

Ch 17A.2

Velocity – Has magnitude and direction (vector)

Speed- measures how fast you are moving (no direction) (scalar)

Given a point P with displacement $s(t)$, then:

- If $v(t) > 0$ and $a(t) > 0$ then the speed is increasing
- If $v(t) < 0$ and $a(t) < 0$ then the speed is increasing
- If $v(t) > 0$ and $a(t) < 0$ then the speed is decreasing
- If $v(t) < 0$ and $a(t) > 0$ then the speed is decreasing

If the signs of $v(t)$ and $a(t)$ are the same - speed increasing
If the signs of $v(t)$ and $a(t)$ are different - speed decreasing

- 1 An object moves in a straight line with position given by $s(t) = t^2 - 4t + 3$ cm from O, where t is in seconds, $t \geq 0$.
- Find expressions for the object's velocity and acceleration, and draw sign diagrams for each function.
 - Find the initial conditions and explain what is happening to the object at that instant.
 - Describe the motion of the object at time $t = 2$ seconds.
 - At what time does the object reverse direction? Find the position of the object at this instant.
 - Draw a motion diagram for the object.
 - For what time intervals is the speed of the object decreasing?

A) $v(t) = s'(t) = 2t - 4$ $a(t) = v'(t) = 2$

$0 = 2t - 4$ $a(t) = 2$

$t = 2$

B) initial conditions mean $t = 0$

$s(0) = 3\text{cm}$
 $v(0) = -4\text{cm/s}$
 $a(0) = 2\text{cm/s}^2$

The object is 3cm to the right of the origin moving to the left and slowing down.

- c Describe the motion of the object at time $t = 2$ seconds.
- d At what time does the object reverse direction? Find the position of the object at this instant.
- e Draw a motion diagram for the object.
- f For what time intervals is the speed of the object decreasing?

c)

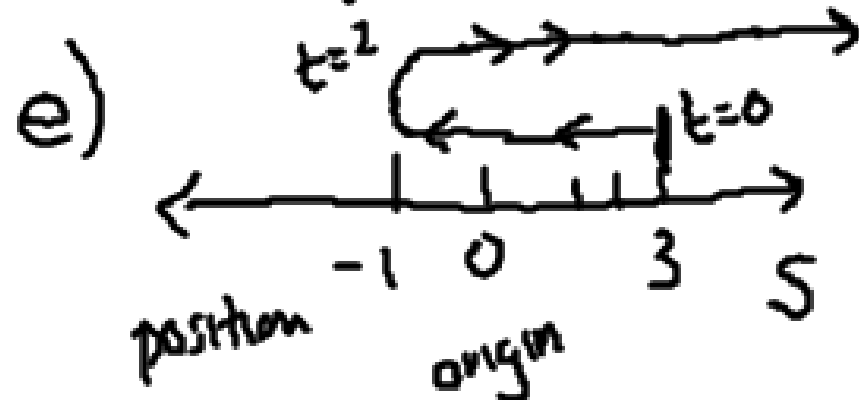
$$s(2) = -1 \text{ cm}$$

$$v(2) = 0 \text{ cm/s}$$

$$a(2) = 2 \text{ cm/s}^2$$

the object is 1 cm to the left of the origin, instantaneously stationary (turning around) accelerating to the right

d) object reverses direction when $v(t) = 0$, at $t = 2$ when object is 1 cm to the left of origin



f) speed decreasing : $0 \leq t \leq 2$
 increasing : $t \geq 2$

Ch 17 B – Rates of Change

There are many examples in the real world where quantities vary with respect to time *or some other quantity*

- Displacement
- Temperature
- Height
- Prices

All means the slope of the tangent line for some function $f(x)$

Example: pg 424

1 The estimated future profits of a small business are given by $P(t) = 2t^2 - 12t + 118$ thousand dollars, where t is the time in years from now.

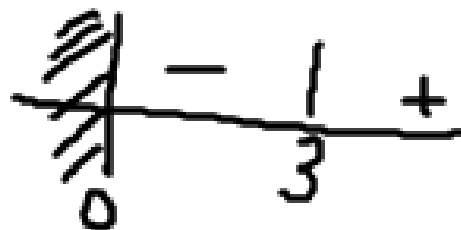
- a What is the current annual profit?
- b Find $\frac{dP}{dt}$ and state its units.
- c Explain the significance of $\frac{dP}{dt}$.
- d For what values of t will the profit:
 - i decrease
 - ii increaseon the previous year?
- e What is the minimum profit and when does it occur?
- f Find $\frac{dP}{dt}$ when $t = 4, 10$ and 25 . What do these figures represent?

a) $P(0) = 118$
Profit = \$118,000

b) $\frac{dP}{dt} = 4t - 12$ \$/yr

c) $\frac{dP}{dt}$ is the rate of change of the profit with time

d) $\frac{dP}{dt} = 0$ $0 = 4t - 12$
 $t = 3$ dec. $t \in (0, 3)$



inc: $t \in (3, \infty)$

Profit inc:

e) min profit happens at $t = 3$

$$P(3) = 2(3)^2 - 12(3) + 118$$

$$= 100$$

min Profit \$100,000

e What is the minimum profit and when does it occur?

f Find $\frac{dP}{dt}$ when $t = 4, 10$ and 25 . What do these figures represent?

$$\frac{dP}{dt} \quad \underline{at \ t = 4}$$

$$\frac{dP}{dt} = 4t - 12$$

$$= 4(4) - 12$$
$$= 4$$

Our profit is increasing
4000/yr after 4 yrs

$$\underline{t = 10}$$

$$\frac{dP}{dt} = 4(10) - 12$$
$$= 28$$

Profit is increasing
\$28000/yr after
10 yrs

$$\underline{t = 25}$$

$$\frac{dP}{dt} = 4(25) - 12$$
$$= 88$$

Profit is
increasing
\$88000/yr after
25 yrs

HW: ch 17A.2

#6, 7, 8

ch 17B

#2, 3, 4, 5