Ch 15B The Chain Rule Day 2

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Find $\frac{dy}{dx}$ for $y = \sqrt[3]{2x^3 - x^2}$

$y = (2x^3 - x^2)^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}(2x^3 - x^2)^{\frac{-2}{3}}(6x^2 - 2x)$

Do not need to simplify :)

$g(f(h(x)))$

Example: Differentiate $y = \left(\left(4x^2 - 5\right)^3 - 1\right)^5$

let $u = (4x^2-5)^3-1$

$u' = 3(4x^2-5)^2(8x)$

$y = u^5$

$y' = 5u^4 \cdot u'$

$y' = 5\left((4x^2-5)^3-1\right)^4\left(3(4x^2-5)^2(8x)\right)$

3 Find the gradient of the tangent to:

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b $y = (3x + 2)^6$ at $x = -1$

$\frac{dy}{dx} = 6(3x+2)^5(3)$

$= 18(3x+2)^5$

$m_T = 18(3(-1)+2)^5$

$= 18(-1)^5$

$= -18$

4 The gradient function of $f(x) = (2x - b)^a$ is $f'(x) = 24x^2 - 24x + 6$.
Find the constants a and b.

$f'(x) = a(2x-b)^{a-1}(2)$

$f'(x) = 2a(2x-b)^{a-1}$

put in factored form

$f'(x) = 6(4x^2 - 4x + 1)$

$= 6(2x-1)^2$

decomposition

$4x^2 - 4x + 1$

$1 \times 4 = 4$

$-2 \times -2 = 4$

$-2 + -2 = -4$

$4x^2 - 2x - 2x + 1$

$2x(2x-1) - 1(2x-1)$

$(2x-1)(2x-1)$

$(2x-1)^2$

→ Since these are the same gradient function, we see that

$6 = 2a \therefore a = 3$

$a - 1 = 2 \therefore a = 3$

$b = 1$

5 Suppose $y = \frac{a}{\sqrt{1+bx}}$ where a and b are constants. When $x = 3$, $y = 1$ and $\frac{dy}{dx} = -\frac{1}{8}$.

Find a and b.

$y = a(1+bx)^{-\frac{1}{2}}$

When $x = 3$, $y = 1$

$1 = a(1+3b)^{-1/2}$

$1 = \frac{a}{(1+3b)^{1/2}}$

$a = (1+3b)^{1/2}$

Find y'

$y' = -\frac{1}{2}a(1+bx)^{-\frac{3}{2}}(b)$

$y' = -\frac{1}{2}ab(1+bx)^{-3/2}$

When $x = 3$, $y' = -\frac{1}{8}$

$-\frac{1}{8} = -\frac{1}{2}ab(1+3b)^{-3/2}$

Substitution

$-\frac{1}{8} = -\frac{1}{2}\left((1+3b)^{1/2}\right)b(1+3b)^{-3/2}$

$$-\frac{1}{8} = -\frac{1}{2}\left((1+3b)^{1/2}\right) b (1+3b)^{-3/2}$$

$$\frac{1}{4} = b(1+3b)^{-1}$$

$$\frac{1}{4} = \frac{b}{1+3b}$$

$$1+3b = 4b$$

$$\boxed{1 = b}$$

$$a = \sqrt{1+3b}$$

$$a = \sqrt{1+3}$$

$$\boxed{a = \sqrt{4} = 2}$$

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Sheet.