B) $\lim_{h \to 0} \frac{(h+3)^2 - 9}{h} = \frac{3^2-9}{0} = \frac{0}{0}$ what?!?

-expand then factor

$$= \lim_{h \to 0} \frac{(h^2+6h+9)-9}{h}$$

$$= \lim_{h \to 0} \frac{h^2+6h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+6)}{\cancel{h}}$$

$$= \lim_{h \to 0} h+6 = 0+6$$

$$= 6$$

Challenge:

C) $\lim_{h \to 16} \frac{\sqrt{h} - 4}{h - 16}$

$$= \lim_{h \to 16} \frac{\cancel{\sqrt{h}-4}}{(\cancel{\sqrt{h}-4})(\sqrt{h}+4)}$$

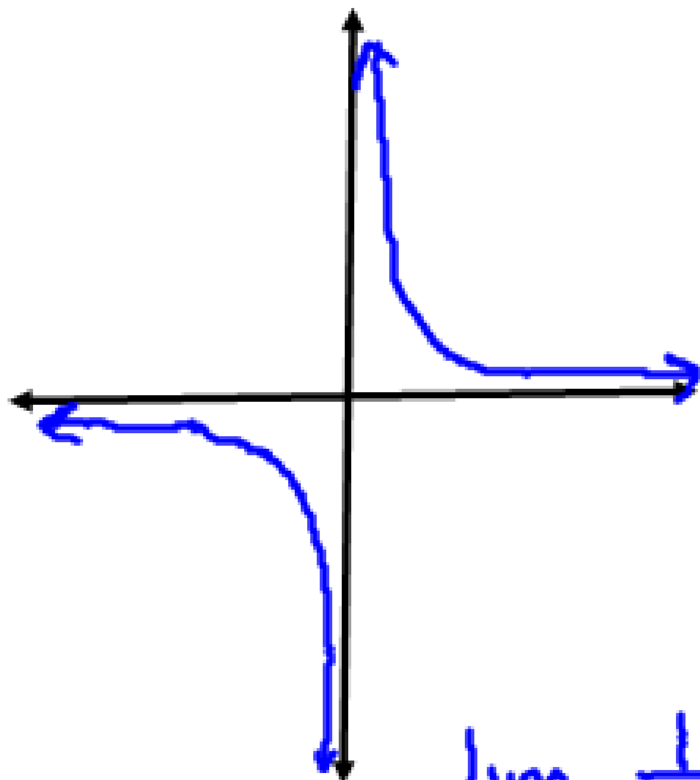
$$= \lim_{h \to 16} \frac{1}{\sqrt{h}+4} = \frac{1}{\sqrt{16}+4}$$

$$= \frac{1}{4+4} = \frac{1}{8}$$

CH 14 B – Limits at Infinity

$$f(x) = \frac{1}{x}$$

x	y
-2	-½
-1	-1
0	undefined
1	1
2	½



x=0 Vertical Asymptote

y=0 Horizontal Asymptote

↳ describes long term behaviour

$$\lim_{x\to\infty} \frac{1}{x} = 0 \quad , \quad \lim_{x\to-\infty} \frac{1}{x} = 0$$

Horizontal Asymptotes –

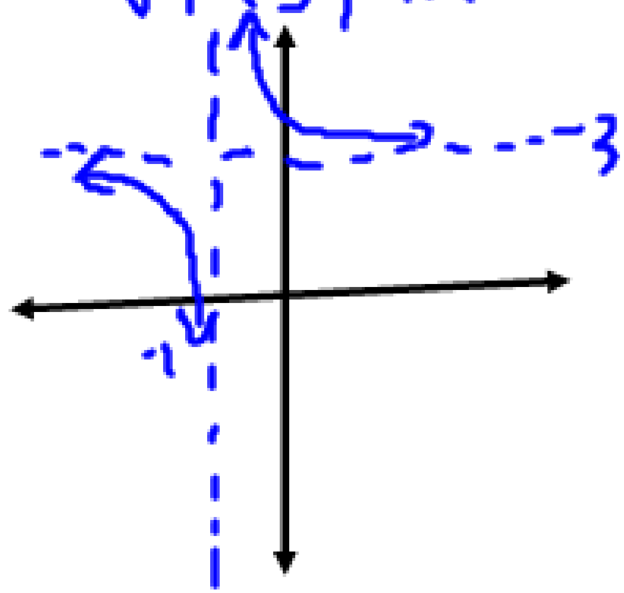
The line $y = b$ is a horizontal asymptotes of the graph of the function $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = b \qquad \text{or} \qquad \lim_{x\to-\infty} f(x) = b$$

Example: find $\lim\limits_{x\to\infty} f(x)$ if $f(x) = \dfrac{1}{x+1} + 3$

$$\lim_{x\to\infty} \frac{1}{x+1} + 3 = 0 + 3$$

VT +3 , HT-1



Example: find $\lim_{x \to \infty} f(x)$ if $f(x) = \dfrac{4x-5}{2x+1}$

$$\lim_{x \to \infty} \frac{4x-5}{2x+1}$$

$$= \lim_{x \to \infty} \frac{\cancel{x}\left(4 - \frac{5}{x}\right)}{\cancel{x}\left(2 + \frac{1}{x}\right)}$$

$$= \lim_{x \to \infty} \frac{4 - 5/x}{2 + 1/x} = \frac{4-0}{2+0} = 2$$

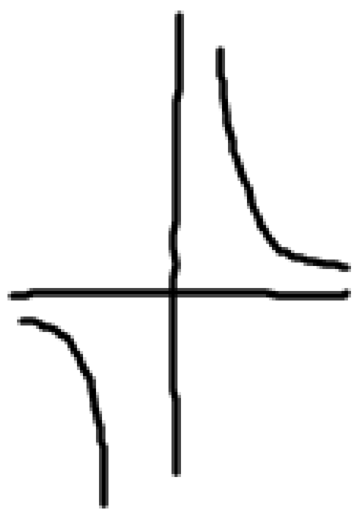
$$= \frac{\cancel{x}\left(4 - \frac{5}{x}\right)}{\cancel{x}\left(2 + \frac{1}{x}\right)}$$

Vertical Asymptotes –

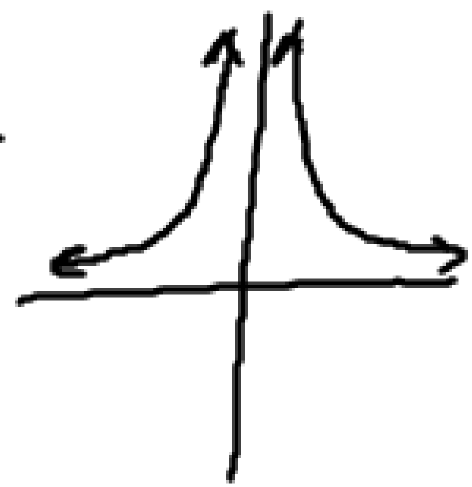
The line $x = a$ is a vertical asymptotes of the graph of the function $y = f(x)$ if either

$\lim\limits_{x \to a^+} f(x) = \pm\infty$ or $\lim\limits_{x \to a^-} f(x) = \pm\infty$

$y = \frac{1}{x}$



$y = \frac{1}{x^2}$



Example: find $\lim\limits_{x \to \frac{-1}{2}^{+}} f(x)$ and $\lim\limits_{x \to \frac{-1}{2}^{-}} f(x)$ if $f(x) = \dfrac{4x-5}{2x+1}$

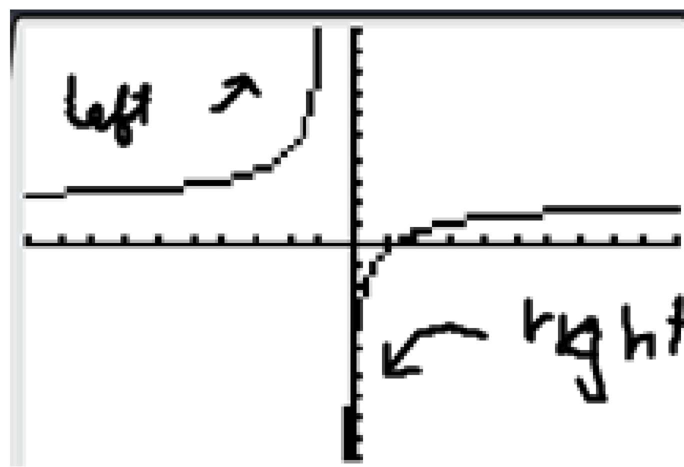
$$\lim_{x \to \frac{-1}{2}^{-}} \frac{4x-5}{2x+1} = +\infty$$

VA come from the denominator

X	Y1	
-1	9	
-.9	10.75	
-.8	13.667	
-.7	19.5	
-.6	37	
-.5	ERROR	
-.4	-33	

Press + for ΔTbl

$$\lim_{x \to \frac{-1}{2}^{+}} \frac{4x-5}{2x+1} = -\infty$$



$Y = e^x$

EXERCISE 14B

1 For each of the following functions:

i discuss the behaviour near the asymptotes and hence deduce their equations

ii state the values of $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

a $f(x) = \frac{1}{x}$ **b** $f(x) = \frac{3x-2}{x+3}$ **c** $f(x) = \frac{1-2x}{3x+2}$

d $f(x) = \frac{x}{1-x}$ **e** $f(x) = \frac{x^2-1}{x^2+1}$ **f** $f(x) = \frac{x}{x^2+1}$

2 **a** Sketch the graph of $y = e^x - 6$.

b Hence discuss the value and geometric interpretation of:

i $\lim_{x \to -\infty} (e^x - 6)$ **ii** $\lim_{x \to \infty} (e^x - 6)$

3 Find, if possible, $\lim_{x \to -\infty} (2e^{-x} - 3)$ and $\lim_{x \to \infty} (2e^{-x} - 3)$.

+ read ch 14C