Chapter 17

Applications of differential calculus

Syllabus reference: 6.3, 6.6

Contents:

Ch 17 A – Kinematics (Motion)

Displacement: $s(t)$

- position of an object from an origin, O, as a function of time.

When given an origin we know:

$s(t) = 0$ when a point P is located at the origin.

$s(t) < 0$ when a point P is located to the left of the origin.

$s(t) > 0$ when a point P is located to the right of the origin.

Average Velocity: how long it takes to go a certain distance. It does not look at the details between the start and end times

$$\text{Average Velocity} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Instantaneous Velocity: What your velocity is at a particular moment in time

$$\text{Instantaneous Velocity} = \text{s}'(\text{t}) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

Average Acceleration: how much your velocity changes over time.

$$\text{Average Acceleration} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}$$

Instantaneous Acceleration: What your acceleration is at a particular moment in time

$$\text{Instantaneous Accelearion} = \mathrm{v}'(\mathrm{t}) = s''(t) = \lim_{h \to 0} \frac{v(t+h) - v(t)}{h}$$

Example: A particle moves according to the displacement function $f(t) = t^2 - 6t + 9$ metres, where t = seconds.

A) Find the average velocity for the time interval from $t = 2$ to $t = 5$ seconds.

$$v_{Ave} = \frac{f(5) - f(2)}{5-2}$$

$$= \frac{[(5)^2 - 6(5) + \cancel{9}] - [(2)^2 - 6(2) + \cancel{9}]}{3}$$

$$= \frac{(25-30) - (4-12)}{3}$$

$$= \frac{(-5)-(-8)}{3}$$

$$v = \frac{3}{3} = 1 \text{ m/s}$$

B) Find the velocity at time t.

Find $f'(t)$

$$f'(t) = 2t - 6$$

C) What is the velocity at $t = 2$ s?

$$v(2) = f'(2) = 2(2) - 6$$

$$= +4 - 6$$

$$= -2 \text{ m/s}$$

D) When is the particle at rest?

$v(t)$	*Interpretation*
$= 0$	P is instantaneously at rest
> 0	P is moving to the right
< 0	P is moving to the left

$v(t) = f'(t) = 2t - 6$

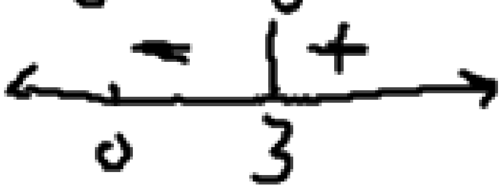
$0 = 2t - 6$

$3 = t$

E) When is the particle moving backwards?

$v(t) < 0$

sign diagram of $v(t)$



$t \in (0, 3)$

F)Find the acceleration at time t

$a(t)$	*Interpretation*
>0	velocity is increasing
<0	velocity is decreasing
$=0$	velocity may be a maximum or minimum or possibly constant

$a(t) = v'(t) = s''(t)$

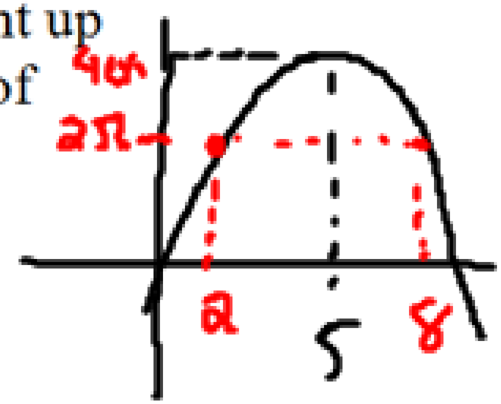
$v(t) = 2t - 6$

$a(t) = 2\ m/s^2$

G)When is the particle decelerating?

never.

Example: A dynamite blast propels a heavy rock straight up with a launch velocity of 160ft/sec. It reaches a height of $f(t) = 160t - 16t^2$.



A) How high does the rock go?

local maximum, find $f'(t) = 0$

$f'(t) = 160 - 32t$

$0 = 160 - 32t$

$32t = 160 \rightarrow t = \frac{160}{32}$

$t = 5$ sec

$f(5) = 160(5) - 16(5)^2$

$= 400$ft

B) What is the velocity of the rock when it is 256 ft above the ground?

→ at what time is the rock at 256ft?

$256 = 160t - 16t^2$

$16t^2 - 160t + 256 = 0$

$t^2 - 10t + 16 = 0$

$(t-8)(t-2) = 0$

$t = 2$

$t = 8$

$v(2) = 160 - 32(2)$

$= 96$ ft/sec

$v(8) = 160 - 32(8)$

$= -96$ ft/sec

C) What is the acceleration function and what does it represent?

$a(t) = v'(t) = s''(t)$

$v(t) = 160 - 32t$

$v'(t) = -32$

$a(t) = -32 \text{ ft/sec}^2$

acceleration due to gravity

Do pg 417 #1-4 Read All of section 17A and do pg421 #1,2