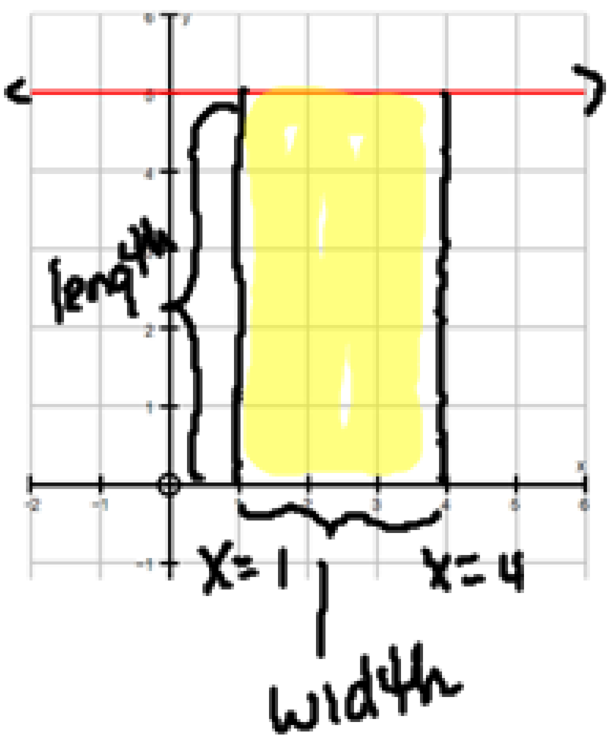
18A The Area Under a Curve

Consider the function $y = 5$. Find the area under the curve for $1 \le x \le 4$.



rectangle

$A = l \times w$

$= (5)(3)$

$4-1$

$= 15$

Now evaluate $\int_1^4 5\,dx$

$= 5x \Big]_1^4$

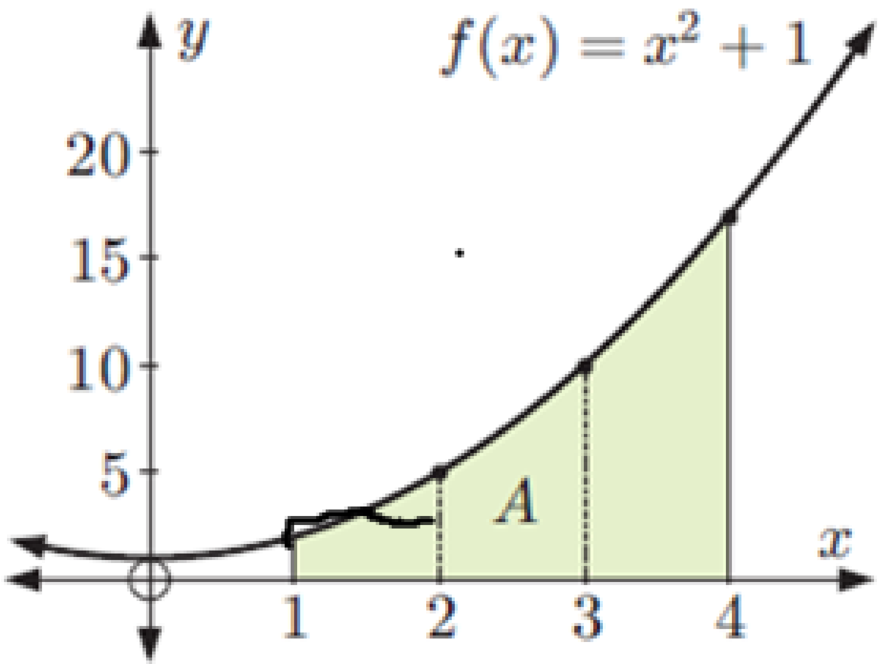
$= 5(4) - 5(1)$

$= 20 - 5$

$= 15$

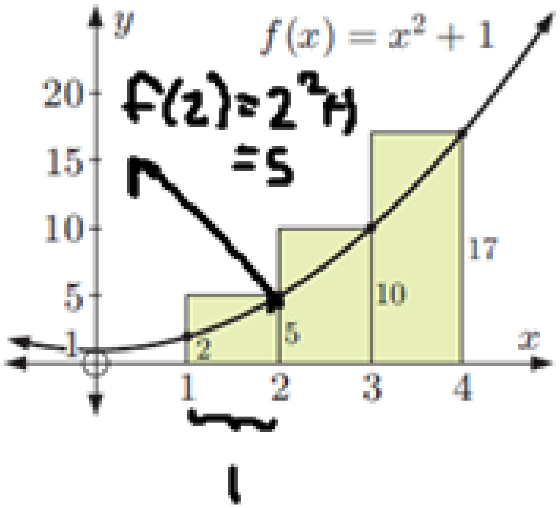
Next, consider the function $f(x) = x^2 + 1$

Want to estimate the area under the curve for $1 \le x \le 4$.
Divide the interval into three strips of equal width:



Extend these strips to make rectangles.

Upper Rectangles:



What is the area?

$A_T = A_1 + A_2 + A_3$

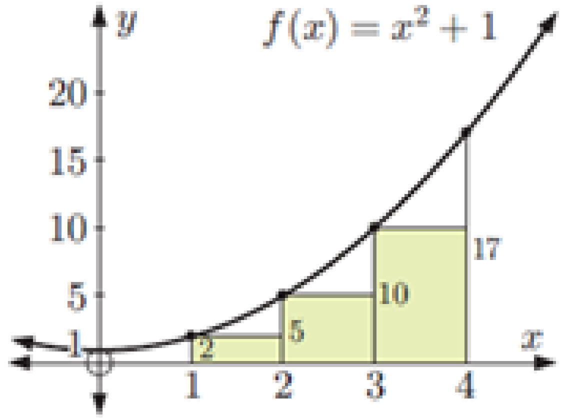
$= (1)(5) + (1)(10) + (1)(17)$

$= 5 + 10 + 17$

$= 32$

over estimate

Lower Rectangles:



What is the area?

$A_T = A_1 + A_2 + A_3$

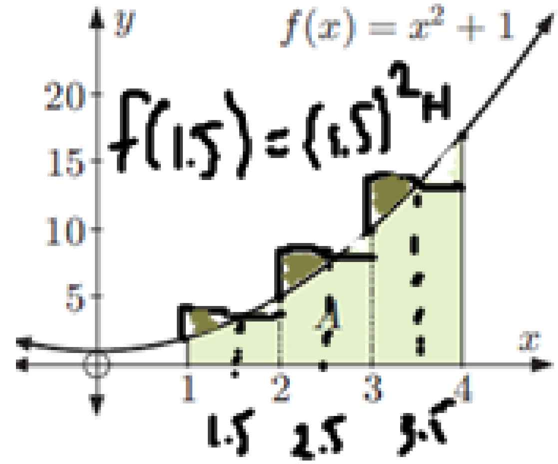
$= (1)(2) + (1)(5) + (1)(10)$

$= 2 + 5 + 10$

$= 17$

under estimate

Middle Rectangles



What is the area?

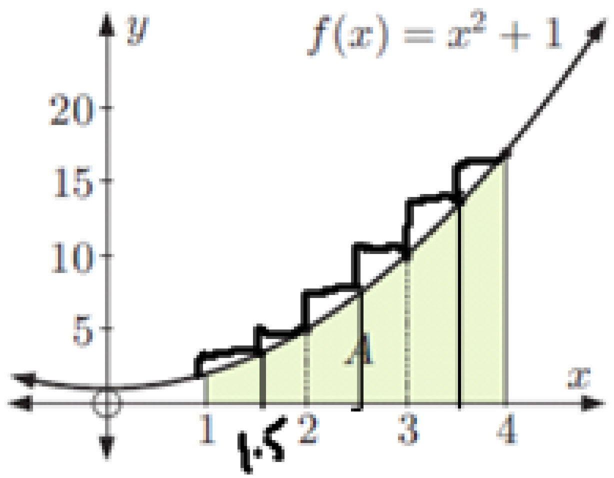
$A_T = A_1 + A_2 + A_3$

$= (1)(3.25) + (1)(7.25)$

$+ (1)(13.25)$

$= 23.75$

Now change the width of each rectangle to $\frac{1}{2}$, so there are 6 subintervals. Calculate the area of the upper and lower rectangles.



UPPER

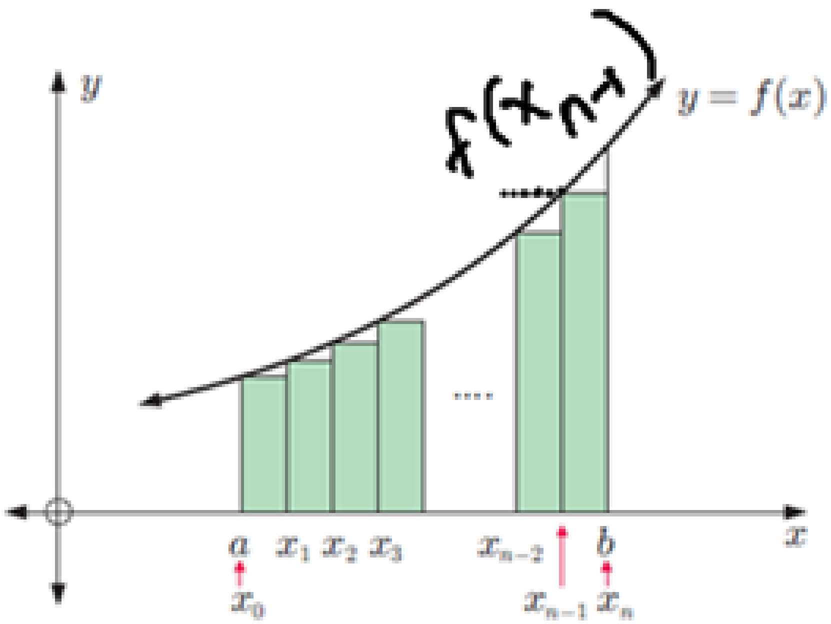
$A_T = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

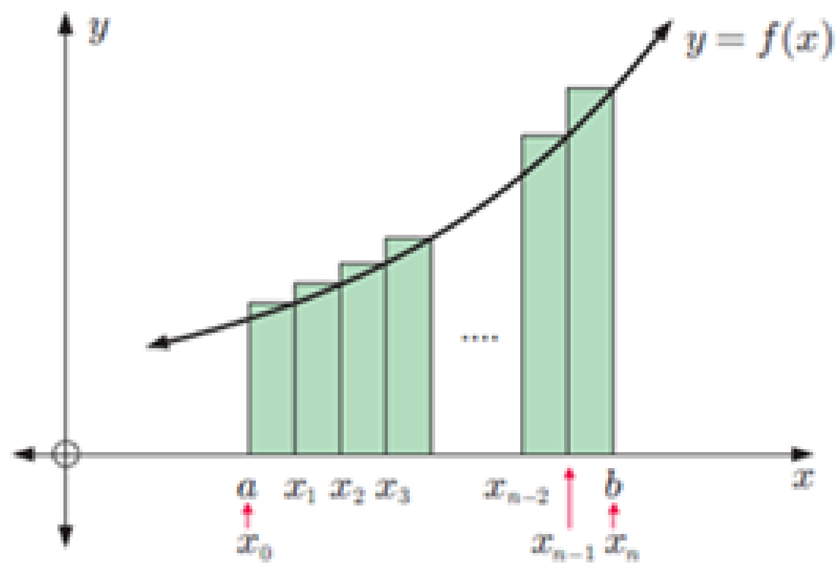
$= \left(\frac{1}{2}\right)(3.25) + \left(\frac{1}{2}\right)(5) + \dots$

As more subintervals are created, the values of the lower and upper rectangle areas become closer and will converge to the value of A, the area under the curve.

Consider the lower and upper rectangle sums for a function which is positive and increasing on the interval $a \le x \le b$. Divide the interval into n subintervals, each of width

$$w = \frac{b-a}{n}. \quad = \frac{4-1}{3} = 1$$





The area of the lower and upper rectangles are:

Upper Area

$$A_T = A_1 + A_2 + \dots + A_n$$
$$= wf(x_1) + wf(x_2) + \dots + wf(x_n)$$
$$= w[f(x_1) + f(x_2) + \dots + f(x_n)]$$
$$= w\sum_{i=1}^{n} f(x_i)$$

Lower Area

$$A_T = A_1 + A_2 + \dots + A_n$$
$$= wf(x_0) + wf(x_1) + \dots + wf(x_{n-1})$$
$$= w[f(x_0) + f(x_1) + \dots + f(x_{n-1})]$$
$$= w\sum_{i=0}^{n-1} f(x_i)$$

Find:

$$A_U - A_L = w\sum_{i=1}^{n} f(x_i) - w\sum_{i=0}^{n-1} f(x_i)$$
$$= w[f(x_n) - f(x_0)] = \frac{b-a}{n}[f(b) - f(a)]$$

$$\lim_{n\to\infty}(A_u - A_L) = \lim_{n\to\infty} \frac{b-a}{n}\left(f(b)-f(a)\right)$$

$$= 0$$

$$A_U - A_L = 0 \implies A_U = A_L$$

$$A = \lim_{n\to\infty} A_U$$

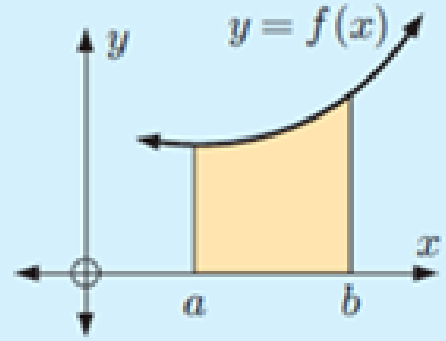
$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, w$$

width of the rectangle

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \int_a^b f(x)\,dx$$

Fundamental Theorem of Calculus (18C)

If $f(x)$ is a continuous positive function on an interval $a \leqslant x \leqslant b$ then the area under the curve between $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$.



For a continuous function $f(x)$ with antiderivative $F(x)$, $\int_a^b f(x)\,dx = F(b) - F(a)$.

3 Use the fundamental theorem of calculus to find the area between the x-axis and:

Pg 453

a $y = x^3$ from $x = 1$ to $x = 2$

$$A = \int_1^2 \cancel{x}^3 \, dx$$

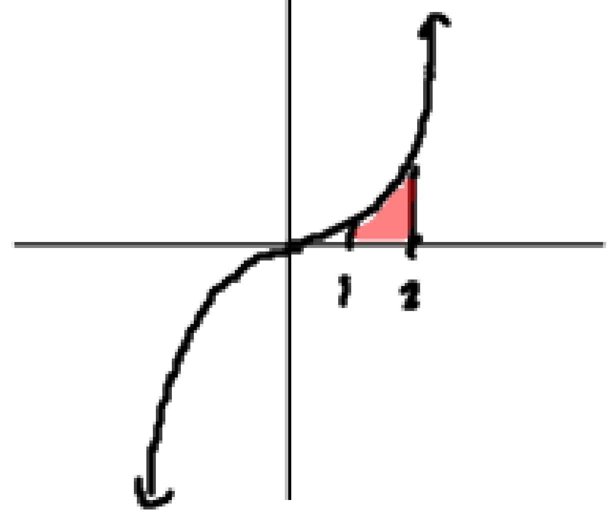
$$= \frac{1}{4}x^4 \Big]_1^2$$

$$= \frac{1}{4}(2)^4 - \frac{1}{4}(1)^4$$

$$= \frac{16}{4} - \frac{1}{4}$$

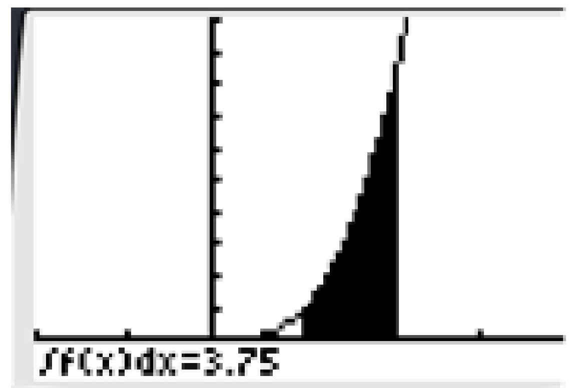
$$= \frac{15}{4} \quad (3.75)$$

$\boxed{\text{math}}$ $\boxed{9}$



$\boxed{\text{2nd}}$ $\boxed{\text{trace}}$

$\boxed{7}$



e $y = \frac{1}{\sqrt{x}}$ from $x = 1$ to $x = 4$

$A = \int_1^4 \frac{1}{\sqrt{x}} dx$

$= \int_1^4 (x)^{-\frac{1}{2}} dx$

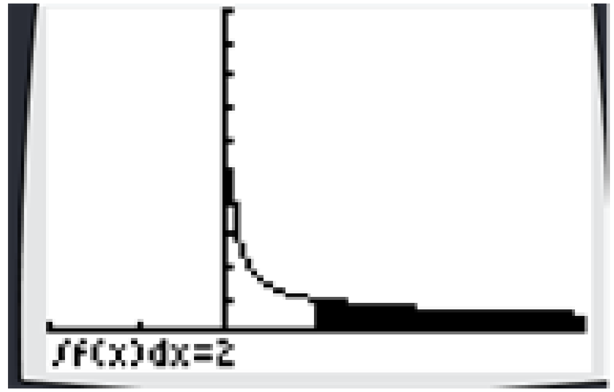
$= 2x^{\frac{1}{2}} \Big]_1^4$

$= 2\sqrt{x} \Big]_1^4$

$= 2\sqrt{4} - 2\sqrt{1}$

$= 4 - 2$

$= 2$



HW pg 453 ch18C

1, 3, 4, 5,