Chapter 15 Wrap Up….

$y = e^x$ $\quad$ $y = e^u$

$y' = e^x$ $\quad$ $y' = e^u \cdot \frac{du}{dx}$

Pg 375 #6

6 The tangent to $f(x) = x^2e^{-x}$ at point P is horizontal. Find the possible coordinates of P.

Product Rule

$u = x^2 \quad v = e^{-x}$

$u' = 2x \quad v' = -e^{-x}$

$f'(x) = uv' + vu'$

$= (x^2)(-e^{-x}) + (e^{-x})(2x)$

$0 = 2xe^{-x} - x^2e^{-x}$

$0 = xe^{-x}(2-x)$

$\boxed{x=0}$ $\quad$ $e^{-x} = 0$ NOT POSSIBLE $\quad$ $2 - x = 0$ $\boxed{x=2}$

When $x=0$: $f(0) = 0^2(e^{-0})$

$= 0$

$(0,0)$

When $x=2$: $f(2) = (2)^2e^{-2}$

$= 4e^{-2}$

$(2, 4e^{-2})$

Pg 378 #5

5 Suppose $f(x) = a\ln(2x+b)$ where $f(e) = 3$ and $f'(e) = \frac{6}{e}$. Find the constants a and b.

$$f'(x) = a\left[\frac{1}{2x+b}\right]\left[2\right]$$

$$f'(x) = \frac{2a}{2x+b}$$

when $x=e$ $\quad f'(e) = \frac{6}{e}$

$$\frac{6}{e} = \frac{2a}{2e+b}$$

$$a = \frac{6(2e+b)}{2e}$$

$$a = \frac{3(2e+b)}{e}$$

when $x=e$, $f(e)=3$

$$3 = a\ln(2e+b)$$

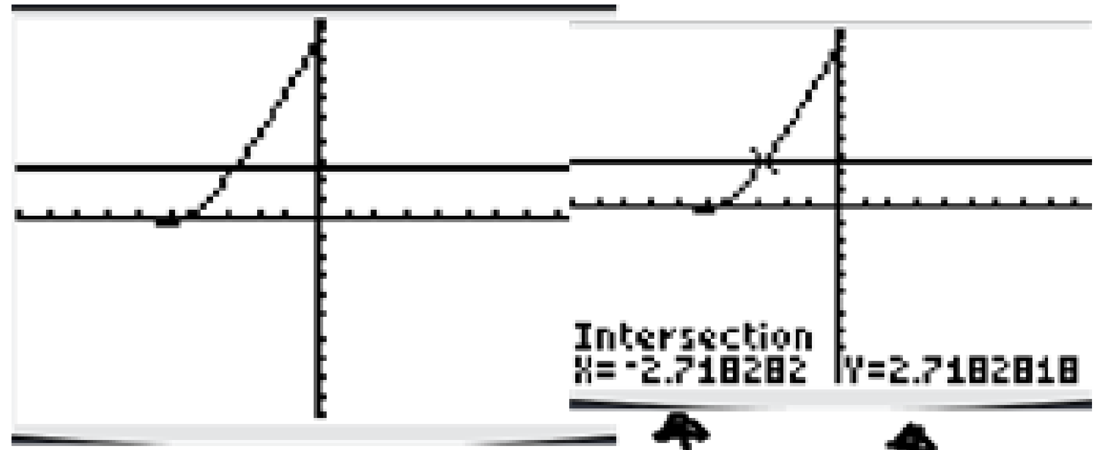
$$a = \frac{3}{\ln(2e+b)}$$

$$a = a$$

$$\frac{\cancel{3}(2e+b)}{e} = \frac{\cancel{3}\;1}{\ln(2e+b)}$$

$$(2e+b)\ln(2e+b) = e$$

$$(2e+b)\ln(2e+b)=e$$



$-e$ e

$$y_1=e$$

$$y_2=(2e+x)\ln(2e+x)$$

$$b=-e$$

$$a=\frac{3}{\ln(2e+b)} \rightarrow a=3$$

$$a=\frac{3}{\ln(2e-e)}$$

$$a=\frac{3}{\ln e}$$

Pg 382 #14

14 If $y = 2\sin x + 3\cos x$, show that $y'' + y = 0$ where y'' represents $\frac{d^2y}{dx^2}$.

$y' = 2(\cos x) + 3(-\sin x)$

$y' = 2\cos x - 3\sin x$

$y'' = 2(-\sin x) - 3(\cos x)$

$y'' = -2\sin x - 3\cos x$

$y'' + y = (-2\sin x - 3\cos x) + (2\sin x + 3\cos x)$

$= -2\sin x + 2\sin x - 3\cos x + 3\cos x$

$= 0$

Review set 15C #9

9 The function f is defined by $f : x \mapsto -10\sin 2x \cos 2x$, $0 \leqslant x \leqslant \pi$.

a Write down an expression for $f(x)$ in the form $k \sin 4x$.

b Solve $f'(x) = 0$, giving exact answers.

$f(x) = -10\sin 2x \cos 2x$

let $\theta = 2x$

$= -10\sin\theta\cos\theta$

$= -5(2\sin\theta\cos\theta)$

$= -5\sin 2\theta$

$= -5\sin(2(2x))$

$= -5\sin 4x$

$\sin 2\theta = 2\sin\theta\cos\theta$

$f'(x) = -5(\cos 4x)\cdot(4)$

$f'(x) = -20\cos 4x$

$\dfrac{0}{-20} = \dfrac{-20\cos 4x}{-20}$

$\cos 4x = 0$

$\cos 4x = 0$ radians $\left[0, \frac{8\pi}{8}\right]$

$\cos^{-1}(\cos 4x) = \cos^{-1}(0)$

$$4x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi k, k \in I$$

$$x = \begin{cases} \frac{\pi}{2}\left(\frac{1}{4}\right) \\ \frac{3\pi}{2}\left(\frac{1}{4}\right) \end{cases} + \frac{2\pi}{4} k, k \in I$$

$$x = \begin{cases} \frac{\pi}{8} \\ \frac{3\pi}{8} \end{cases} + \frac{4\pi}{8} k, k \in I$$

Interval

$$x = \begin{cases} \frac{\pi}{8}, \frac{5\pi}{8}, \\ \frac{3\pi}{8}, \frac{7\pi}{8}, \end{cases}$$