Ch 15B The Chain Rule Day 2

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Find $\frac{dy}{dx}$ for $y = \sqrt[3]{2x^3 - x^2}$

$y = g(f(x))$

$y' = g'(f(x)) \cdot f'(x)$

Power Rule

$y = x^n$

$y' = n \cdot x^{n-1}$

$y = (2x^3 - x^2)^{1/3}$

$\frac{dy}{dx} = \frac{1}{3}(2x^3 - x^2)^{\frac{1}{3} - \frac{3}{3}}(6x^2 - 2x)$

$= \frac{1}{3}(2x^3 - x^2)^{-2/3}(6x^2 - 2x)$

Example: Differentiate $y = \left(\left(4x^2 - 5\right)^3 - 1\right)^5$

$y = f(g(h(x)))$

- use chain Rule twice

$y' = 5\left((4x^2-5)^3-1\right)^4\left(3(4x^2-5)^2(8x)\right)$

3 Find the gradient of the tangent to:

b $y = (3x+2)^6$ at $x = -1$

derivative of $3x+2$

$$\frac{dy}{dx} = 6(3x+2)^5(3)$$

$$\frac{dy}{dx} = 18(3x+2)^5$$

at $x = -1$

$$m_T = 18(3(-1)+2)^5$$
$$= 18(-3+2)^5$$
$$= 18(-1)^5$$
$$= -18$$

4 The gradient function of $f(x) = (2x - b)^a$ is $f'(x) = 24x^2 - 24x + 6$.
Find the constants a and b.

$$f'(x) = a(2x - b)^{a-1}(2)$$

$$f'(x) = 2a(2x - b)^{a-1}$$

put in factored form

$$f'(x) = 6(4x^2 - 4x + 1)$$

$$f'(x) = 6(2x - 1)^2$$

→ since these are the same gradient function we see that $2a = 6$ $\quad a - 1 = 2$

$b = 1$

$a = 3 \quad b = 1$

5 Suppose $y = \frac{a}{\sqrt{1+bx}}$ where a and b are constants. When $x = 3$, $y = 1$ and $\frac{dy}{dx} = -\frac{1}{8}$. Find a and b.

$$y = a(1+bx)^{-1/2}$$

$$\frac{dy}{dx} = -\frac{1}{2}a(1+bx)^{-\frac{3}{2}}(b)$$

$$\frac{dy}{dx} = \frac{-ab}{2}(1+bx)^{-3/2}$$

$$-\frac{1}{8} = \frac{-ab}{2}(1+3b)^{-3/2}$$

$$\frac{1}{4b} = a(1+3b)^{-3/2}$$

$$a = \frac{(1+3b)^{3/2}}{4b}$$

when $x = 3$ $y = 1$

$$1 = \frac{a}{(1+3b)^{1/2}}$$

$$a = (1+3b)^{1/2}$$

$$a = a$$

$$(1+3b)^{1/2} = \frac{(1+3b)^{3/2}}{4b}$$

$$4b = \frac{(1+3b)^{3/2}}{(1+3b)^{1/2}}$$

$$4b = 1+3b$$

$$4b - 3b = 1$$

$$b = 1$$

$$a = (1+3b)^{1/2}$$

$$a = (4)^{1/2}$$

$$a = 2$$

Ch 15C - Product Rule

Product Rule: The product of two differentiable functions u and v is differentiable and

$y = (f(x))(g(x))$

$u = f(x) \quad v = g(x)$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$(uv)' = uv' + vu'$$

Example: Find the derivative of $f(x) = (x^2 - 4)(x^3 + 2)$

$u = x^2 - 4$ $\quad$ $v = x^3 + 2$ $\quad$ $f(x) = x^5 + 2x^2 - 4x^3 - 8$

$u' = 2x$ $\quad$ $v' = 3x^2$ $\quad$ $f(x) = x^5 - 4x^3 + 2x^2 - 8$

$f'(x) = 5x^4 - 12x^2 + 4x$

$f'(x) = uv' + vu'$

$f'(x) = (x^2 - 4)(3x^2) + (x^3 + 2)(2x)$

$= 3x^4 - 12x^2 + 2x^4 + 4x$

$= 5x^4 - 12x^2 + 4x$

Example: Find the derivative of $f(x) = (2x^5 - 2x)(2x^3 + 2x + 1)$

$u = 2x^5 - 2x$ $\quad$ $v = 2x^3 + 2x + 1$

$u' = 10x^4 - 2$ $\quad$ $v' = 6x^2 + 2$

$$f'(x) = uv' + vu'$$

$$= (2x^5 - 2x)(6x^2 + 2) + (2x^3 + 2x + 1)(10x^4 - 2)$$

Don't Simplify

Example: Find the derivative of $f(x) = 3x^4(2x+1)^2$

$u = 3x^4$

$u' = 12x^3$

$v = (2x+1)^2$ ← Chain Rule :)

$v' = 2(2x+1)^1(2)$

$v' = 4(2x+1)$

$f'(x) = uv' + vu'$

$= (3x^4)(4(2x+1)) + (2x+1)^2(12x^3)$

* Don't Simplify

:)

Example: Find the derivative of $f(x) = x^2\sqrt{2x+1}$

$u = x^2$

$u' = 2x$

$v = (2x+1)^{1/2}$

$v' = \frac{1}{2}(2x+1)^{-1/2}(2)$

$\frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$

$f'(x) = (x^2)(2x+1)^{-1/2} + (2x+1)^{1/2}(2x)$

EXERCISE 15C

1 Use the product rule to differentiate:

a $f(x) = x(x-1)$ b $f(x) = 2x(x+1)$ c $f(x) = x^2\sqrt{x+1}$

2 Find $\frac{dy}{dx}$ using the product rule:

a $y = x^2(2x-1)$ b $y = 4x(2x+1)^3$ c $y = x^2\sqrt{3-x}$

d $y = \sqrt{x}(x-3)^2$ e $y = 5x^2(3x^2-1)^2$ f $y = \sqrt{x}(x-x^2)^3$

3 Find the gradient of the tangent to:

a $y = x^4(1-2x)^2$ at $x = -1$ b $y = \sqrt{x}(x^2-x+1)^2$ at $x = 4$

c $y = x\sqrt{1-2x}$ at $x = -4$ d $y = x^3\sqrt{5-x^2}$ at $x = 1$

Check your answers using technology.

4 Consider $y = \sqrt{x}(3-x)^2$.

a Show that $\frac{dy}{dx} = \frac{(3-x)(3-5x)}{2\sqrt{x}}$.

b Find the x-coordinates of all points on $y = \sqrt{x}(3-x)^2$ where the tangent is horizontal.

$m = 0 \quad \frac{dy}{dx} = 0$

c For what values of x is $\frac{dy}{dx}$ undefined?

d Are there any values of x for which y is defined but $\frac{dy}{dx}$ is not?

e What is the graphical significance of your answer in d?

5 Suppose $y = -2x^2(x+4)$. For what values of x does $\frac{dy}{dx} = 10$?