dx$$

$$= 2\left(\frac{1}{2}x^2\right) + 3x \Big]_{-1}^{4}$$

$$= x^2 + 3x \Big]_{-1}^{4}$$

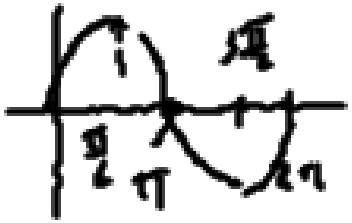
$$= \left[(4)^2 + 3(4)\right] - \left[(-1)^2 + 3(-1)\right]$$

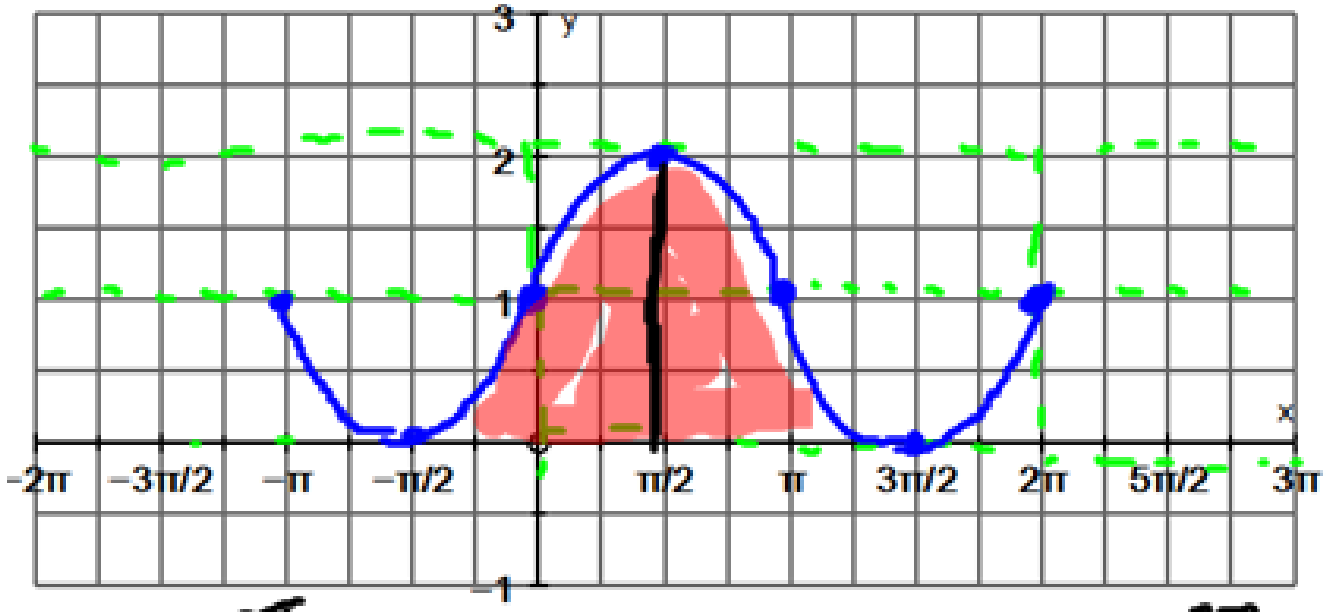
$$= (16 + 12) - (1 - 3)$$

$$= (28) - (-2)$$

$$= 30$$

Find the area enclosed by one arch of the curve $y = \sin x + 1$





VT+1 → SA: $y = 1$

Period $= 2\pi$

Amp 1

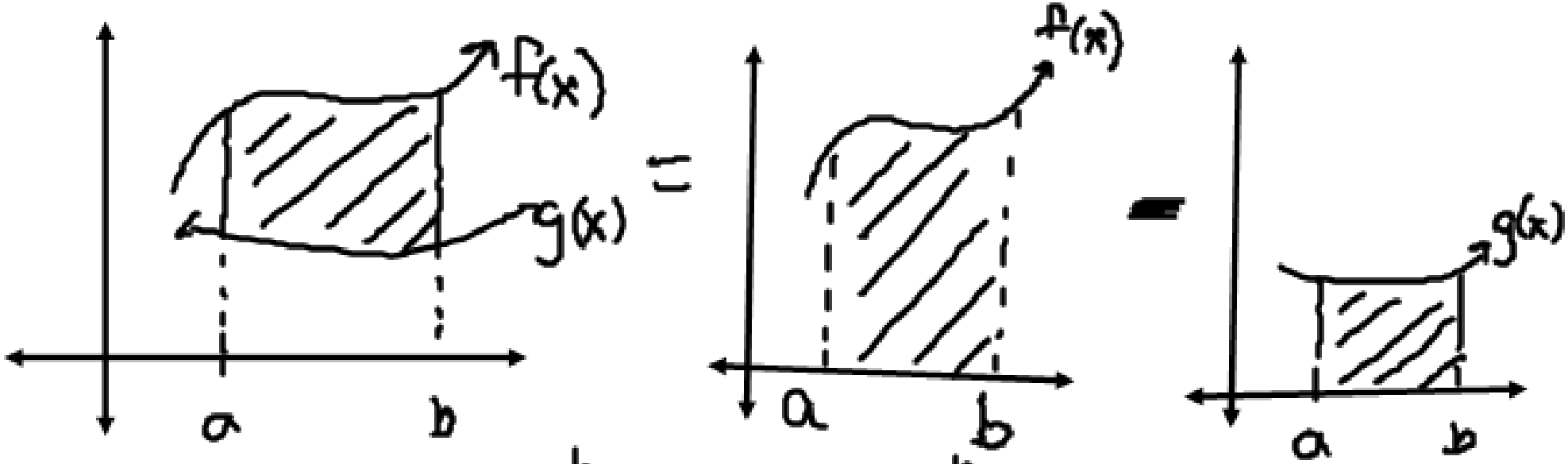
$$A = \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin x + 1)\,dx = -\cos x + x \Big]_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \left(-\cos\left(\tfrac{3\pi}{2}\right) + \tfrac{3\pi}{2}\right) - \left(-\cos\left(-\tfrac{\pi}{2}\right) + \left(-\tfrac{\pi}{2}\right)\right)$$

$$= \left(0 + \tfrac{3\pi}{2}\right) - \left(0 - \tfrac{\pi}{2}\right)$$

$$= \tfrac{3\pi}{2} + \tfrac{\pi}{2} = 2\pi$$

Ch 19B -Area Between TWO Functions Day 1

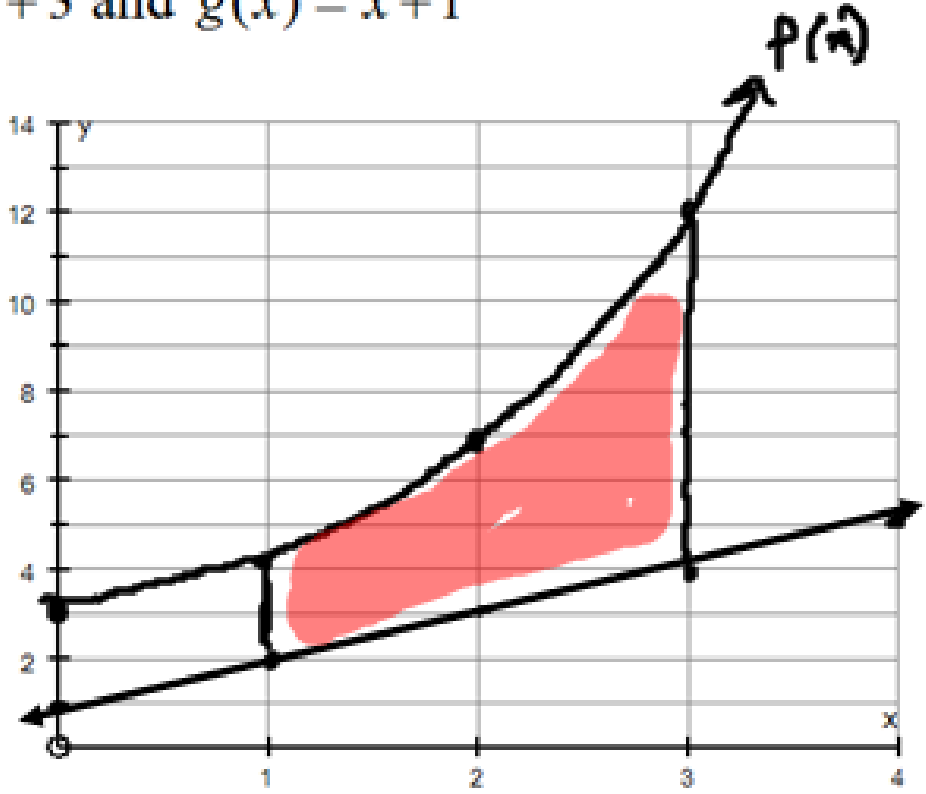


$$\text{Area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b \left(f(x) - g(x)\right)dx$$

$$= \int_a^b \left(y_{upper} - y_{lower}\right)dx$$

Example: Find the area between $f(x) = x^2 + 3$ and $g(x) = x + 1$ between $x = 1$ and $x = 3$.



$$A = \int_1^3 \left(f(x) - g(x)\right) dx$$

$$= \int_1^3 \left((x^2+3) - (x+1)\right) dx$$

$$= \int_1^3 (x^2 - x + 2)\, dx$$

$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \Big]_1^3$$

$$= \left(\frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 + 2(3)\right) - \left(\frac{1}{3} - \frac{1}{2} + 2\right) = \frac{26}{3}$$

Example: Find the area trapped between $f(x) = x^2$ and $g(x) = 3x - 2$.

① Find PoI

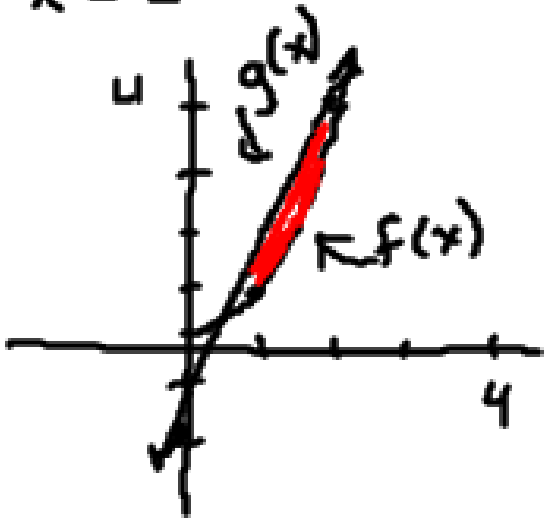
$f(x) = g(x)$

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$



$$A = \int_1^2 (g(x) - f(x))\,dx$$

$$= \int_1^2 ((3x-2) - (x^2))\,dx$$

$$= \int_1^2 (-x^2 + 3x - 2)\,dx$$

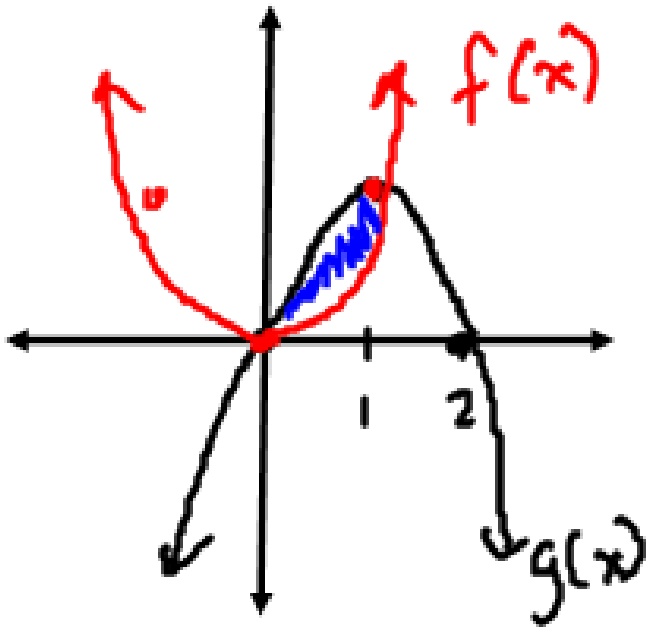
$$= -\frac{1}{3}x^3 + 3\left(\frac{1}{2}x^2\right) - 2(x)\Big]_1^2$$

$$= \left(-\frac{8}{3} + \frac{3}{2}(4) - 4\right) - \left(-\frac{1}{3} + \frac{3}{2} - 2\right)$$

$$= \frac{1}{6}$$

Example: Find the area trapped between $f(x) = x^2$ and $g(x) = 2x - x^2 = (x)(2-x)$ vs 1

x-int $(0,0)$ $(2,0)$



PGI

$2x - x^2 = x^2$

$2x - 2x^2 = 0$

$2x(1-x) = 0$

$x = 0 \quad x = 1$

$$A = \int_0^1 (g(x) - f(x))\,dx$$

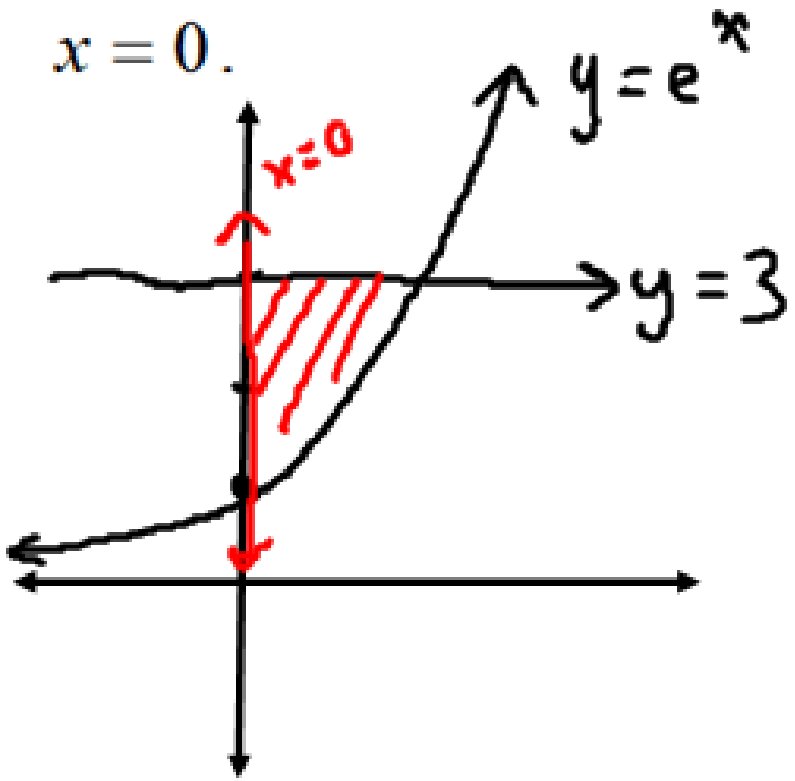
$$= \int_0^1 \left((2x - x^2) - (x^2)\right)dx$$

$$= \int_0^1 (2x - 2x^2)\,dx = 2\left(\frac{1}{2}x^2\right) - 2\left(\frac{1}{3}x^3\right)\Big]_0^1$$

$$= (1)^2 - \frac{2}{3}(1) - 0$$

$$= \frac{1}{3}$$

Example: Find the area trapped between $y = e^x$, $y = 3$, and $x = 0$.



PGI

$3 = e^x$

$x = \ln 3$

$$A = \int_0^{\ln 3} (3 - e^x)\,dx$$

$$= 3x - e^x \Big]_0^{\ln 3}$$

$$= (3\ln 3 - e^{\ln 3}) - (3(0) - e^0)$$

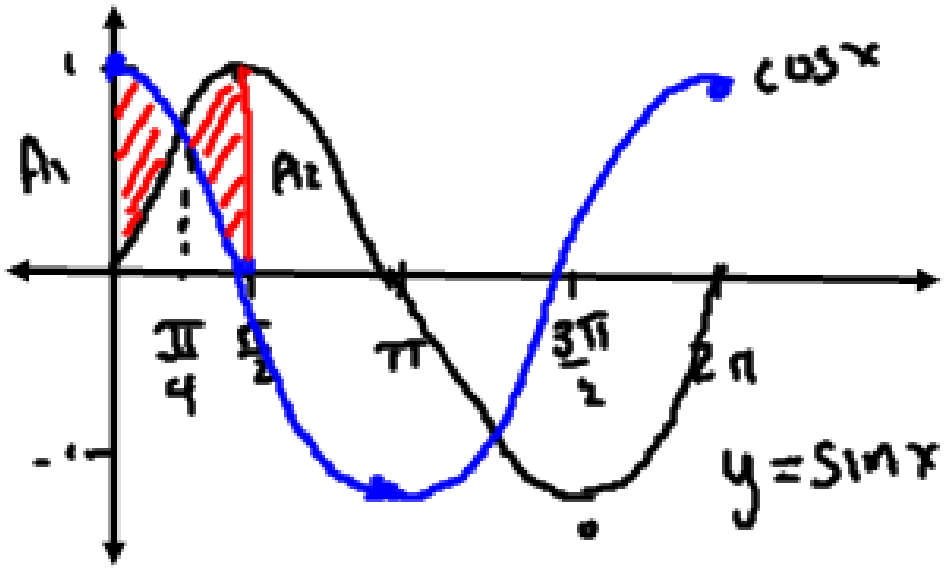
$$= 3\ln 3 - 3 + 1$$

$$= 3\ln 3 - 2$$

$a^{\log_a x} = x$

Example: Find the area of the region bounded by the curves

$y = \sin x,\ y = \cos x,\ x = 0,\ x = \frac{\pi}{2}.$



PG I

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}$

$A = A_1 + A_2$

$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$ $= 2\sqrt{2} - 1$

HW: ch 19 A # 1ac, 2def,3, 4bc

HW: ch 19 B # 2,3,4

HW: ch 19 A # 1ac, 2def,3, 4bc

HW: ch 19 B # 2,3,4