$$\text{B) } \lim_{h\to 0}\frac{(h+3)^2-9}{h} = \frac{0}{0} \leftarrow \text{what!?!}$$

$$= \lim_{h\to 0}\frac{(h^2+6h+9)-9}{h}$$

$$= \lim_{h\to 0}\frac{h^2+6h}{h}$$

$$= \lim_{h\to 0}\frac{h(h+6)}{h}$$

$$= \lim_{h\to 0} h+6 = 0+6 = 6$$

Challenge:

$$\text{C) } \lim_{h\to 16}\frac{\sqrt{h}-4}{h-16} \leftarrow \text{factor}$$

$$= \lim_{h\to 16}\frac{\cancel{\sqrt{h}-4}}{\cancel{(\sqrt{h}-4)}(\sqrt{h}+4)}$$

$$= \lim_{h\to 16}\frac{1}{\sqrt{h}+4} = \frac{1}{\sqrt{16}+4}$$

$$= \frac{1}{8}$$

CH 14 B – Limits at Infinity

$$f(x) = \frac{1}{x}$$

VA at $x=0$

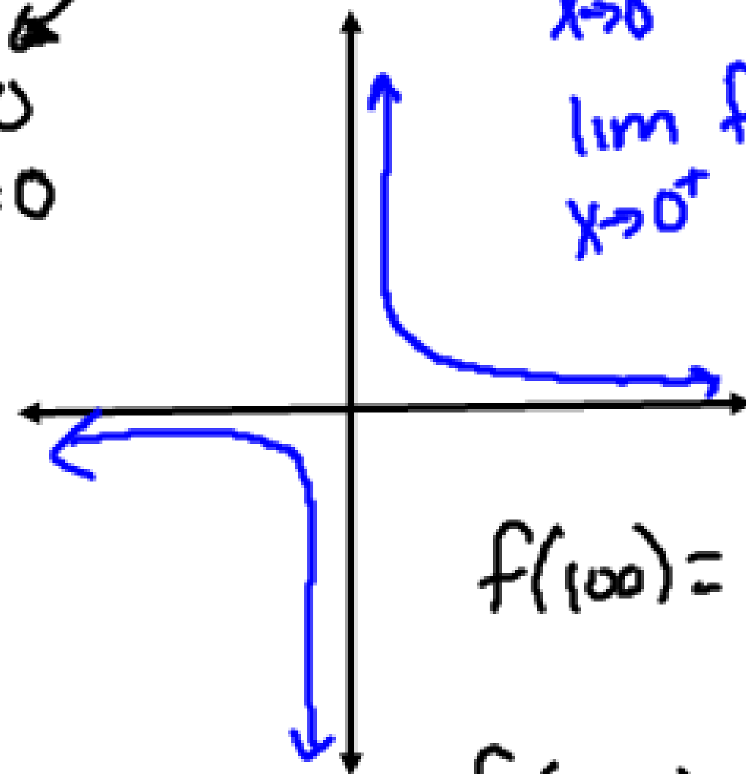
HA at $y=0$

long term behaviour

x	y
-2	$-\frac{1}{2}$
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$

$$\lim_{x\to 0^-} f(x) = -\infty$$

$$\lim_{x\to 0^+} f(x) = +\infty$$



$$f(100) = \frac{1}{100}$$

$$f(1000) = \frac{1}{10000}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to -\infty} \frac{1}{x} = 0$$

Horizontal Asymptotes –

The line $y = b$ is a horizontal asymptotes of the graph of the function $y = f(x)$ if either

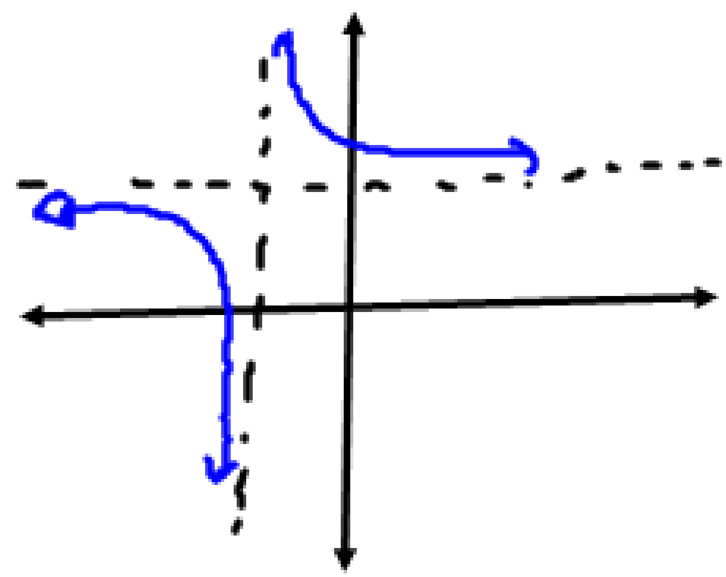
$$\lim_{x\to\infty} f(x) = b \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = b$$

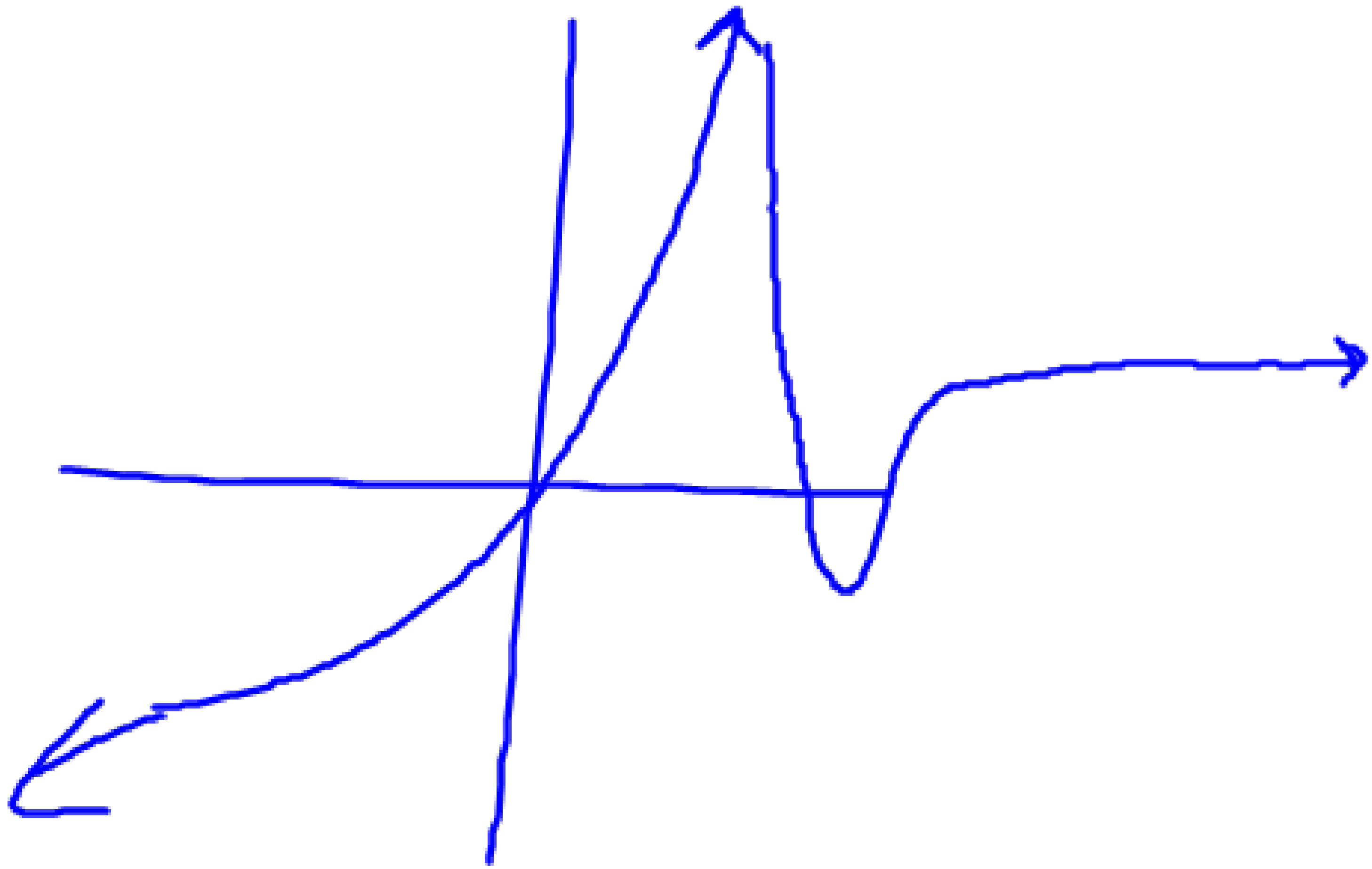
Example: find $\lim\limits_{x\to\infty} f(x)$ if $f(x) = \dfrac{1}{x+1} + 3$

VT+3

HT −1

$$\lim_{x\to\infty} \frac{1}{x+1} + 3 = 0 + 3$$





Example: find $\lim\limits_{x\to\infty} f(x)$ if $f(x) = \dfrac{4x-5}{2x+1}$

$$\lim_{x\to\infty} \frac{4x-5}{2x+1} = \frac{\cancel{x}\left(4-\frac{5}{x}\right)}{\cancel{x}\left(2+\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{4-\frac{5}{x}}{2+\frac{1}{x}} = \frac{4-0}{2+0} = 2$$

Vertical Asymptotes –

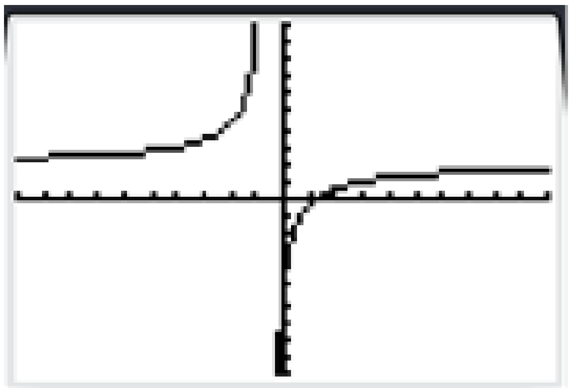
$\frac{1}{x^2}$ 

The line $x = a$ is a vertical asymptotes of the graph of the function $y = f(x)$ if either

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^-} f(x) = \pm\infty$$

Example: find $\lim\limits_{x \to \frac{-1}{2}^+} f(x)$ and $\lim\limits_{x \to \frac{-1}{2}^-} f(x)$ if $f(x) = \dfrac{4x-5}{2x+1}$ $= \dfrac{-7}{0}$

$2\left(-\frac{1}{2}\right)+1 = 0$



X	Y_1
-2	4.3333
-1.5	5.5
-1	9
-.5	ERROR
0	-5
.5	-1.5
1	-.3333

Press + for ΔTbl

$$\lim_{x \to -\frac{1}{2}^-} \frac{4x-5}{2x+1} = +\infty \qquad \lim_{x \to -\frac{1}{2}^+} \frac{4x-5}{2x+1} = -\infty$$

EXERCISE 14B

1 For each of the following functions:

i discuss the behaviour near the asymptotes and hence deduce their equations

ii state the values of $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to\infty} f(x)$.

a $f(x) = \dfrac{1}{x}$

b $f(x) = \dfrac{3x-2}{x+3}$

c $f(x) = \dfrac{1-2x}{3x+2}$

d $f(x) = \dfrac{x}{1-x}$

e $f(x) = \dfrac{x^2-1}{x^2+1}$

f $f(x) = \dfrac{x}{x^2+1}$

2 a Sketch the graph of $y = e^x - 6$.

b Hence discuss the value and geometric interpretation of:

i $\lim\limits_{x\to-\infty} (e^x - 6)$

ii $\lim\limits_{x\to\infty} (e^x - 6)$

3 Find, if possible, $\lim\limits_{x\to-\infty} (2e^{-x} - 3)$ and $\lim\limits_{x\to\infty} (2e^{-x} - 3)$.

+ read ch 14C