$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$y = \ln(g(x))$$

$$y' = \frac{1}{g(x)} \cdot g'(x)$$

$$y = \ln(2x)$$

$$g(x) = 2x$$

$$g'(x) = 2$$

$$y' = \frac{1}{2x}(2)$$

$$= \frac{1}{x}$$

E) $f(x) = \ln(\ln(x))$

let $u = \ln x$

$u' = \frac{1}{x}$

$f(x) = \ln u$

$f'(x) = \frac{1}{u} \cdot u'$

$f'(x) = \left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right)$

F) $f(x) = \frac{x}{\ln(2x)}$

Quotient Rule

$u = x \qquad v = \ln(2x)$

$u' = 1 \qquad v' = \frac{1}{2x}(2) = \frac{1}{x}$

$f'(x) = \frac{vu' - uv'}{v^2}$

$f'(x) = \frac{(\ln(2x))(1) - (x)\left(\frac{1}{x}\right)}{(\ln(2x))^2}$

$= \frac{\ln(2x) - 1}{(\ln(2x))^2}$

G) $f(x) = \ln\left(\dfrac{x+1}{x-2}\right)$ $\rightarrow$ $f(x) = \ln(G(x))$

let $G(x) = \dfrac{x+1}{x-2}$

$u = x+1 \qquad v = x-2$

$u' = 1 \qquad v' = 1$

$$G'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2}$$

$$= \frac{-3}{(x-2)^2}$$

$$f'(x) = \frac{1}{G(x)} \cdot G'(x)$$

$$= \left(\frac{1}{\frac{x+1}{x-2}}\right)\left(\frac{-3}{(x-2)^2}\right)$$

$$= \left(\frac{\cancel{x-2}}{x+1}\right)\left(\frac{-3}{(x-2)^{\cancel{2}}}\right)$$

$$= \frac{-3}{(x+1)(x-2)}$$

H) $f(x) = \ln\left(x^2 (x^2 + 3)\right)$

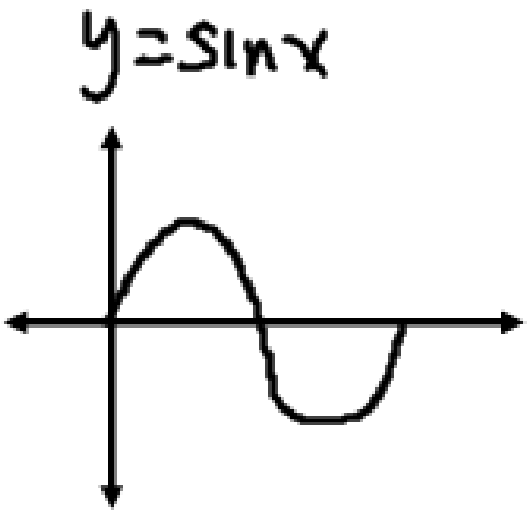
$$f(x) = \ln(x^2) + \ln(x^2+3)$$

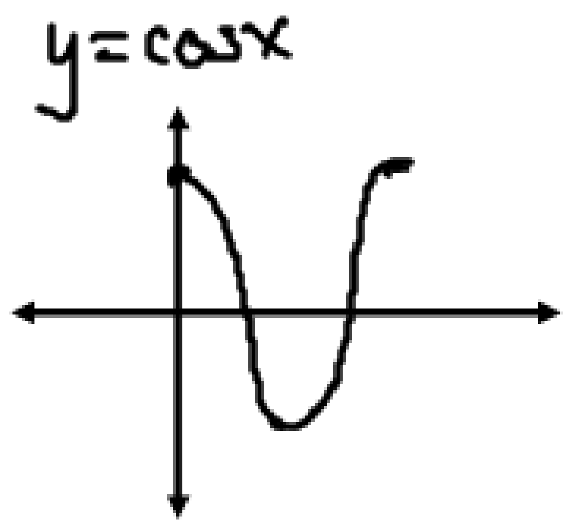
$$f(x) = \ln(x^4 + 3x^2)$$

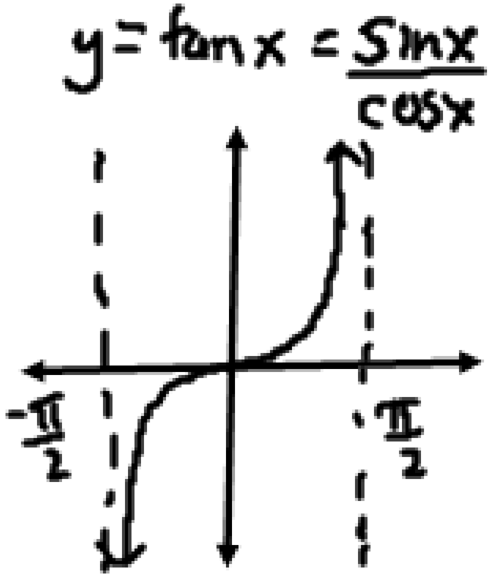
$$f'(x) = \left(\frac{1}{x^4+3x^2}\right)\left(4x^3+6x\right)$$

$$= \frac{4x^3+6x}{x^4+3x^2}$$

$$= \frac{\cancel{x}(4x^2+6)}{\cancel{x}(x^3+3x)} = \frac{4x^2+6}{x^3+3x}$$







15 G – Derivatives of Trigonometric Functions

Desmos:

https://www.desmos.com/calculator/qhapridsc0

For x in **radians**:

If $f(x) = \sin x$, then $f'(x) = \cos x$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

What about $y = \tan x$?

recall: $\tan x = \dfrac{\sin x}{\cos x}$

$\frac{d}{dx}(\tan x)$ use Quotient Rule

$u = \sin x$ $\quad v = \cos x$

$u' = \cos x$ $\quad v' = -\sin x$

$$\frac{d}{dx}(\tan x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

Function	*Derivative*
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\dfrac{1}{\cos^2 x}$

Function	*Derivative*
$\sin[f(x)]$	$\cos[f(x)]\, f'(x)$
$\cos[f(x)]$	$-\sin[f(x)]\, f'(x)$
$\tan[f(x)]$	$\dfrac{f'(x)}{\cos^2[f(x)]}$

Chain Rule!! :)

Examples:

Find the derivative of the following:

A) $y = \sin\left(3x^2 + 5\right)$

let $u = 3x^2 + 5$

$u' = 6x$

$y = \sin u$

$y' = \cos u \cdot u'$

$= \cos(3x^2+5)[6x]$

B) $y = x^2 \cos x$

Product Rule

$u = x^2 \qquad v = \cos x$

$u' = 2x \qquad v' = -\sin x$

$y' = uv' + vu'$

$= (x^2)(-\sin x) + (\cos x)(2x)$

C) $y=\left(x^2-\sin\left(3x^2\right)\right)^{10}$

let $u = x^2 - \sin(3x^2) \Rightarrow u' = 2x - \cos(3x^2)[6x]$

chain Rule

$y = u^{10}$

$y' = 10u^9 \cdot u'$

$y' = 10\left[x^2 - \sin(3x^2)\right]^9\left[2x - 6x\cos(3x^2)\right]$

D) $y = \cos^2\left(x^2 + 3x - 1\right)$

let $u = \cos(x^2+3x-1)$

$u' = -\sin(x^2+3x-1) \cdot (2x+3)$

$y = [\cos(x^2+3x-1)]^2$

$y' = 2[\cos(x^2+3x-1)][-\sin(x^2+3x-1)](2x+3)$

$y = u^2$

$y' = 2u \cdot \frac{du}{dx}$

E) $y = \dfrac{\tan(8x)}{x^2 + \cos(x^2+1)}$

Quotient Rule

$u = \tan(8x)$

↑ Chain Rule

$v = x^2 + \cos(x^2+1)$

↑ Chain Rule

$u' = \dfrac{1}{\cos^2(8x)}(8)$

$v' = 2x - \sin(x^2+1)\,[2x]$

$y' = \dfrac{vu' - uv'}{v^2}$

$$y' = \frac{\left[x^2 + \cos(x^2+1)\right]\left[\frac{8}{\cos^2(8x)}\right] - \left[\tan(8x)\right]\left[2x - 2x\sin(x^2+1)\right]}{\left(x^2 + \cos(x^2+1)\right)^2}$$

Find the gradient of the line tangent to the graph of $y = \sin x + 3$ at $x = \pi$

Step1 – find $\frac{dy}{dx}$

$\frac{dy}{dx} = \cos x$

Step 2 – find slope of the tangent at $x=\pi$

$m_T = \cos(\pi)$

$= -1$

Hw. pg 380 #1-4

EXERCISE 15G

1 Find $\dfrac{dy}{dx}$ for:

- **a** $y = \sin(2x)$
- **b** $y = \sin x + \cos x$
- **c** $y = \cos(3x) - \sin x$
- **d** $y = \sin(x+1)$
- **e** $y = \cos(3 - 2x)$
- **f** $y = \tan(5x)$
- **g** $y = \sin(\frac{x}{2}) - 3\cos x$
- **h** $y = 3\tan(\pi x)$
- **i** $y = 4\sin x - \cos(2x)$

2 Differentiate with respect to x:

- **a** $x^2 + \cos x$
- **b** $\tan x - 3\sin x$
- **c** $e^x \cos x$
- **d** $e^{-x}\sin x$
- **e** $\ln(\sin x)$
- **f** $e^{2x}\tan x$
- **g** $\sin(3x)$
- **h** $\cos(\frac{x}{2})$
- **i** $3\tan(2x)$
- **j** $x\cos x$
- **k** $\dfrac{\sin x}{x}$
- **l** $x\tan x$

3 Differentiate with respect to x:

- **a** $\sin(x^2)$
- **b** $\cos(\sqrt{x})$
- **c** $\sqrt{\cos x}$
- **d** $\sin^2 x$
- **e** $\cos^3 x$
- **f** $\cos x \sin(2x)$
- **g** $\cos(\cos x)$
- **h** $\cos^3(4x)$
- **i** $\dfrac{1}{\sin x}$
- **j** $\dfrac{1}{\cos(2x)}$
- **k** $\dfrac{2}{\sin^2(2x)}$
- **l** $\dfrac{8}{\tan^3(\frac{x}{2})}$

4 Find the gradient of the tangent to:

- **a** $f(x) = \sin^3 x$ at the point where $x = \frac{2\pi}{3}$
- **b** $f(x) = \cos x \sin x$ at the point where $x = \frac{\pi}{4}$.