

**Chapter**

**17**

# **Applications of differential calculus**

**Syllabus reference: 6.3, 6.6**

**Contents:**

- A** Kinematics
- B** Rates of change
- C** Optimisation



## Ch 17 A – Kinematics (Motion)

Displacement:  $s(t)$

- position of an object from an origin, O, as a function of time.

When given an origin we know:

$s(t) = 0$  when a point P is located at the origin.

$s(t) < 0$  when a point P is located to the left of the origin.

$s(t) > 0$  when a point P is located to the right of the origin.

**Average Velocity:** how long it takes to go a certain distance. It does not look at the details between the start and end times

$$\text{Average Velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

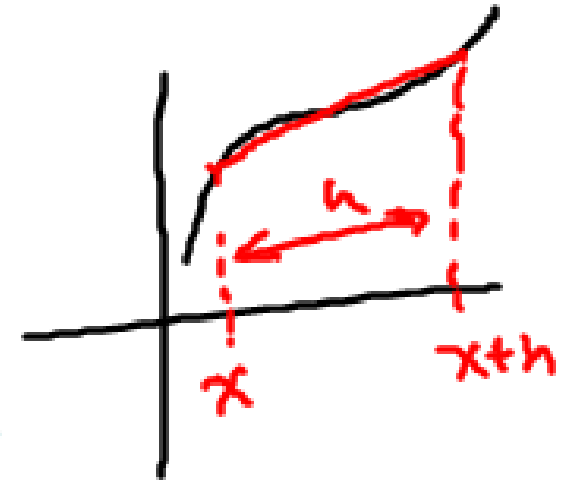


**Instantaneous Velocity:** What your velocity is at a particular moment in time

$$\text{Instantaneous Velocity} = s'(t) = \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h}$$

Average Acceleration: how much your velocity changes over time.

$$\text{Average Acceleration} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$$



Instantaneous Acceleration: What your acceleration is at a particular moment in time

$$\text{Instantaneous Acceleration} = v'(t) = s''(t) = \lim_{h \rightarrow 0} \frac{v(t+h) - v(t)}{h}$$

Example: A particle moves according to the displacement function  $f(t) = t^2 - 6t + 9$  metres, where  $t =$  seconds.

$$(t-3)^2$$



A) Find the average velocity for the time interval from  $t = 2$  to  $t = 5$  seconds.

$$v_{\text{AVE}} = \frac{f(5) - f(2)}{5 - 2}$$

$$v_{\text{AVE}} = \frac{[(5)^2 - 6(5) + 9] - [(2)^2 - 6(2) + 9]}{3}$$

$$= \frac{(25 - 30) - (4 - 12)}{3}$$

$$= \frac{(-5) - (-8)}{3}$$

$$= \frac{3}{3} = 1 \text{ m/s}$$

B) Find the velocity at time  $t$ .

Find  $f'(t)$

$$f'(t) = 2t - 6$$

C) What is the velocity at  $t = 2$  s?

$$v(2) = f'(2) = 2(2) - 6 = -2 \text{ m/s}$$

D) When is the particle at rest?

$$v(t) = 0$$

$$0 = 2t - 6$$

$$3 = t \quad \text{at 3 seconds}$$

E) When is the particle moving backwards?

Sign diagram  $v(t)$



$$t \in (0, 3)$$

time can't be negative 😊

$v(t)$	Interpretation
$= 0$	P is instantaneously at rest
$> 0$	P is moving to the right ←
$< 0$	P is moving to the left

F) Find the acceleration at time  $t$

$$a(t) = v'(t) = s''(t)$$

$$v(t) = 2t - 6$$

$$v'(t) = 2 \text{ m/s}^2$$

$$\text{ms}^{-2}$$

$a(t)$	<i>Interpretation</i>
$> 0$	velocity is increasing
$< 0$	velocity is decreasing
$= 0$	velocity may be a maximum or minimum or possibly constant

G) When is the particle decelerating?

$$a(t) = 2 \text{ m/s}^2 \leftarrow \text{constant}$$

never decelerating

Example: A dynamite blast propels a heavy rock straight up with a launch velocity of 160ft/sec. It reaches a height of

$$f(t) = 160t - 16t^2.$$

$$= -16t^2 + 160t$$

A) How high does the rock go?

local maximum, find  $f'(t) = 0$

$$f'(t) = 160 - 32t$$

$$0 = 160 - 32t$$

$$32t = 160$$

$$t = 5 \text{ sec}$$



$$f(5) = -16(5)^2 + 160(5) = 400 \text{ ft}$$

B) What is the velocity of the rock when it is 256 ft above the ground?

at what time is the rock at 256ft?

$$256 = 160t - 16t^2$$

$$t = 2 \text{ sec}$$

$$16t^2 - 160t + 256 = 0$$

$$t = 8 \text{ sec}$$

$$t^2 - 10t + 16 = 0$$

$$(t - 8)(t - 2) = 0$$

$$v(t) = f'(t) = -32t + 160$$

$$v(2) = -32(2) + 160 = 96 \text{ ft/sec}$$

$$v(8) = -32(8) + 160 = -96 \text{ ft/sec}$$



C) What is the acceleration function and what does it represent?

$$a(t) = v'(t)$$

$$v(t) = 160 - 32t$$

$$v'(t) = -32 \text{ ft/s}^2$$

Acceleration due to gravity

Do pg 417 #1-4 Read All of section 17A and do pg421 #1,2