Probe Friday

Find the derivative

using $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

4 Find the gradient of the tangent to:

a $y = x^2$ at $x = 2$

b $y = \frac{8}{x^2}$ at the point $(9, \frac{8}{81})$

c $y = 2x^2 - 3x + 7$ at $x = -1$

d $y = \frac{2x^2 - 5}{x}$ at the point $(2, \frac{3}{2})$

e $y = \frac{x^2 - 4}{x^2}$ at the point $(4, \frac{3}{4})$

f $y = \frac{x^3 - 4x - 8}{x^2}$ at $x = -1$

Ch 15A

$f(x) = x^n$

$f'(x) = n \cdot x^{n-1}$

$$y = \frac{x^3 - 4x - 8}{x^2} = \frac{x^3}{x^2} - \frac{4x}{x^2} - \frac{8}{x^2}$$

$$y = x^1 - 4x^{-1} - 8x^{-2}$$

$$\frac{dy}{dx} = 1(x^{1-1}) - 4(-1 \cdot x^{-1-1}) - 8(-2x^{-2-1})$$

$$\frac{dy}{dx} = 1 + \frac{4}{x^2} + \frac{16}{x^3}$$

$\underline{x = -1}$

$$\frac{dy}{dx} = 1 + \frac{4}{(-1)^2} + \frac{16}{(-1)^3}$$

$$= 1 + 4 - 16$$

$$= -11$$

7 **a** If $y = 4x - \frac{[illegible]}{x}$, find $\frac{dy}{dx}$ and interpret its meaning.

b The position of a car moving along a straight road is given by $S = 2t^2 + 4t$ metres where t is the time in seconds. Find $\frac{dS}{dt}$ and interpret its meaning.

c The cost of producing x toasters each week is given by $C = 1785 + 3x + 0.002x^2$ dollars. Find $\frac{dC}{dx}$ and interpret its meaning.

$S = t^n$

$\frac{dS}{dt} = n \cdot t^{n-1}$

$S = 2t^2 + 4t$

$\frac{dS}{dt} = 2(2t) + 4(1)$

$\frac{dS}{dt} = 4t + 4$

$\frac{dS}{dt}$: represents slope of the tangent, or the instantaneous rate of change $\Rightarrow$ velocity

speed

Ch 15 B – The Chain Rule

$$f(g(x)) \neq g(f(x))$$

Review of composite functions:

Ex: Find $g(f(x))$ if $g(x) = x^3$ and $f(x) = 2x+1$

$$g(f(x)) = (f(x))^3$$
$$= (2x+1)^3$$

$$g(f(x)) = g(2x+1)$$
$$= (2x+1)^3$$

1

1 1

1 2 1

1 3 3 1

$$g(f(x)) = 1(2x)^3(1)^0 + 3(2x)^2(1)^1 + 3(2x)^1(1)^2 + 1(2x)^0(1)^3$$
$$= 8x^3 + 12x^2 + 6x + 1$$

Ex: Write $y = 10x + 15$ as a composite function:

$y = 5(2x+3)$ let $f(x) = 2x+3$

$y = 5(f(x))$ $g(x) = 5x$

$y = g(f(x))$

Ex: Write y= $(x+15)^3$ as a composite function:

$y = (x+15)^3$ $f(x) = x+15$

$y = (f(x))^3$ $g(x) = x^3$

$= g(f(x))$

Ex: Expand $y = (3x-2)^2$ and find its derivative.

$y = (3x-2)(3x-2)$

$y = 9x^2 - 12x + 4$

$\frac{dy}{dx} = 18x - 12$

Compare this with w hen $y = g(u)$ where $u = f(x)$ $y = (3x-2)^2$

$$\frac{dy}{dx} = \left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)$$

$= (2u)(3)$

$= 6u$

$= 6(3x-2)$

$\frac{dy}{dx} = 18x - 12$

$y = g(f(x))$

$y = g(u)$

$y = u^2$

$\frac{dy}{du} = 2u$

$u = f(x)$

$u = 3x-2$

$\frac{du}{dx} = 3$

Example: Find the derivative of $y=(2x+3)^3$

If $y=f(u)$ and $u=g(x)$, then $\frac{dy}{dx}=\left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)$

$y=(u)^3$

$y=g(f(x))$

$y=g(u)$

$y=u^3$

$\frac{dy}{du}=3u^2$

$u=f(x)$

$u=2x+3$

$\frac{du}{dx}=2$

$$\frac{dy}{dx}=\left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)$$
$$=(3u^2)(2)$$
$$=6u^2$$
$$=6(2x+3)^2$$

If $F(x) = f(g(x))$, then $F'(x) = f'(g(x)) \cdot g'(x)$

$$f(\text{☺}) \qquad\qquad f'(\text{☺}) \cdot \text{☺}'$$

Differentiate A) $y = \left(-x^3 - 3\right)^5 = (\text{☺})^5$

$$\text{☺} = -x^3 - 3$$

$$\text{☺}' = -3x^2$$

$$\frac{dy}{dx} = 5(\text{☺})^4 \cdot \text{☺}'$$

$$= 5(-x^3-3)^4(-3x^2)$$

$$= -15x^2(-x^3-3)^4$$

B) $y=(2x+5)^5$

$y' = 5(2x+5)^4 \cdot (2)$

$y' = 10(2x+5)^4$

C) $y=\sqrt[4]{x^3-2x}$

$y=(x^3-2x)^{1/4}$

$y' = \frac{1}{4}(x^3-2x)^{\frac{1}{4}-\frac{4}{4}}(3x^2-2)$

$y' = \frac{1}{4}(x^3-2x)^{\frac{-3}{4}}(3x^2-2)$

EXERCISE 15B.1

1 Find $g(f(x))$ if:

a $g(x) = x^2$ and $f(x) = 2x + 7$
b $g(x) = 2x + 7$ and $f(x) = x^2$
c $g(x) = \sqrt{x}$ and $f(x) = 3 - 4x$
d $g(x) = 3 - 4x$ and $f(x) = \sqrt{x}$
e $g(x) = \dfrac{2}{x}$ and $f(x) = x^2 + 3$
f $g(x) = x^2 + 3$ and $f(x) = \dfrac{2}{x}$

2 Find $g(x)$ and $f(x)$ such that $g(f(x))$ is:

a $(3x + 10)^3$
b $\dfrac{1}{2x + 4}$
c $\sqrt{x^2 - 3x}$
d $\dfrac{10}{(3x - x^2)^3}$

EXERCISE 15B.2

1 Write in the form au^n, clearly stating what u is:

$1u^{-2}$ **a** $\dfrac{1}{(2x - 1)^2} = 1(2x-1)^{-2}$
$u = 2x - 1$
b $\sqrt{x^2 - 3x}$
c $\dfrac{2}{\sqrt{2 - x^2}}$
d $\sqrt[3]{x^3 - x^2}$
e $\dfrac{4}{(3 - x)^3}$
f $\dfrac{10}{x^2 - 3}$

2 Find the gradient function $\dfrac{dy}{dx}$ for:

a $y = (4x - 5)^2$
b $y = \dfrac{1}{5 - 2x}$
c $y = \sqrt{3x - x^2}$
d $y = (1 - 3x)^4$
e $y = 6(5 - x)^3$
f $y = \sqrt[3]{2x^3 - x^2}$
g $y = \dfrac{6}{(5x - 4)^2}$
h $y = \dfrac{4}{3x - x^2}$
i $y = 2\left(x^2 - \dfrac{2}{x}\right)^3$

$$\sqrt{9x^2+5} = 20$$

$$\sqrt{9x^2} = 15$$

1.1, 1.01, 1.001

$$-\lim_{x\to 5} \frac{x^2-6x+5}{x-5}$$

$$\lim_{x\to 5} \frac{\cancel{(x-5)}(x-1)}{\cancel{x-5}}$$

$$\lim_{x\to 5} x-1 = 5-1$$
$$= 4$$

$$\lim_{x\to\infty} \frac{x+2}{x-1} = 1$$

$$= \lim_{x\to\infty} \frac{x\left(1+\frac{2}{x}\right)}{x\left(1-\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{1+2/x}{1-1/x}$$

$$= \frac{1+0}{1-0}$$

$$= 1$$

$$\lim_{x\to 1^+} \frac{x+2}{1-x} = -\infty$$

$$\left\{ \begin{array}{l} y = \frac{x+2}{1-x} \\ VA: x=1 \\ HA: y=-1 \\ x\text{-int}: x=-2 \end{array} \right.$$