Ch 15 A- Simple Rules of Differentiation

Rates of change occur in so many different fields of study (science, engineering, rates of reaction in chemistry, marginal costs in economics…) that it was given a special name and notation.

The DERIVATIVE of a function f at a number a is denoted by $f'(a)$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If we are not given a value a, use: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

NOTATION: there are many ways to denote the derivative

Notation	How you say it ☺
y' or $f'(x)$	"y prime" or "f prime of x"
$\frac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"
$\frac{df}{dx}$	"df dx" or "the derivative of f with respect to x"
$\frac{d}{dx}f(x)$	"the derivative of f at x"

The Power Rule:

Find the derivative of $f(x) = x^n$

$(x+h)^n = (\ )(\ )(\ )(\ )\cdots(\ )$ Ch7 textbook

Pascal's Triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

binomial coefficent $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - (x^n)}{h}$$

$$= \lim_{h \to 0} \frac{\left[x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \cdots + \binom{n}{n-1}x h^{n-1} + \binom{n}{n}h^n\right] - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left[\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n\right] - \cancel{x^n}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\left[\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + nxh^{n-1} + h^n\right] - \cancel{x^n}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \cdots nxh^{n-2} + h^{n-1}\right]}{\cancel{h}}$$

$$= \lim_{h \to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \cdots + nxh^{n-2} + h^{n-1}$$

$$= nx^{n-1}$$

$$f(x) = x^n$$

$$f'(x) = n \cdot x^{n-1}$$

Find the derivative of $f(x) = x^{12}$

$f'(x) = 12x^{11}$

Find the derivative of $f(x) = x^{-4}$

$f'(x) = -4(x^{-4-1})$

$= -4x^{-5}$

Derivative of a Constant Rule: If f is a constant function of value c, then

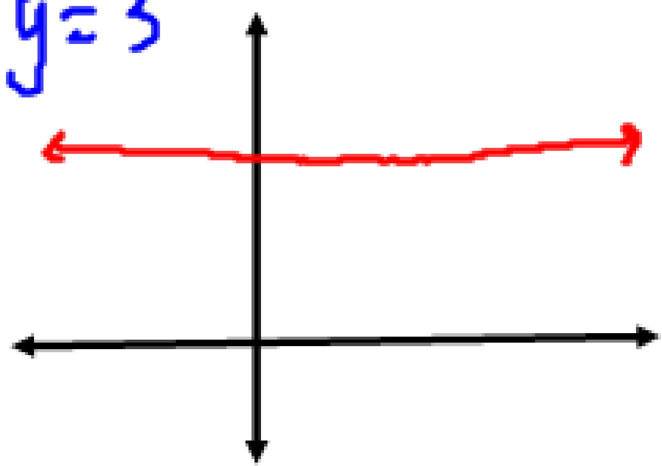
$y = c.$

$$\frac{d}{dx}c = 0$$

→ Think about a horizontal line

$y = 3$

the derivative of "c" with respect to x



slope of the tangent line is zero ∴ the deriv

The constant multiple Rule: If c is a constant and f is differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Example: let $g(x) = cf(x)$. Find $g'(x)$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h}$$

$$= \lim_{h\to 0} c\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$g'(x) = c \cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= c \cdot f'(x)$$

Example: $f(x) = 9x^7$

$$f'(x) = 9[7x^6]$$
$$= 63x^6$$

The Sum Rule: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

let $p(x)=f(x)+g(x)$

$$p'(x)=\lim_{h\to 0}\frac{p(x+h)-p(x)}{h}=\lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)+g(x+h)-g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$=f'(x)+g'(x)$$

The difference Rule: $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$

Example: Find the derivative of $f(x) = 3x^4 + 6x^2 - 12x$

- sum/diff. rule
- constant mult rule
- Power Rule

$$f'(x) = 3(4x^3) + 6(2x) - 12(1)$$
$$= 12x^3 + 12x - 12$$

Find the derivative of

$y = \sqrt{x}$

$$y = x^{\frac{1}{2}}$$
$$y' = \frac{1}{2}\left(x^{\frac{1}{2}-1}\right)$$
$$= \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

B) $y = 4\sqrt[5]{x^3}$

$$y = 4 \cdot x^{3/5}$$
$$y' = 4\left(\frac{3}{5}x^{\frac{3}{5}-\frac{5}{5}}\right)$$
$$= \frac{12}{5}x^{-2/5} = \frac{12}{5\sqrt[5]{x^2}}$$

EXERCISE 15A

1 Find $f'(x)$ given that $f(x)$ is:

a x^3 **b** $2x^3$ **c** $7x^2$ **d** $6\sqrt{x}$

e $3\sqrt[3]{x}$ **f** $x^2 + x$ **g** $4 - 2x^2$ **h** $x^2 + 3x - 5$

i $\frac{1}{2}x^4 - 6x^2$ **j** $\dfrac{3x - 6}{x}$ $= \dfrac{3x}{x} - \dfrac{6}{x}$ **k** $\dfrac{2x - 3}{x^2}$ **l** $\dfrac{x^3 + 5}{x}$

m $\dfrac{x^3 + x - 3}{x}$ **n** $\dfrac{1}{\sqrt{x}}$ $= 3 - 6x^{-1}$ **o** $(2x - 1)^2$ **p** $(x + 2)^3$

$(2x-1)(2x-1)$

$4x^2 - 4x + 1$

2 Find $\dfrac{dy}{dx}$ for:

a $y = 2.5x^3 - 1.4x^2 - 1.3$ **b** $y = \pi x^2$ **c** $y = \dfrac{1}{5x^2}$

d $y = 100x$ **e** $y = 10(x + 1)$ **f** $y = 4\pi x^3$

3 Differentiate with respect to x:

a $6x + 2$ **b** $x\sqrt{x}$ **c** $(5 - x)^2$ **d** $\dfrac{6x^2 - 9x^4}{3x}$

e $(x + 1)(x - 2)$ **f** $\dfrac{1}{x^2} + 6\sqrt{x}$ **g** $4x - \dfrac{1}{4x}$ **h** $x(x + 1)(2x - 5)$

4 Find the gradient of the tangent to: (derivative)

a $y = x^2$ at $x = 2$

b $y = \frac{8}{x^2}$ at the point $(9, \frac{8}{81})$

c $y = 2x^2 - 3x + 7$ at $x = -1$

d $y = \frac{2x^2 - 5}{x}$ at the point $(2, \frac{3}{2})$

e $y = \frac{x^2 - 4}{x^2}$ at the point $(4, \frac{3}{4})$

f $y = \frac{x^3 - 4x - 8}{x^2}$ at $x = -1$

Check your answers using technology.

5 Suppose $f(x) = x^2 + (b+1)x + 2c$, $f(2) = 4$, and $f'(-1) = 2$. system of equations

Find the constants b and c.

6 Find the gradient function of $f(x)$ where $f(x)$ is:

a $4\sqrt{x} + x$

b $\sqrt[3]{x}$

c $-\frac{2}{\sqrt{x}}$

d $2x - \sqrt{x}$

e $\frac{4}{\sqrt{x}} - 5$

f $3x^2 - x\sqrt{x}$

g $\frac{5}{x^2\sqrt{x}}$

h $2x - \frac{3}{x\sqrt{x}}$

7 **a** If $y = 4x - \frac{3}{x}$, find $\frac{dy}{dx}$ and interpret its meaning.

b The position of a car moving along a straight road is given by $S = 2t^2 + 4t$ metres where t is the time in seconds. Find $\frac{dS}{dt}$ and interpret its meaning.

c The cost of producing x toasters each week is given by $C = 1785 + 3x + 0.002x^2$ dollars. Find $\frac{dC}{dx}$ and interpret its meaning.