

$$y = \ln x \quad y' = \frac{1}{x} \quad y = \ln u$$

$$y' = \frac{1}{u} \frac{du}{dx}$$

E)  $f(x) = \ln(\ln(x))$

$$f(x) = \ln u \quad u = \ln x$$

$$f'(x) = \frac{1}{u} \frac{du}{dx} \quad u' = \frac{1}{x}$$

$$= \left(\frac{1}{\ln x}\right) \left(\frac{1}{x}\right)$$

F)  $f(x) = \frac{x}{\ln(2x)}$

$$u = x \quad v = \ln(2x) \quad \leftarrow \text{Chain Rule}$$

$$u' = 1 \quad v' = \left(\frac{1}{2x}\right)(2)$$

$$f'(x) = \frac{v u' - u v'}{v^2}$$

$$f'(x) = \frac{(\ln(2x))(1) - (x)\left(\frac{1}{x}\right)}{[\ln(2x)]^2}$$

$$G) f(x) = \ln\left(\frac{x+1}{x-2}\right)$$

$$\rightarrow f(x) = \ln(G(x))$$

Inside is Quotient Rule

$$G(x) = \frac{x+1}{x-2} \quad u = x+1 \quad v = x-2 \\ u' = 1 \quad v' = 1$$

$$G'(x) = \frac{v u' - u v'}{v^2} \\ = \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2}$$

$$G'(x) = \frac{-3}{(x-2)^2}$$

$$f'(x) = \frac{1}{G(x)} \cdot G'(x)$$

$$= \frac{1}{\frac{x+1}{x-2}} \left( \frac{-3}{(x-2)^2} \right)$$

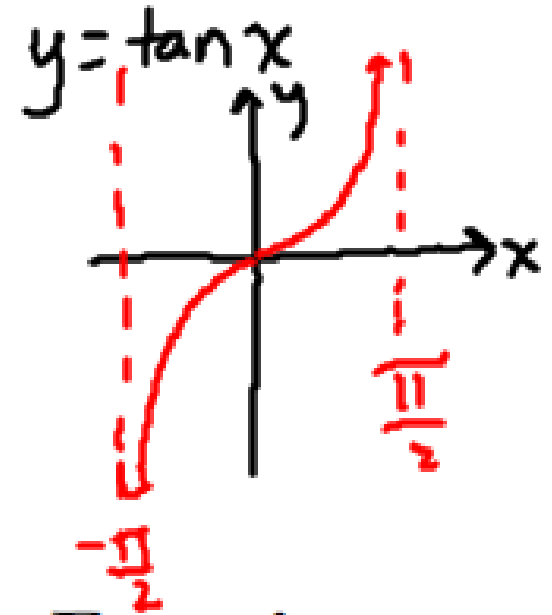
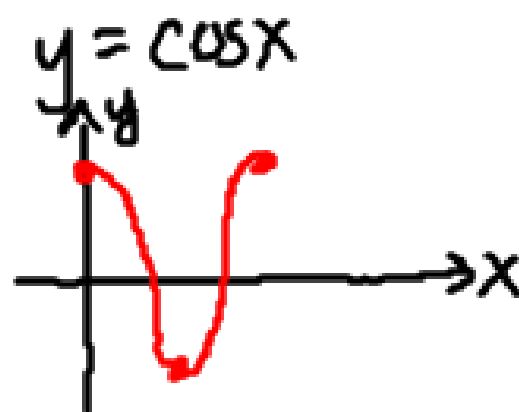
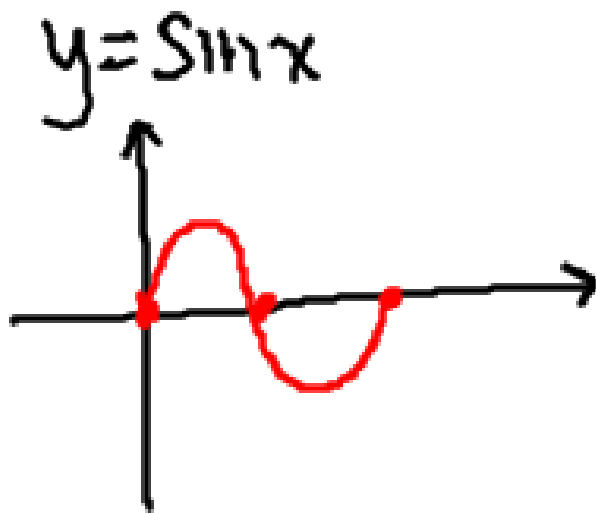
$$= \left( \frac{x-2}{x+1} \right) \left( \frac{-3}{(x-2)^2} \right)$$

$$= \frac{-3}{(x+1)(x-2)}$$

$$H) f(x) = \ln(x^2(x^2 + 3))$$

$$f(x) = \ln[x^4 + 3x^2]$$

$$f'(x) = \left( \frac{1}{x^4 + 3x^2} \right) (4x^3 + 6x)$$



## 15 G – Derivatives of Trigonometric Functions

Desmos:

<https://www.desmos.com/calculator/qhapridsc0>

For  $x$  in radians:

If  $f(x) = \sin x$ , then  $f'(x) = \cos x$

If  $f(x) = \cos x$ , then  $f'(x) = -\sin x$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x}$$

What about  $y = \tan x$ ?

recall  $\tan x = \frac{\sin x}{\cos x}$  ← Quotient Rule!

$$u = \sin x$$
$$u' = \cos x$$

$$v = \cos x$$
$$v' = -\sin x$$

$$\frac{d}{dx} (\tan x) = \frac{v u' - u v'}{v^2}$$
$$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

<i>Function</i>	<i>Derivative</i>
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$

<i>Function</i>	<i>Derivative</i>
$\sin[f(x)]$	$\cos[f(x)] f'(x)$
$\cos[f(x)]$	$-\sin[f(x)] f'(x)$
$\tan[f(x)]$	$\frac{f'(x)}{\cos^2[f(x)]}$

Examples:

Find the derivative of the following:

$$A) y = \sin(3x^2 + 5)$$

Chain Rule  $u = 3x^2 + 5$   
 $u' = 6x$

$$y = \sin u$$

$$y' = \cos(u) \cdot u'$$
$$= \cos(3x^2 + 5) [6x]$$

$$B) y = x^2 \cos x$$

Product Rule

$$u = x^2 \quad v = \cos x$$
$$u' = 2x \quad v' = -\sin x$$

$$y' = uv' + vu'$$
$$= (x^2)(-\sin x) + (\cos x)(2x)$$

$$C) y = (x^2 - \sin(3x^2))^{10}$$

$$\text{let } u = x^2 - \sin(3x^2)$$

$$u' = 2x - \cos(3x^2)(6x)$$

$$y = u^{10}$$

$$y' = 10u^9 \cdot u'$$

$$y' = 10 [x^2 - \sin(3x^2)]^9 [2x - 6x \cos(3x^2)]$$



$$D) y = \cos^2(x^2 + 3x - 1)$$

$$y = [\cos(x^2 + 3x - 1)]^2$$

$$\text{let } u = \underbrace{\cos(x^2 + 3x - 1)}_{\text{chain Rule}}$$

$$y' = 2[\cos(x^2 + 3x - 1)]^1 \cdot [-\sin(x^2 + 3x - 1)][2x + 3]$$

$$y = u^2$$

$$u' = -\sin(x^2 + 3x - 1) \cdot (2x + 3)$$

$$y' = 2u \cdot \frac{du}{dx}$$

$$E) y = \frac{\tan(8x)}{x^2 + \cos(x^2 + 1)}$$

Quotient Rule

$$u = \tan(8x)$$

$$v = x^2 + \cos(x^2 + 1)$$

$$u' = \frac{1}{\cos^2(8x)} (8)$$

$$v' = 2x - \sin(x^2 + 1) [2x]$$

$$y' = \frac{v u' - u v'}{v^2}$$

$$y' = \frac{(x^2 + \cos(x^2 + 1)) \left( \frac{8}{\cos^2(8x)} \right) - \tan(8x) (2x - 2x \sin(x^2 + 1))}{(x^2 + \cos(x^2 + 1))^2}$$

Find the gradient of the line tangent to the graph of  
 $y = \sin x + 3$  at  $x = \pi$

Step 1 - find  $\frac{dy}{dx}$

$$\frac{dy}{dx} = \cos x$$

Step 2 - find slope of tangent

$$m_T = \cos(\pi)$$

$$m_T = -1$$

HW: pg 380  
#1-4

## EXERCISE 15G

1 Find  $\frac{dy}{dx}$  for:

**a**  $y = \sin(2x)$

**b**  $y = \sin x + \cos x$

**c**  $y = \cos(3x) - \sin x$

**d**  $y = \sin(x + 1)$

**e**  $y = \cos(3 - 2x)$

**f**  $y = \tan(5x)$

**g**  $y = \sin\left(\frac{\pi}{2}\right) - 3 \cos x$

**h**  $y = 3 \tan(\pi x)$

**i**  $y = 4 \sin x - \cos(2x)$

2 Differentiate with respect to  $x$ :

**a**  $x^2 + \cos x$

**b**  $\tan x - 3 \sin x$

**c**  $e^x \cos x$

**d**  $e^{-x} \sin x$

**e**  $\ln(\sin x)$

**f**  $e^{2x} \tan x$

**g**  $\sin(3x)$

**h**  $\cos\left(\frac{\pi}{2}\right)$

**i**  $3 \tan(2x)$

**j**  $x \cos x$

**k**  $\frac{\sin x}{x}$

**l**  $x \tan x$

3 Differentiate with respect to  $x$ :

**a**  $\sin(x^2)$

**b**  $\cos(\sqrt{x})$

**c**  $\sqrt{\cos x}$

**d**  $\sin^2 x$

**e**  $\cos^3 x$

**f**  $\cos x \sin(2x)$

**g**  $\cos(\cos x)$

**h**  $\cos^3(4x)$

**i**  $\frac{1}{\sin x}$

**j**  $\frac{1}{\cos(2x)}$

**k**  $\frac{2}{\sin^2(2x)}$

**l**  $\frac{8}{\tan^3\left(\frac{\pi}{2}\right)}$

4 Find the gradient of the tangent to:

**a**  $f(x) = \sin^3 x$  at the point where  $x = \frac{2\pi}{3}$

**b**  $f(x) = \cos x \sin x$  at the point where  $x = \frac{\pi}{4}$ .

