

Power Rule

$$y = x^n$$

$$y' = n \cdot x^{n-1}$$

Chain Rule:

$$y = f(g(x))$$

$$= f'(g(x)) \cdot g'(x)$$

Ch 15C - Product Rule

Product Rule: The product of two differentiable functions u and v is differentiable and

$$y = (f(x))(g(x)) \quad \begin{array}{l} u = f(x) \\ v = g(x) \end{array}$$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$(uv)' = uv' + vu'$$

Example: Find the derivative of $f(x) = (x^2 - 4)(x^3 + 2)$

$$u = x^2 - 4$$

$$u' = 2x$$

$$v = x^3 + 2$$

$$v' = 3x^2$$

$$\begin{aligned} f(x) &= x^5 + 2x^2 - 4x^3 - 8 \\ &= x^5 - 4x^3 + 2x^2 - 8 \end{aligned}$$

$$f'(x) = 5x^4 - 12x^2 + 4x$$

$$f'(x) = uv' + vu'$$

$$= (x^2 - 4)(3x^2) + (x^3 + 2)(2x)$$

$$= 3x^4 - 12x^2 + (2x^4 + 4x)$$

$$= 5x^4 - 12x^2 + 4x$$

Don't need to simplify

Example: Find the derivative of $f(x) = (2x^5 - 2x)(2x^3 + 2x + 1)$

$$u = 2x^5 - 2x$$

$$v = 2x^3 + 2x + 1$$

$$u' = 10x^4 - 2$$

$$v' = 6x^2 + 2$$

$$f'(x) = uv' + vu'$$

$$= (2x^5 - 2x)(6x^2 + 2) + (2x^3 + 2x + 1)(10x^4 - 2)$$

Don't
simplify

Example: Find the derivative of $f(x) = 3x^4(2x+1)^2$

$$u = 3x^4 \quad v = (2x+1)^2 \quad \& \text{ need to use the Chain Rule } \therefore$$
$$u' = 12x^3 \quad v' = 2(2x+1)'(2)$$

$$f'(x) = uv' + vu'$$
$$= (3x^4)(4(2x+1)) + (2x+1)^2(12x^3)$$

Example: Find the derivative of $f(x) = x^2 \sqrt{2x+1}$

$$u = x^2 \quad v = (2x+1)^{\frac{1}{2}} \leftarrow \text{chain Rule.}$$

$$u' = 2x \quad v' = \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)$$

$$\begin{aligned} f'(x) &= uv' + vu' \\ &= (x^2)(2x+1)^{-\frac{1}{2}} + (2x+1)^{\frac{1}{2}}(2x) \\ &= \frac{x^2}{\sqrt{2x+1}} + (\sqrt{2x+1})(2x) \end{aligned}$$

EXERCISE 15C

1 Use the product rule to differentiate:

a $f(x) = x(x - 1)$

b $f(x) = 2x(x + 1)$

c $f(x) = x^2\sqrt{x + 1}$

2 Find $\frac{dy}{dx}$ using the product rule:

a $y = x^2(2x - 1)$

b $y = 4x(2x + 1)^3$

c $y = x^2\sqrt{3 - x}$

d $y = \sqrt{x}(x - 3)^2$

e $y = 5x^2(3x^2 - 1)^2$

f $y = \sqrt{x}(x - x^2)^3$

3 Find the gradient of the tangent to:

a $y = x^4(1 - 2x)^2$ at $x = -1$

b $y = \sqrt{x}(x^2 - x + 1)^2$ at $x = 4$

c $y = x\sqrt{1 - 2x}$ at $x = -4$

d $y = x^3\sqrt{5 - x^2}$ at $x = 1$

Check your answers using technology.

4 Consider $y = \sqrt{x}(3 - x)^2$.

a Show that $\frac{dy}{dx} = \frac{(3 - x)(3 - 5x)}{2\sqrt{x}}$.

b Find the x -coordinates of all points on $y = \sqrt{x}(3 - x)^2$ where the tangent is horizontal.

c For what values of x is $\frac{dy}{dx}$ undefined?

d Are there any values of x for which y is defined but $\frac{dy}{dx}$ is not?

e What is the graphical significance of your answer in d?

5 Suppose $y = -2x^2(x + 4)$. For what values of x does $\frac{dy}{dx} = 10$?

$m = 0$