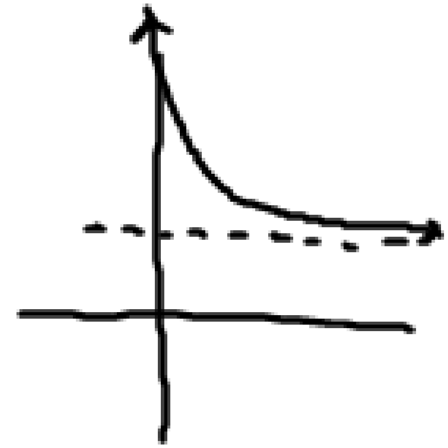
17B Continued... Pg 426

10 The temperature of a liquid after being placed in a refrigerator is given by $T = 5 + 95e^{-kt}$ °C where k is a positive constant and t is the time in minutes.

- **a** Find k if the temperature of the liquid is 20°C after 15 minutes.
- **b** What was the temperature of the liquid when it was first placed in the refrigerator?
- **c** Show that $\frac{dT}{dt} = c(T - 5)$ for some constant c. Find the value of c.
- **d** At what rate is the temperature changing at:
  - **i** $t = 0$ mins
  - **ii** $t = 10$ mins
  - **iii** $t = 20$ mins?



A) $T = 20 \quad t = 15$

$$20 = 5 + 95e^{-(k)(15)}$$

$$\frac{15}{95} = \frac{95e^{-15k}}{95}$$

$$\frac{3}{19} = e^{-15k}$$

$$\ln\left(\frac{3}{19}\right) = -15k$$

$$k = -\frac{1}{15}\ln\left(\frac{3}{19}\right)$$

$$K = \frac{1}{15}\ln\left(\left(\frac{3}{19}\right)^{-1}\right)$$

$$k = \frac{1}{15}\ln\left(\frac{19}{3}\right)$$

$$\approx 0.123$$

Power Rule

$$b \cdot \log_a x = \log_a(x^b)$$

Ch3 in the textbook

b What was the temperature of the liquid when it was first placed in the refrigerator?

c Show that $\frac{dT}{dt} = c(T - 5)$ for some constant c. Find the value of c.

d At what rate is the temperature changing at:

i $t = 0$ mins   ii $t = 10$ mins   iii $t = 20$ mins?

$T = 5 + 95e^{-kt}$

Chain Rule

B) $t = 0$

$T = 5 + 95e^{-k(0)}$

$T = 5 + 95(1)$

$T = 100°C$

c) $\frac{dT}{dt} = 0 + 95\left(-ke^{-kt}\right)$

$\frac{dT}{dt} = -95ke^{-kt}$

$= -k[\underbrace{95e^{-kt} + 5}_{T} \overbrace{-5}^{\text{fancy 2010}}]$

$\frac{dT}{dt} = -k[T - 5]$

$c = -k$

$c = -(0.123)$

$c = -0.123$

d At what rate is the temperature changing at:

i $t = 0$ mins ii $t = 10$ mins iii $t = 20$ mins?

at $t = 10$ min

$$\frac{dT}{dt} = -95ke^{-kt}$$

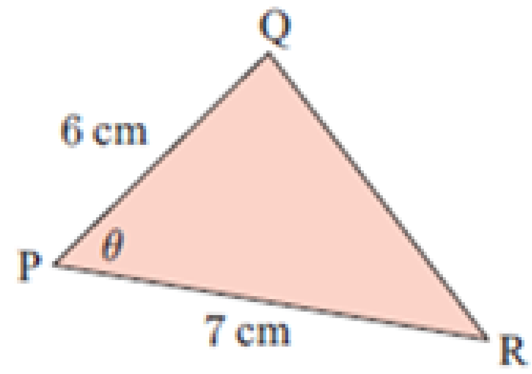
$$= -95(0.123)e^{-0.123(10)}$$

$$= -3.415$$

the temp in decreasing at a rate of 3.4°C/min

13 Find exactly the rate of change in the area of triangle PQR as θ changes, at the time when $\theta = 45°$.

$\hookrightarrow \frac{\pi}{4}$



Area of a triangle $A = \frac{1}{2}ab\sin C$

two side lengths and the enclosed angle.

$$A = \frac{1}{2}(6)(7)\sin\theta$$

$$A = 21\sin\theta \text{ cm}^2$$

θ must be in radians

$$\frac{dA}{d\theta} = 21(\cos\theta)$$

$$\frac{dA}{d\theta} = 21\cos\theta \text{ cm}^2/\text{radian}$$

$$= 21\cos\left(\frac{\pi}{4}\right)$$

$$= 21\left(\frac{\sqrt{2}}{2}\right)$$

Ch 17 C -Optimization

- To optimize somethings means to maximize or minimize some aspect of it.

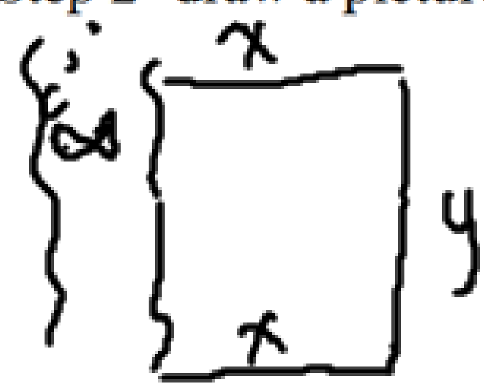
Example: A farmer has 2400ft of fencing and wants to fence off a rectangular field that boarders a straight river. He does not need fencing along the river. What are the dimensions of the field that has the largest area?

Without Calculus:

Step 1- Understand the problem

want maximized Area

Step 2- draw a picture



Step 3- Introduce notation

let x be the width

y be the length

Step 4- set up algebraic situation

Perimeter

$$2400 = 2x + y$$

$$y = 2400 - 2x$$

Area

$$A = l \times w$$

$$A = xy$$

make a substitution so we are only dealing with 1 variable.

$A = x \cdot y$

$= x(2400 - 2x)$

$A = -2x^2 + 2400x$

Step 5- solve the problem

max value occurs at the vertex $x = \frac{-b}{2a}$

$x = \frac{-2400}{2(-2)}$

$= \frac{2400}{4}$

$x = 600$

$y = 2400 - 2(600)$

$= 1200$

The demensions should be 600ft x 1200ft to obtain max Area.

Example: A farmer has 2400ft of fencing and wants to fence off a rectangular field that boarders a straight river. He does not need fencing along the river. What are the dimensions of the field that has the largest area?

With Calculus

→ 1^{st} four steps are the same

$A = -2x^2 + 2400x$

<u>Step 5</u> - solve → find $\frac{dA}{dx}$

$\frac{dA}{dx} = -4x + 2400$

$0 = -4x + 2400$

$4x = 2400$

$x = 600$

→ Sign diagram

$$\begin{array}{ccccc} & + & & - & \\ 0 & & 600 & & \end{array}$$

There is a local max at $x = 600$ to show that it is the global max look at concavity

$\frac{d^2A}{dx^2} = -4$ ← The function is concave down everywhere so $x = 600$ is the max

HW pg 426 (17b)

#11, 14

pg 431

#2, 4, 6 (17c)