$y = \cos(\cos x)$ let $u = \cos x$

$y = \cos u \leftarrow$ Chain Rule $\downarrow$

$u' = -\sin x$

$y' = -\sin u \cdot \frac{du}{dx}$

$y' = -\sin[\cos x] \cdot (-\sin x)$

$= +\sin x(\sin(\cos x))$

$$y = \cos^2(x^2+5x-2)$$

$$y = \left[\cos(x^2+5x-2)\right]^2$$

let $u = \cos(x^2+5x-2)$

$u' =$

$-\sin(x^2+5x-2)[2x+5]$

$$y = u^2$$

$$y' = 2u \cdot u'$$

$$y' = 2\cos(x^2+5x-2)\left[-(2x+5)\sin(x^2+5x-2)\right]$$

→ Monday Unit circle Probe

Ch 15 H- Second and Higher Derivatives

First Derivative: $f'(x)$ or $\dfrac{dy}{dx}$

Second Derivative: The derivative of the derivative.

$$f''(x) \text{ or } \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

The n^{th} Derivative: $f^{(n)}(x)$ or $= \dfrac{d^ny}{dx^n}$

Example: Find the second derivative of $y = 2x^3 - 5x$

$$y' = 6x^2 - 5$$

$$y'' = 12x$$

each time we take the derivative, the degree of the function goes down by 1

…Third derivative

$$y''' = 12$$

4th… $y^{(4)} = 0$

5th $y^{(5)} = 0 \rightarrow \frac{d^5y}{dx^5}$

Example: Given $y = \frac{x+1}{x^2}$ find $\frac{d^2y}{dx^2}$

Quotient Rule:

$u = x+1 \quad v = x^2$

$u' = 1 \quad v' = 2x$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2)(1) - (x+1)(2x)}{(x^2)^2}$$

$$= \frac{x^2 - 2x^2 - 2x}{x^4}$$

$$\frac{dy}{dx} = \frac{-x^2 - 2x}{x^4}$$

$$\frac{dy}{dx} = \frac{-x-2}{x^3}$$

$$\frac{dy}{dx} = \frac{-x-2}{x^3}$$

Quotient Rule again :)

$u = -x-2 \quad v = x^3$

$u' = -1 \quad v' = 3x^2$

$$\frac{d^2y}{dx^2} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^3)(-1) - (-x-2)(3x^2)}{(x^3)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-x^3 + 3x^2(x+2)}{x^6}$$

Example: Given $y = (x^2 - 4)^3$ find $\frac{d^2y}{dx^2}$

Chain Rule:

$y' = 3(x^2-4)^2(2x)$

$y' = (6x)(x^2-4)^2$

Product Rule + Chain Rule

$u = 6x \qquad v = (x^2-4)^2$

$u' = 6 \qquad v' = 2(x^2-4)(2x)$

$v' = 4x(x^2-4)$

$$\frac{d^2y}{dx^2} = uv' + vu'$$

$$= (6x)(4x(x^2-4)) + (x^2-4)^2(6)$$

Example: Given $y = 5e^{2x}$, show that $\frac{d^2y}{dx^2} = 4y$

$$\frac{dy}{dx} = 5(e^{2x})(2)$$

$$\frac{dy}{dx} = 10e^{2x}$$

$$\frac{d^2y}{dx^2} = 10(e^{2x})(2)$$

$$= 20e^{2x}$$

$$= 4(5e^{2x})$$

$$= 4y$$

$$y = e^u$$

$$y' = e^u \cdot u'$$

$$u = f(x)$$

Example: Given $y = \cos x$, find the 22^{nd} derivative

$y' = -\sin x$

$y'' = -\cos x$

$y''' = -(-\sin x)$

$y'''' = \sin x$

$y''''' = \cos x$

$y^{(5)} = -\sin x$

The 4th derivative is the same as the original function. So this pattern will repeat every 4th.

$$\frac{22}{4} = 5\frac{2}{4}$$

The 22nd derivative is the same as the 2nd derivative

$$\frac{d^{22}y}{dx^{22}} = -\cos x$$

5 Find x when $f''(x) = 0$ for:

a $f(x) = 2x^3 - 6x^2 + 5x + 1$

$f'(x) = 6x^2 - 12x + 5$

$f''(x) = 12x - 12$

$0 = 12x - 12$

$1 = x$

8 Suppose $f(x) = 2\sin^3 x - 3\sin x$.

a Show that $f'(x) = \underbrace{-3\cos x}\cos 2x$.

$3^3 = 9$

$f(x) = 2[\sin x]^3 - 3\sin x$

$f'(x) = 2[3(\sin x)^2(\cos x)] - 3[\cos x]$

$f'(x) = 6\cos x \sin^2 x - 3\cos x$

$f'(x) = 3\cos x[2\sin^2 x - 1]$

$= 3\cos x[-\cos 2x]$

$= -3\cos x \cos 2x$

looks real close to

$\cos 2\theta = 1 - 2\sin^2\theta$

$-\cos 2\theta = -1 + 2\sin^2\theta$

$-\cos 2\theta = 2\sin^2\theta - 1$

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EXERCISE 15H

1 Find $f''(x)$ given that:

a $f(x) = 3x^2 - 6x + 2$

b $f(x) = \dfrac{2}{\sqrt{x}} - 1$

c $f(x) = 2x^3 - 3x^2 - x + 5$

d $f(x) = \dfrac{2 - 3x}{x^2}$

e $f(x) = (1 - 2x)^3$

f $f(x) = \dfrac{x + 2}{2x - 1}$

2 Find $\dfrac{d^2y}{dx^2}$ given that:

a $y = x - x^3$

b $y = x^2 - \dfrac{5}{x^2}$

c $y = 2 - \dfrac{3}{\sqrt{x}}$

d $y = \dfrac{4 - x}{x}$

e $y = (x^2 - 3x)^3$

f $y = x^2 - x + \dfrac{1}{1 - x}$

3 Given $f(x) = x^3 - 2x + 5$, find:

a $f(2)$

b $f'(2)$

c $f''(2)$

d $f^{(3)}(2)$

4 Suppose $y = Ae^{kx}$ where A and k are constants.

Show that:

a $\dfrac{dy}{dx} = ky$

b $\dfrac{d^2y}{dx^2} = k^2y$

c $\dfrac{d^3y}{dx^3} = k^3y$

5 Find x when $f''(x) = 0$ for:

a $f(x) = 2x^3 - 6x^2 + 5x + 1$

b $f(x) = \dfrac{x}{x^2 + 2}$