Ch 15D - Quotient Rule

Quotient Rule: The quotient of two differentiable functions u and v is differentiable and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$y = \frac{f(x)}{g(x)} \qquad u = f(x) \qquad v = g(x)$$

$$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$$

Example: differentiate $f(x) = \dfrac{x^5}{2x^3}$

$u = x^5 \qquad v = 2x^3$

$u' = 5x^4 \qquad v' = 6x^2$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(2x^3)(5x^4) - (x^5)(6x^2)}{(2x^3)^2}$$

$$f'(x) = \frac{10x^7 - 6x^7}{4x^6}$$

$$= \frac{4x^7}{4x^6} = x$$

$$f(x) = \frac{x^2}{2}$$

$$f'(x) = \frac{1}{2}(2x)$$

$$f'(x) = x$$

Example: differentiate $f(x) = \dfrac{1-x}{2+x}$

$u = 1 - x \qquad v = 2 + x$

$u' = -1 \qquad v' = 1$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(2+x)(-1) - (1-x)(1)}{(2+x)^2}$$

leave like this

Example: differentiate $f(x) = \dfrac{2x^2 - x}{x^2 + 1}$

$u = 2x^2 - x$ $\qquad$ $v = x^2 + 1$

$u' = 4x - 1$ $\qquad$ $v' = 2x$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2+1)(4x-1) - (2x^2-x)(2x)}{(x^2+1)^2}$$

Find the derivative of $F(x)=\dfrac{3x^2-2\sqrt{x}}{x}$

don't "need" to use Quotient Rule

$$F(x)=\frac{3x^2}{x}-2\frac{\sqrt{x}}{x}$$

$$=3x-2x^{-1/2}$$

$$F'(x)=3-2\left(\frac{-1}{2}x^{-3/2}\right)$$

$$=3+x^{-3/2}$$

$$=3+\frac{1}{\sqrt{x^3}}$$

$u=3x^2-2\sqrt{x}$ $\quad v=x$

$u'=6x-2\left(\frac{1}{2}x^{-1/2}\right)$ $\quad v'=1$

$=6x-x^{-1/2}$

$$F'(x)=\frac{vu'-uv'}{v^2}$$

$$=\frac{(x)(6x-x^{-1/2})-(3x^2-2\sqrt{x})(1)}{x^2}$$

Find the derivative of $p(x) = \dfrac{x^2}{\sqrt{1-3x}}$

$u = x^2 \qquad v = (1-3x)^{1/2}$ ← Chain Rule :)

$u' = 2x \qquad v' = \frac{1}{2}(1-3x)^{-\frac{1}{2}}(-3 \quad)$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1-3x)^{1/2}(2x) - (x^2)\left(\frac{-3}{2}(1-3x)^{-1/2}\right)}{1-3x}$$

Find the derivative of $p(x) = \dfrac{x}{\sqrt{x}(x+3)^2}$

$u = x$ $\quad$ $v = (x)^{1/2}(x+3)^2$ ← Product Rule + Chain Rule!! :)

$u' = 1$

$a = x^{1/2}$ $\quad$ $b = (x+3)^2$

$a' = \frac{1}{2}x^{-1/2}$ $\quad$ $b' = 2(x+3)(1)$

$v' = ab' + ba'$

$v' = (x^{1/2})(2(x+3)) + (x+3)^2\left(\frac{1}{2}x^{-1/2}\right)$

$$p'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{\sqrt{x}(x+3)^2(1) - (x)\left[\sqrt{x}(2(x+3)) + (x+3)^2\left(\frac{1}{2}x^{-1/2}\right)\right]}{\left(\sqrt{x}(x+3)^2\right)^2}$$

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3 **a** If $y = \frac{2\sqrt{x}}{1-x}$, show that $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}(1-x)^2}$.

b For what values of x is $\frac{dy}{dx}$ **i** zero **ii** undefined?

$u = 2x^{1/2}$ $\quad$ $v = 1-x$

$u' = x^{-1/2}$ $\quad$ $v' = -1$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1-x)(x^{-1/2}) - (2x^{1/2})(-1)}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{x^{-1/2} - x^{1/2} + 2x^{1/2}}{(1-x)^2}$$

$$= \frac{x^{-1/2} + x^{1/2}}{(1-x)^2} = \frac{\frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{1}}{(1-x)^2}$$

$$= \frac{\frac{1}{\sqrt{x}}(1+x)}{(1-x)^2} = \frac{(1+x)}{\sqrt{x}(1-x)^2}$$

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3 **a** If $y = \frac{2\sqrt{x}}{1-x}$, show that $\frac{dy}{dx} = \frac{x+1}{\sqrt{x}(1-x)^2}$.

b For what values of x is $\frac{dy}{dx}$ **i** zero **ii** undefined?

$$0 = \frac{x+1}{\sqrt{x}(1-x)^2}$$

$$0 = x+1$$

$$x = -1$$

use graphing calc to check :)

↳ when the denominator is zero.

$$\sqrt{x}(1-x)^2 = 0$$

$$\sqrt{x} = 0 \qquad (1-x)^2 = 0$$

$$x = 0 \qquad x = 1$$

EXERCISE 15D

1 Use the quotient rule to find $\dfrac{dy}{dx}$ if:

a $y = \dfrac{1+3x}{2-x}$ b $y = \dfrac{x^2}{2x+1}$ c $y = \dfrac{x}{x^2-3}$

d $y = \dfrac{\sqrt{x}}{1-2x}$ e $y = \dfrac{x^2-3}{3x-x^2}$ f $y = \dfrac{x}{\sqrt{1-3x}}$

2 Find the gradient of the tangent to:

a $y = \dfrac{x}{1-2x}$ at $x = 1$ b $y = \dfrac{x^3}{x^2+1}$ at $x = -1$

c $y = \dfrac{\sqrt{x}}{2x+1}$ at $x = 4$ d $y = \dfrac{x^2}{\sqrt{x^2+5}}$ at $x = -2$

Check your answers using technology.

3 a If $y = \dfrac{2\sqrt{x}}{1-x}$, show that $\dfrac{dy}{dx} = \dfrac{x+1}{\sqrt{x}(1-x)^2}$.

b For what values of x is $\dfrac{dy}{dx}$ i zero ii undefined?

4 a If $y = \dfrac{x^2-3x+1}{x+2}$, show that $\dfrac{dy}{dx} = \dfrac{x^2+4x-7}{(x+2)^2}$.

b For what values of x is $\dfrac{dy}{dx}$ i zero ii undefined?

c What is the graphical significance of your answers in b?