Chapter 19

Applications of integration

Syllabus reference: 6.5, 6.6

Contents:

Ch 19 A – Area under a curve… again

Use a graph of the integrand and areas to evaluate the integral:

$$\int_{-1}^{4} (2x+3)\,dx$$

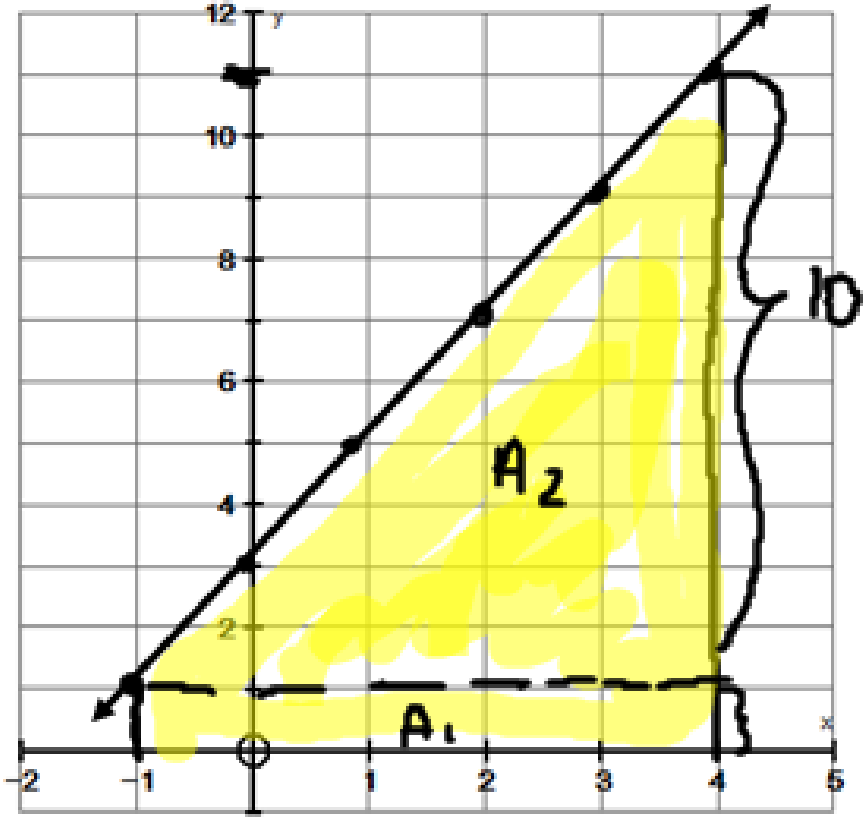
$A_{TOTAL} = A_1 + A_2$

$= l \cdot w + \frac{1}{2} b \cdot h$

$= (5)(1) + \frac{1}{2}(5)(10)$

$= 5 + 25$

$= 30$



$$A = \int_{-1}^{4} (2x+3)\,dx$$

$$= 2\left(\frac{1}{2}x^2\right) + 3x \Big]_{-1}^{4}$$

$$= x^2 + 3x \Big]_{-1}^{4}$$

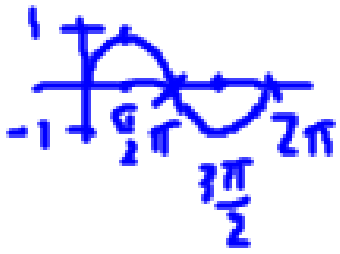
$$= \left[(4)^2 + 3(4)\right] - \left[(-1)^2 + 3(-1)\right]$$

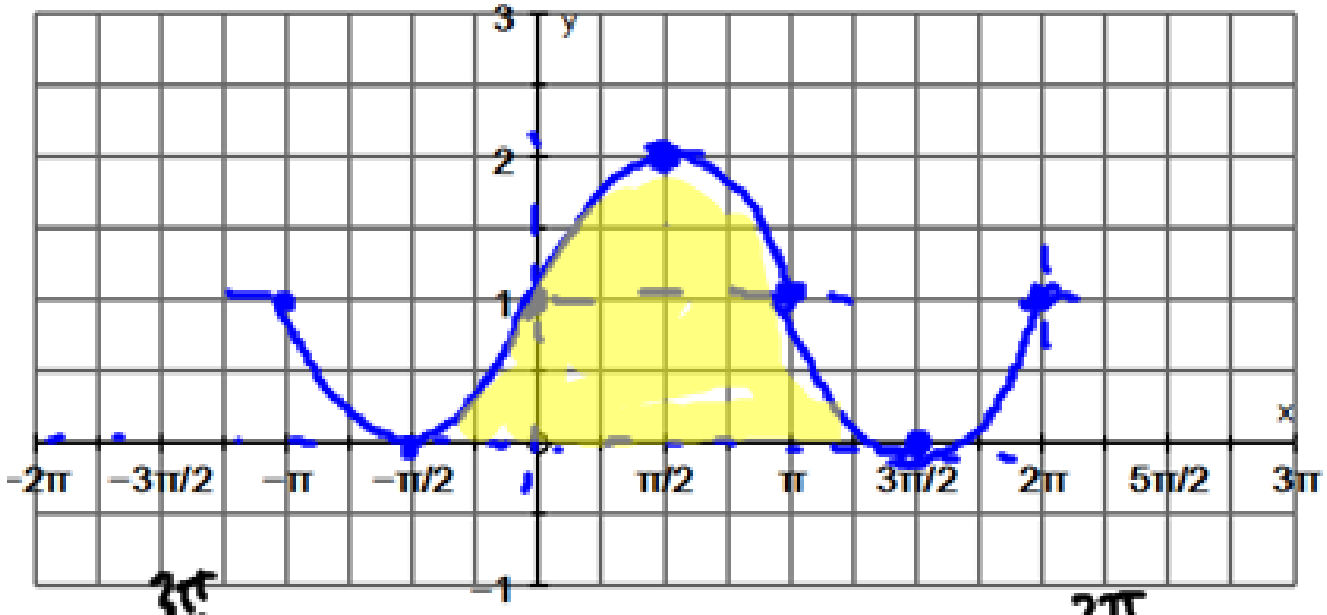
$$= (16+12) - (1-3)$$

$$= 28 - (-2)$$

$$= 30$$

Find the area enclosed by one arch of the curve $y = \sin x + 1$





VT+1 $\Rightarrow$ SA $y = 1$

Period 2π

Amp 1

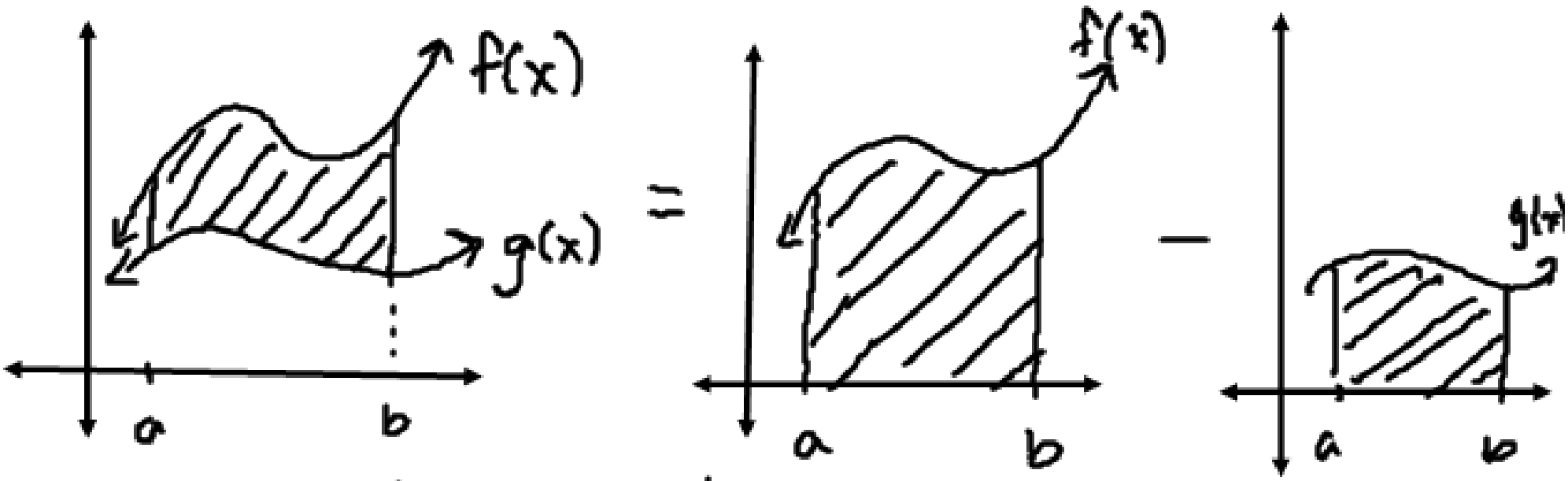
$$A = \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin x + 1)\,dx = -\cos x + x \Big]_{-\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \left(-\cos\left(\tfrac{3\pi}{2}\right) + \tfrac{3\pi}{2}\right) - \left(-\cos\left(-\tfrac{\pi}{2}\right) + \left(-\tfrac{\pi}{2}\right)\right)$$

$$= \left(0 + \tfrac{3\pi}{2}\right) - \left(0 - \tfrac{\pi}{2}\right)$$

$$= \tfrac{3\pi}{2} + \tfrac{\pi}{2} = \tfrac{4\pi}{2} = 2\pi$$

Ch 19B -Area Between TWO Functions Day 1

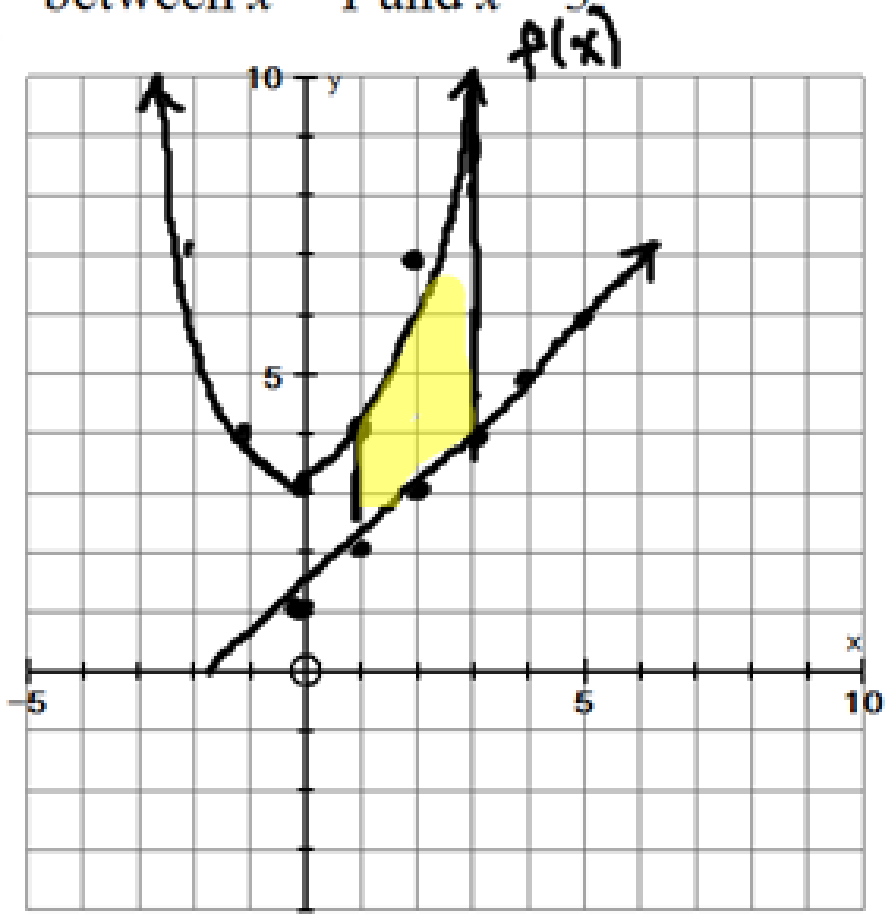


$$\text{Area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b \left(f(x) - g(x)\right)dx = \int_a^b \left(\begin{matrix}\text{upper}\\ \text{function}\end{matrix} - \begin{matrix}\text{lower}\\ \text{function}\end{matrix}\right)dx$$

Example: Find the area between $f(x) = x^2 + 3$ and $g(x) = x + 1$ between $x = 1$ and $x = 3$

upper ↑ $f(x)$ lower ↑ $g(x)$



$$A = \int_1^3 \left((x^2+3) - (x+1)\right) dx$$

$$= \int_1^3 (x^2 - x + 2)\, dx$$

$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x \Big]_1^3$$

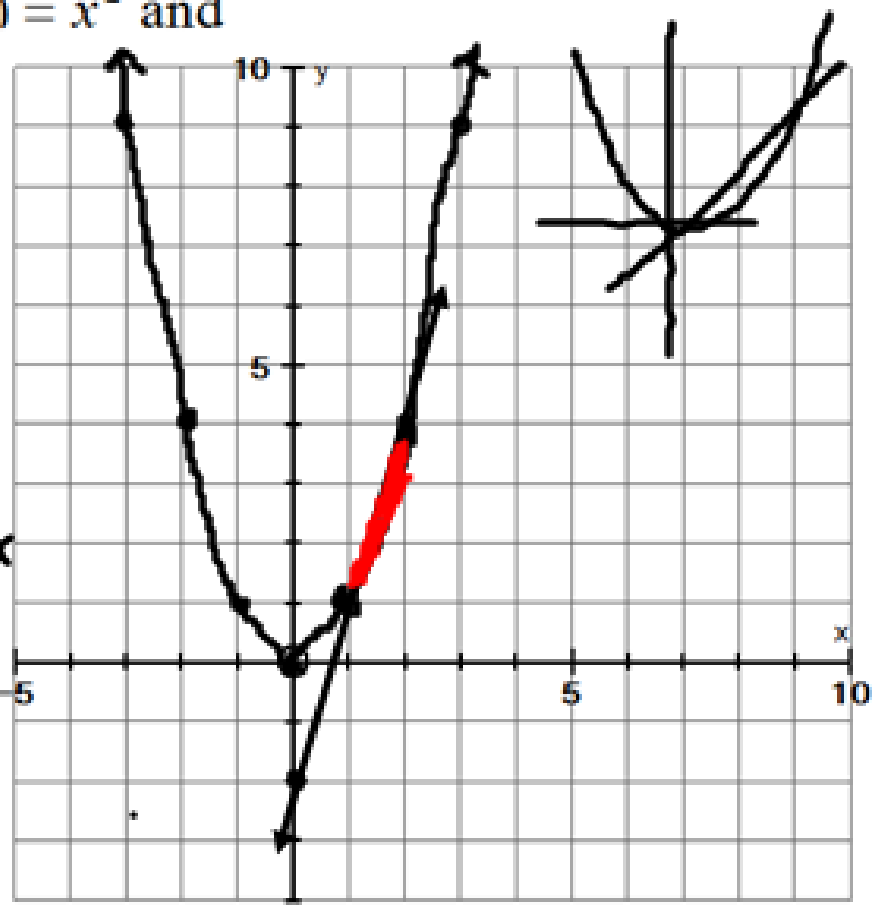
$$= \left[\frac{1}{3}(3)^3 - \frac{1}{2}(3)^2 + 2(3)\right] - \left[\frac{1}{3} - \frac{1}{2} + 2\right]$$

$$= \left(9 + 6 - \frac{9}{2}\right) - \left(-\frac{1}{6} + 2\right)$$

$$= 15 - \frac{9}{2} + \frac{1}{6} - 2$$

$$= 13 - 4.5 + \frac{1}{6} = \frac{26}{3}$$

Example: Find the area trapped between $f(x) = x^2$ and $g(x) = 3x - 2$.



① find POI

$f(x) = g(x)$

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

$$A = \int_1^2 (g(x) - f(x))\,dx$$

$$= \int_1^2 ((3x-2) - (x^2))\,dx$$

$$= \int_1^2 (-x^2 + 3x - 2)\,dx$$

$$A = -\frac{1}{3}x^3 + 3\left(\frac{1}{2}x^2\right) - 2(x)\Big]_1^2$$

$$= \left(-\frac{8}{3} + \frac{3}{2}(4) - 4\right) - \left(-\frac{1}{3} + \frac{3}{2} - 2\right)$$

$$= \frac{1}{6}$$

Example: Find the area trapped between $f(x) = x^2$ and $g(x) = 2x - x^2$.

x-int: $0 = x(2-x)$

$(0,0)$ $(2,0)$

POI: $x^2 = 2x - x^2$

$0 = 2x - 2x^2$

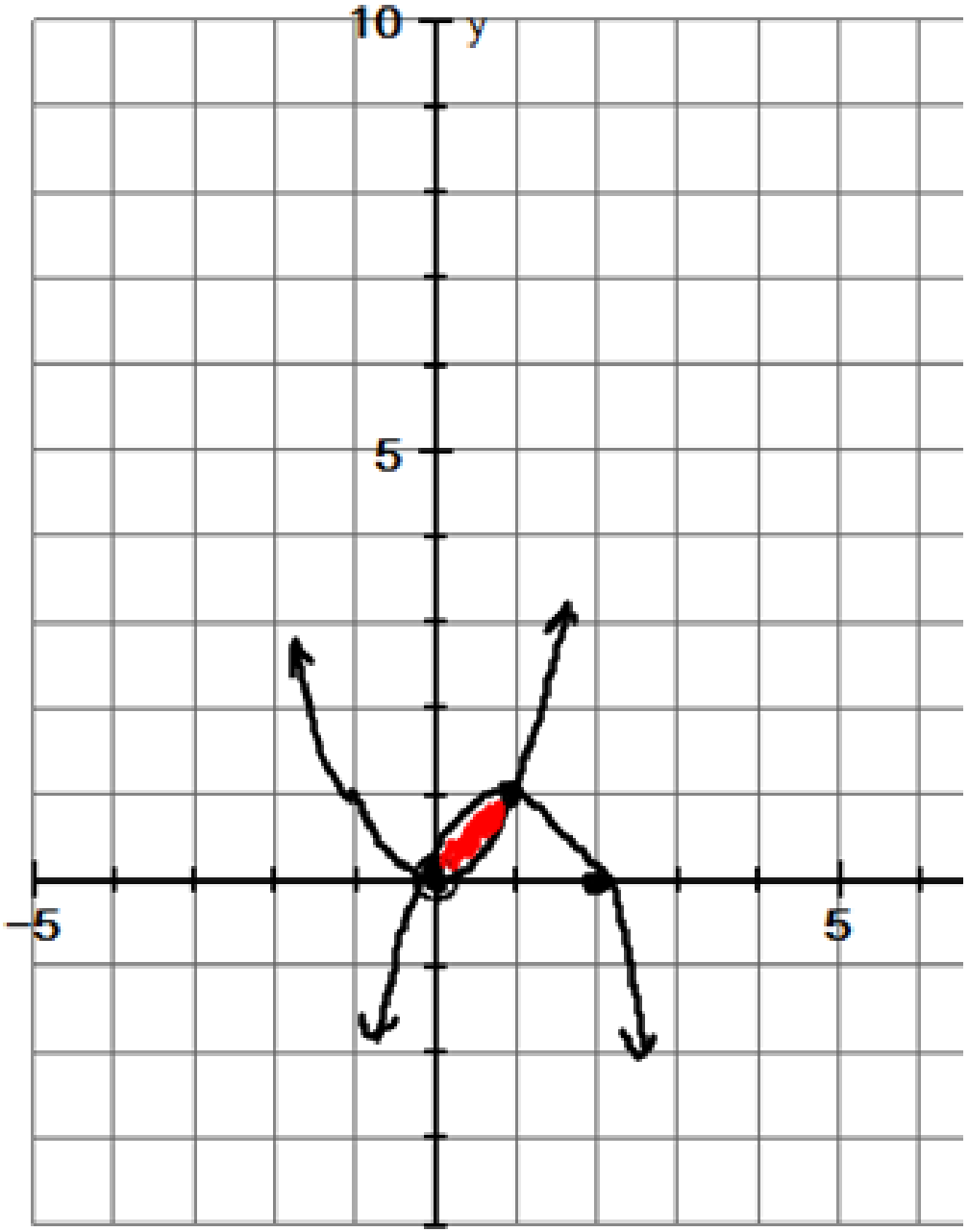
$0 = 2x(1-x)$

$x = 0$ $x = 1$

$$A = \int_0^1 (g(x) - f(x))\,dx$$

$$= \int_0^1 ((2x - x^2) - (x^2))\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx$$



$$A = \int_0^1 (2x - 2x^2)\,dx$$

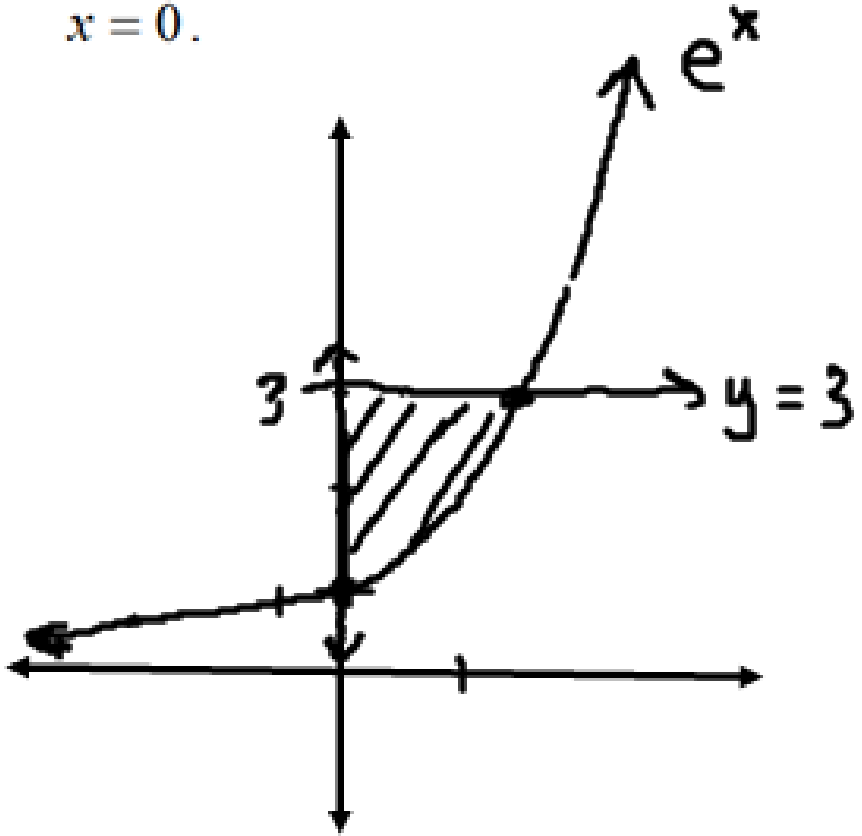
$$= 2\left(\frac{1}{2}x^2\right) - 2\left(\frac{1}{3}x^3\right)\Big]_0^1$$

$$= x^2 - \frac{2}{3}x^3\Big]_0^1$$

$$= \left(1 - \frac{2}{3}\right) - (0 - 0)$$

$$= \frac{1}{3}$$

Example: Find the area trapped between $y = e^x$, $y = 3$, and $x = 0$.



$$A = \int_0^{\ln 3} (3 - e^x)\,dx$$

$$= 3x - e^x \Big]_0^{\ln 3}$$

$$= (3\ln 3 - e^{\ln 3}) - (3(0) - e^0)$$

$$= (3\ln 3 - 3) - (0 - 1)$$

$$= 3\ln 3 - 2$$

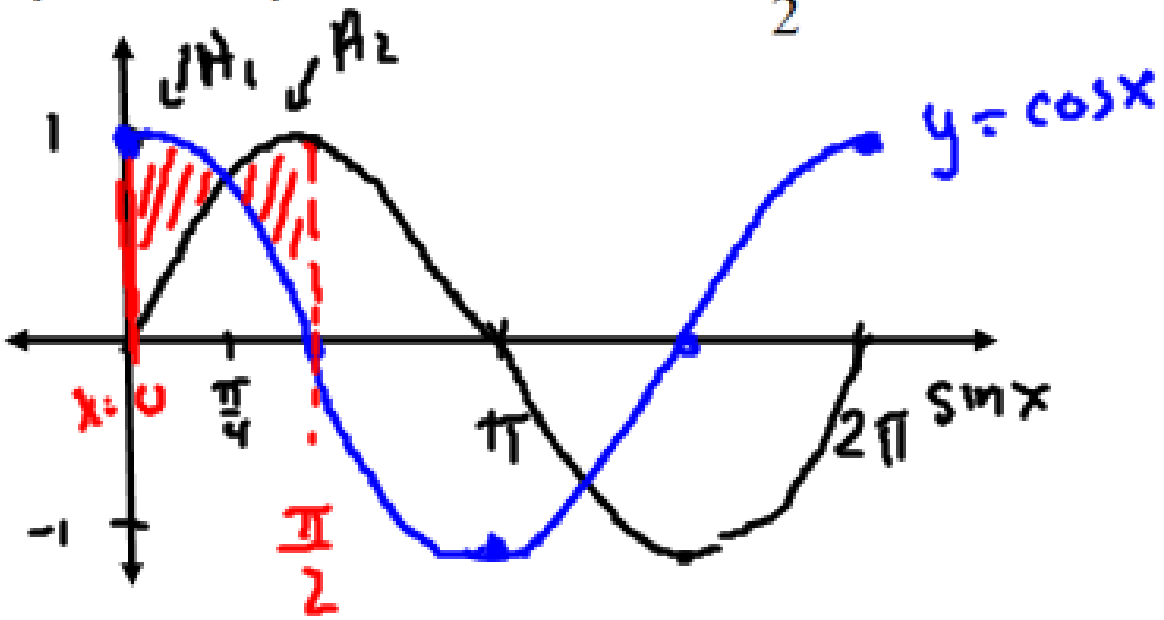
POI:

$e^x = 3$

$x = \ln 3$

Example: Find the area of the region bounded by the curves

$y = \sin x,\ y = \cos x,\ x = 0,\ x = \frac{\pi}{2}.$



$A = 2\sqrt{2} - 1$

$$A_T = A_1 + A_2$$

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\pi/2} (\sin x - \cos x)\,dx$$

HW: ch 19 A # 1ac, 2def,3, 4bc

HW: ch 19 B # 2,3,4