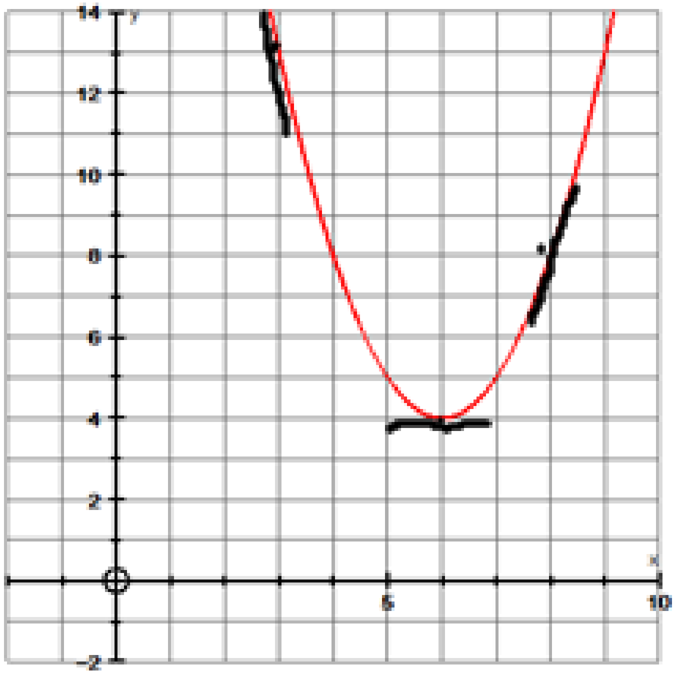
Ch 14D – The Derivative Function

For non-linear functions the slopes (gradients) of the tangent lines are different for different values of x along the curve.

Example:



The Gradient Function is a function that gives the slope of the tangent line of a curve $y = f(x)$ at some value $x = a$, for any point in the domain of $f(x)$. The gradient function of $y = f(x)$ is called the **Derivative Function** and is labelled $y = f'(x)$

say "f prime of x"

The value of $y = f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$

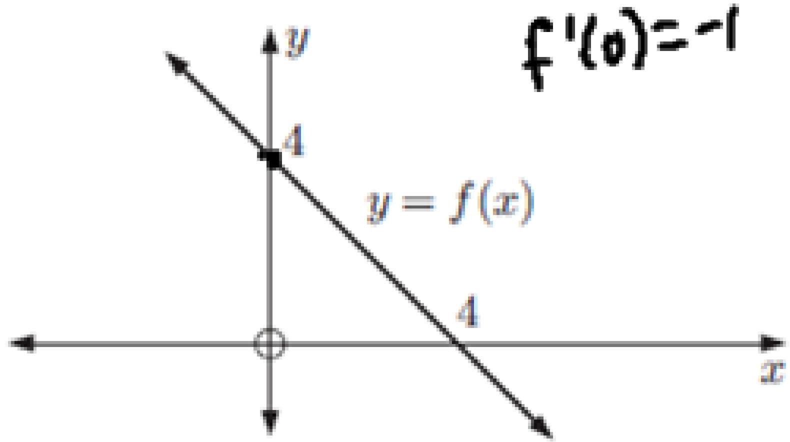
Pg 354

2 Using the graph below, find:

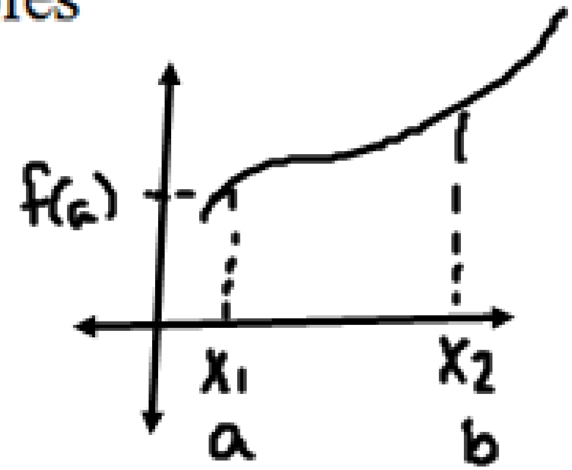
a $f(0) = 4$ b $f'(0)$

$f'(0) = -1$

$m = \frac{\Delta y}{\Delta x} = \frac{0-4}{4-0} = -1$



Ch 14 E – Differentiation from First Principles



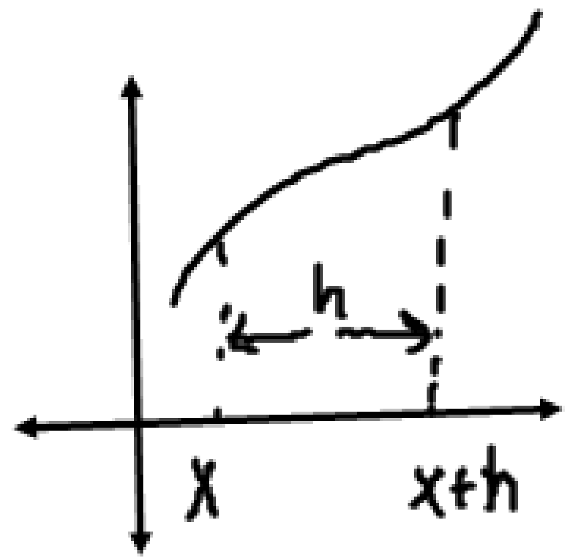
Recall the slope of a line:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

Slope of a tangent line:

The general formula for the slope (gradient) tangent line of a curve $f(x)$ is given as

$$m_{\text{tangent}} = \lim_{h \to 0} m_{\text{secant}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$



The Derivative Function (or just derivative) of $y = f(x)$ is :

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

Example: Find the derivative of $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x)^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$f'(x) = \lim_{h \to 0} 2x + h$$

$$= 2x$$

Example: Find the derivative of $f(x) = -x^3 + x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(-(x+h)^3 + (x+h)\right) - \left(-x^3 + x\right)}{h}$$

$$= \lim_{h\to 0} \frac{-(x^3 + 3x^2h + 3xh^2 + h^3) + x + h + x^3 - x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-x^3} - 3x^2h - 3xh^2 - h^3 + \cancel{x} + h + \cancel{x^3} \cancel{- x}}{h}$$

$$= \lim_{h\to 0} \frac{h(-3x^2 - 3xh - h^2 + 1)}{h}$$

$$= \lim_{h\to 0} -3x^2 - 3xh - h^2 + 1$$

$$= -3x^2 + 1$$

Pascal's Δ

1

1 1

1 2 1

1 3 3 1

Example: Find the slope of the tangent line to the curve $y = -5x + 2$ at x=1.

$$m_{tangent} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{(-5(1+h) + 2) - (-5(1) + 2)}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{-5} - 5h \cancel{+2}) \cancel{+5} \cancel{-2}}{h}$$

$$= \lim_{h \to 0} \frac{-5h}{h}$$

$$= \lim_{h \to 0} -5$$

$$= -5$$

Example: Find the slope of the tangent line to the curve $y = x^2 - 6x + 11$ at x=5.

$$m_{tangent} = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h \to 0} \frac{((5+h)^2 - 6(5+h) + 11) - (5^2 - 6(5) + 11)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{25} + 10h + h^2 - \cancel{30} - 6h + \cancel{11} - \cancel{25} + \cancel{30} - \cancel{11}}{h}$$

$$= \lim_{h \to 0} \frac{4h + h^2}{h}$$

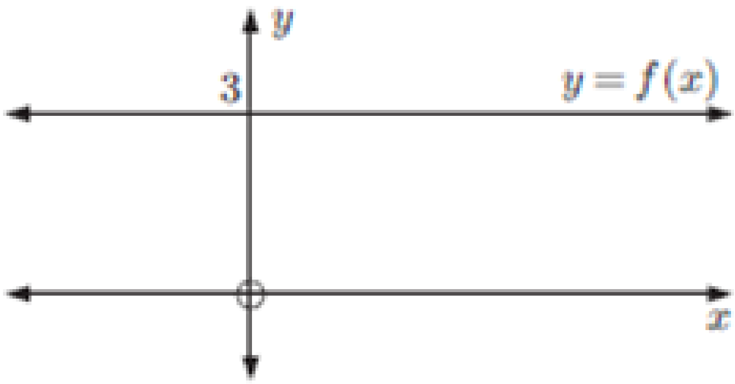
$$= \lim_{h \to 0} 4 + h$$

$$= 4$$

EXERCISE 14D

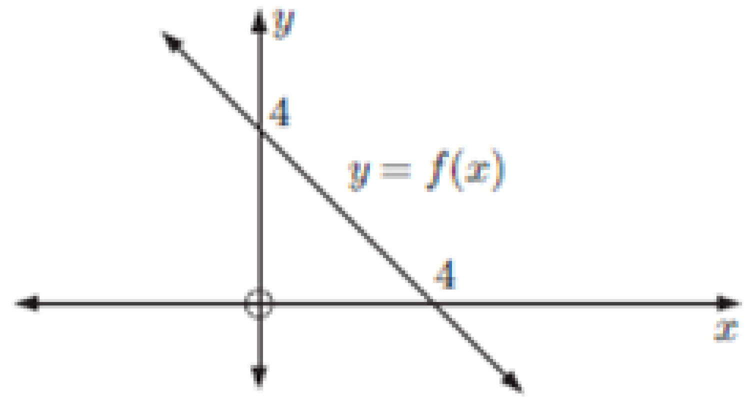
1 Using the graph below, find:

a $f(2)$ b $f'(2)$

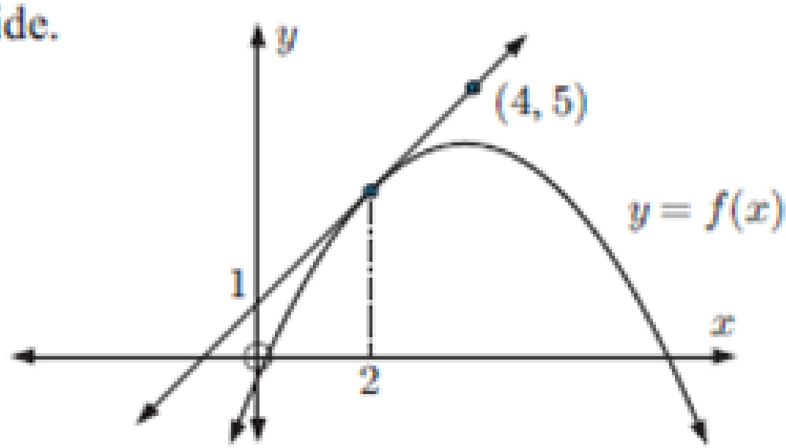


2 Using the graph below, find:

a $f(0)$ b $f'(0)$



3 Consider the graph alongside.
Find $f(2)$ and $f'(2)$.



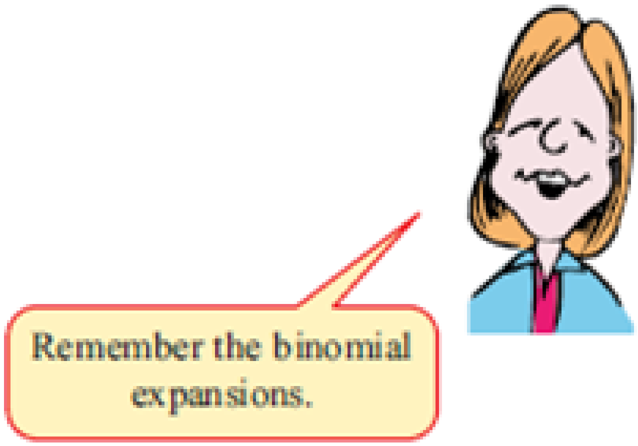
EXERCISE 14E

1 a Find, from first principles, the gradient function of $f(x)$ where $f(x)$ is:

i x ii 5

iii x^3 iv x^4

b Hence predict a formula for $f'(x)$ where $f(x) = x^n$, $n \in \mathbb{N}$.



2 Find $f'(x)$ from first principles, given that $f(x)$ is:

a $2x + 5$ b $x^2 - 3x$ c $-x^2 + 5x - 3$

3 Find $\frac{dy}{dx}$ from first principles given:

a $y = 4 - x$ b $y = 2x^2 + x - 1$ c $y = x^3 - 2x^2 + 3$

4 Use the first principles formula $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find:

a $f'(2)$ for $f(x) = x^3$ b $f'(3)$ for $f(x) = x^4$.

5 Use the first principles formula to find the gradient of the tangent to:

a $f(x) = 3x + 5$ at $x = -2$ b $f(x) = 5 - 2x^2$ at $x = 3$

c $f(x) = x^2 + 3x - 4$ at $x = 3$ d $f(x) = 5 - 2x - 3x^2$ at $x = -2$

6 a Given $y = x^3 - 3x$, find $\frac{dy}{dx}$ from first principles.

b *Hence* find the points on the graph at which the tangent has zero gradient.