Chapter 15 Wrap Up….

Pg 375 #6

derivative

(x, y)

6 The tangent to $f(x) = x^2e^{-x}$ at point P is horizontal. Find the possible coordinates of P.

$f'(x) = ?$

Product Rule

$u = x^2 \qquad v = e^{-x}$

$u' = 2x \qquad v' = -e^{-x}$

$f'(x) = uv' + vu'$

$= (x^2)(-e^{-x}) + (e^{-x})(2x)$

$0 = 2xe^{-x} - x^2e^{-x}$

$0 = xe^{-x}(2 - x)$

$\boxed{x = 0} \qquad e^{-x} = 0 \qquad 2 - x = 0$

$\boxed{x = 2}$

not possible

$f(0) = 0$

$(0, 0)$

$f(2) = (2)^2e^{-2}$

$= 4e^{-2}$

$(2, 4e^{-2})$

Pg 378 #5

$y = \ln u \qquad y' = \frac{1}{u} \cdot u'$

5 Suppose $f(x) = a\ln(2x+b)$ where $f(e) = 3$ and $f'(e) = \frac{6}{e}$. Find the constants a and b.

$x = e,\ y = 3$

$3 = a\ln(2e+b)$

$a = \frac{3}{\ln(2e+b)}$

$f'(x) = a\left(\frac{1}{2x+b}\right)(2)$

$f'(x) = \frac{2a}{2x+b}$ when $x = e,\ f'(e) = \frac{6}{e}$

$\frac{6}{e} = \frac{2a}{2e+b} \Rightarrow a = \frac{6(2e+b)}{2e}$

$a = \frac{3(2e+b)}{e}$

$a = a$

$\frac{3}{\ln(2e+b)} = \frac{3(2e+b)}{e} \Rightarrow \frac{e}{\ln(2e+b)} = 2e+b$

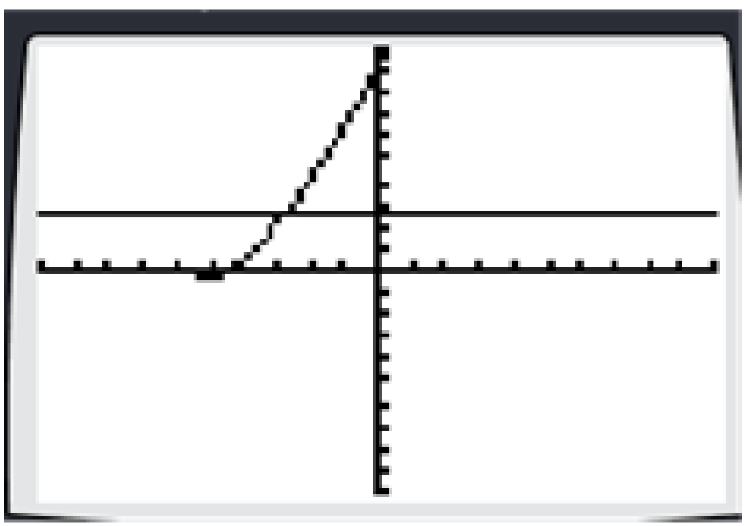
$$\frac{e}{\ln(2e+b)} = 2e+b$$

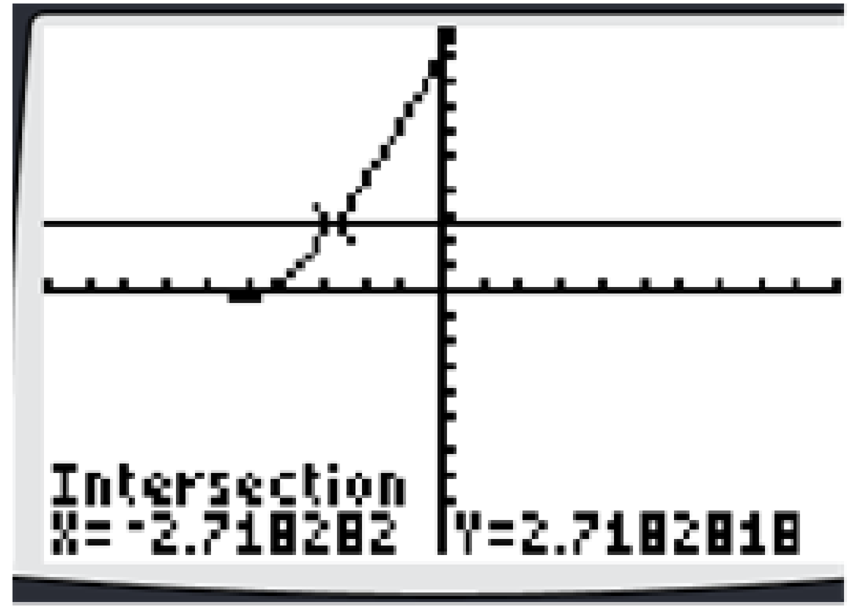
$$e = (2e+b)\cdot\ln(2e+b)$$

graphing technology :)

$y_1 = e$

$y_2 = (2e+x)\cdot\ln(2e+x)$





$x = -e$

$\therefore$ we know $\boxed{b = -e}$

$$a = \frac{3}{\ln(2e+b)}$$

$$a = \frac{3}{\ln(e)}$$

$$\boxed{a = 3}$$

Pg 382 #14

14 If $y = 2\sin x + 3\cos x$, show that $y'' + y = 0$ where y'' represents $\frac{d^2y}{dx^2}$

$y' = 2(\cos x) + 3(-\sin x)$

$y' = 2\cos x - 3\sin x$

$y'' = 2(-\sin x) - 3(\cos x)$

$y'' = -2\sin x - 3\cos x$

$y'' + y = (-2\sin x - 3\cos x) + (2\sin x + 3\cos x)$

$= -2\sin x + 2\sin x - 3\cos x + 3\cos x$

$= 0$

Review set 15C #9 pg384

9 The function f is defined by $f : x \mapsto -10\sin 2x \cos 2x$, $0 \leqslant x \leqslant \pi$.

a Write down an expression for $f(x)$ in the form $k \sin 4x$.

b Solve $f'(x) = 0$, giving exact answers.

trig identity:

$\sin 2\theta = 2\sin\theta\cos\theta$

$f(x) = -10\sin 2x \cos 2x$

let $\theta = 2x$

$= -10\sin\theta\cos\theta$

$= -5(2\sin\theta\cos\theta)$

$= -5\sin(2\theta)$

$f(x) = -5\sin(4x)$

$f'(x) = -5(\cos(4x)\cdot 4)$

$\frac{0}{-20} = \frac{-20\cos(4x)}{-20}$

$\cos 4x = 0$

$4x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi n, n \in I$

$$4x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi n, \; n \in I$$

Interval $0 \leq x \leq \pi$ $\frac{8\pi}{8}$

$$x = \begin{cases} \frac{\pi}{2}\left(\frac{1}{4}\right) \\ \frac{3\pi}{2}\left(\frac{1}{4}\right) \end{cases} + 2\pi n\left(\frac{1}{4}\right), \; n \in I$$

$$x = \begin{cases} \frac{\pi}{8}, \frac{5\pi}{8}, \\ \frac{3\pi}{8}, \frac{7\pi}{8}, \end{cases}$$

$\frac{4\pi}{8}$

$$x = \begin{cases} \frac{\pi}{8} \\ \frac{3\pi}{8} \end{cases} + \frac{\pi}{2} n, \; n \in I$$

General soln

at these angles the slope of the tangent line is zero.