On Monday October 1, 2018, you and your classmates will be presenting you Math IAs to the grade 11 IB class (and teachers). The requirements for the assignment are listed below. In this session, you will be expected to talk about your IA; it is not a test, but rather an opportunity to seek feedback, clarify and revise your thinking, and finish up your final draft of your exploration.

Please note: this is not an optional exercise. All students are expected to attend and the work will go towards your predicted grade.

Your IA presentation should be laid out on a piece of paper/bristol board, so that someone coming by can see it and engage with it. Your board should include:

- What is the project is on (brief description of your topic, but don't be too vague).
- What was the question you tried to answer through this IA?
- The reason that you chose this particular project (personal engagement)
- Outline the math (what you learned)
- Areas of math, the process of learning a "new to you" math
- Limitations of what you learned and where you could take this project

Ch 15 A- Simple Rules of Differentiation

Rates of change occur in so many different fields of study (science, engineering, rates of reaction in chemistry, marginal costs in economics…) that it was given a special name and notation.

The DERIVATIVE of a function f at a number a is denoted by $f'(a)$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If we are not given a value a, use: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

NOTATION: there are many ways to denote the derivative

Notation	How you say it ☺
y' or $f'(x)$	"y prime" or "f prime of x"
$\frac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"
$\frac{df}{dx}$	"df dx" or "the derivative of f with respect to x"
$\frac{d}{dx}f(x)$	"the derivative of f at x"

The Power Rule:

Find the derivative of $f(x) = x^n$

ch7 Binomial coeff

$(x+h)^n = (\ )(\ )\cdots(\ )$

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$

1
1 1
1 2 1
1 3 3 1

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^n - (x^n)}{h}$

$= \lim_{h\to 0} \frac{\left[\cancel{x^n} + \binom{n}{1}x^{n-1}\cdot h + \binom{n}{2}x^{n-2}h^2 + \cdots + \binom{n}{n-1}xh^{n-1} + \binom{n}{n}h^n\right] - \cancel{x^n}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}\left[nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}\cdot h + \cdots + nxh^{n-2} + h^{n-1}\right]}{\cancel{h}}$

$= \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}\cdot h + \cdots + nxh^{n-2} + h^{n-1}$

$= nx^{n-1}$

$$f(x) = x^n \qquad f'(x) = n \cdot x^{n-1}$$

Find the derivative of $f(x) = x^{12}$

$$f'(x) = 12(x^{12-1})$$
$$= 12x^{11}$$

Find the derivative of $f(x) = x^{-4}$

$$f'(x) = -4(x^{-4-1})$$
$$= -4x^{-5}$$

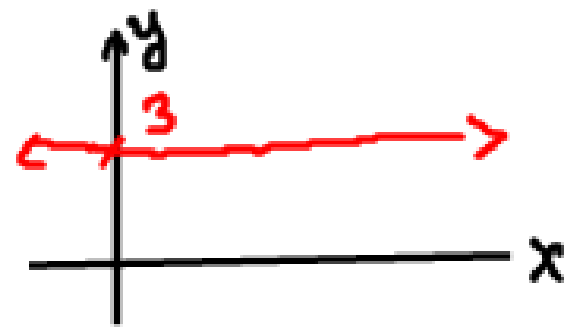
Derivative of a Constant Rule: If f is a constant function of value c, then

$$\frac{d}{dx}c = 0$$

$y=c$

$y=3$

- Horizontal line



The slope of the tangent line is zero

The constant multiple Rule: If c is a constant and f is differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Example: let $g(x) = cf(x)$. Find $g'(x)$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \frac{c\cdot f(x+h) - c\cdot f(x)}{h}$$

$$= \lim_{h\to 0} c\cdot\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$g'(x) = c\cdot \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= c\cdot f'(x)$$

Example: $f(x) = 9x^7$

$$f'(x) = 9(7x^6)$$
$$= 63x^6$$

The Sum Rule: If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$ let $p(x) = f(x)+g(x)$

$$p'(x) = \lim_{h\to 0} \frac{p(x+h)-p(x)}{h}$$

$$= \lim_{h\to 0} \frac{[f(x+h)+g(x+h)] - [f(x)+g(x)]}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x) + g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = f'(x) + g'(x)$$

The difference Rule:

$$\frac{d}{dx}\left(f(x)-g(x)\right) = f'(x) - g'(x)$$

Example: Find the derivative of $f(x) = 3x^4 + 6x^2 - 12x$

- sum/diff Rule
- constant mult. Rule
- Power Rule

$$f'(x) = 3(4x^3) + 6(2x) - 12(1)$$
$$= 12x^3 + 12x - 12$$

Find the derivative of

$y = \sqrt{x}$

$$y = x^{\frac{1}{2}}$$
$$y' = \frac{1}{2}\left(x^{\frac{1}{2}-1}\right)$$
$$= \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

B) $y = 4\sqrt[5]{x^3}$

$$y = 4(x^3)^{1/5} = 4x^{\frac{3}{5}}$$
$$y' = 4\left(\frac{3}{5}x^{\frac{3}{5}-\frac{5}{5}}\right)$$
$$= \frac{12}{5}x^{\frac{-2}{5}} = \frac{12}{5\sqrt[5]{x^2}}$$

EXERCISE 15A

1 Find $f'(x)$ given that $f(x)$ is:

- **a** x^3
- **b** $2x^3$
- **c** $7x^2$
- **d** $6\sqrt{x}$
- **e** $3\sqrt[3]{x}$
- **f** $x^2 + x$
- **g** $4 - 2x^2$
- **h** $x^2 + 3x - 5$
- **i** $\frac{1}{2}x^4 - 6x^2$
- **j** $\dfrac{3x - 6}{x}$ $= \dfrac{3x}{x} - \dfrac{6}{x}$ $= 3 - 6x^{-1}$
- **k** $\dfrac{2x - 3}{x^2}$
- **l** $\dfrac{x^3 + 5}{x}$
- **m** $\dfrac{x^3 + x - 3}{x}$
- **n** $\dfrac{1}{\sqrt{x}}$
- **o** $(2x - 1)^2$
  $(2x-1)(2x-1)$
  $4x^2 - 4x + 1$
- **p** $(x + 2)^3$

2 Find $\dfrac{dy}{dx}$ for:

- **a** $y = 2.5x^3 - 1.4x^2 - 1.3$
- **b** $y = \pi x^2$
- **c** $y = \dfrac{1}{5x^2}$
- **d** $y = 100x$
- **e** $y = 10(x + 1)$
- **f** $y = 4\pi x^3$

3 Differentiate with respect to x:

- **a** $6x + 2$
- **b** $x\sqrt{x}$
- **c** $(5 - x)^2$
- **d** $\dfrac{6x^2 - 9x^4}{3x}$
- **e** $(x + 1)(x - 2)$
- **f** $\dfrac{1}{x^2} + 6\sqrt{x}$
- **g** $4x - \dfrac{1}{4x}$
- **h** $x(x + 1)(2x - 5)$

4 Find the gradient of the tangent to: (derivative)

a $y = x^2$ at $x = 2$ $\quad m_T = 2(2)$

$y' = 2x \quad = 4$

b $y = \dfrac{8}{x^2}$ at the point $(9, \frac{8}{81})$

c $y = 2x^2 - 3x + 7$ at $x = -1$

d $y = \dfrac{2x^2 - 5}{x}$ at the point $(2, \frac{3}{2})$

e $y = \dfrac{x^2 - 4}{x^2}$ at the point $(4, \frac{3}{4})$

f $y = \dfrac{x^3 - 4x - 8}{x^2}$ at $x = -1$

Check your answers using technology.

5 Suppose $f(x) = x^2 + (b+1)x + 2c$, $f(2) = 4$, and $f'(-1) = 2$. system of equations

Find the constants b and c.

6 Find the gradient function of $f(x)$ where $f(x)$ is:

ch3

a $4\sqrt{x} + x$ **b** $\sqrt[3]{x}$ **c** $-\dfrac{2}{\sqrt{x}}$ **d** $2x - \sqrt{x}$

e $\dfrac{4}{\sqrt{x}} - 5$ **f** $3x^2 - x\sqrt{x}$ **g** $\dfrac{5}{x^2\sqrt{x}}$ **h** $2x - \dfrac{3}{x\sqrt{x}}$

7 **a** If $y = 4x - \dfrac{3}{x}$, find $\dfrac{dy}{dx}$ and interpret its meaning.

b The position of a car moving along a straight road is given by $S = 2t^2 + 4t$ metres where t is the time in seconds. Find $\dfrac{dS}{dt}$ and interpret its meaning.

c The cost of producing x toasters each week is given by $C = 1785 + 3x + 0.002x^2$ dollars. Find $\dfrac{dC}{dx}$ and interpret its meaning.