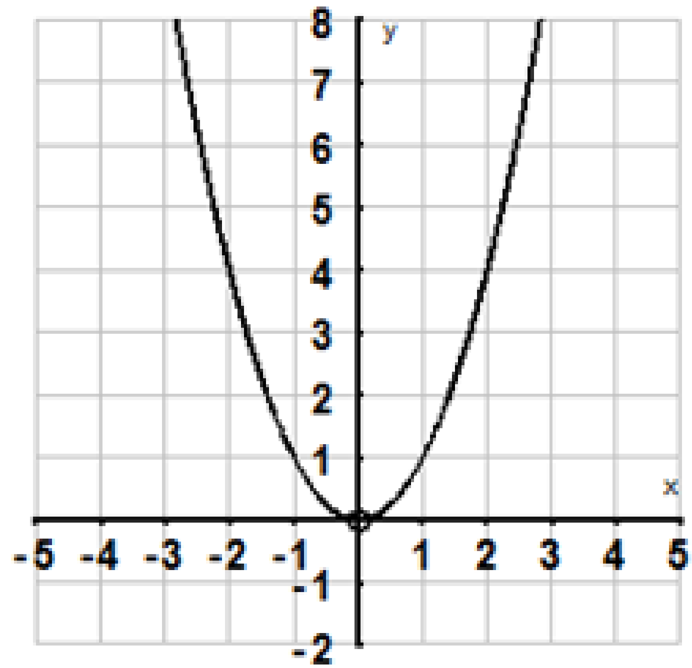
# CH 14 A- Limits

A limit is the value that a function approaches as the input approaches some value.

Notation: $\lim_{x \to a} f(x) = A$

Reads as : the limit of f(x) as x approaches *a* is *A*

Example: $\lim_{x \to 2} x^2 = 4$



We can make the values of f(x) arbitrarily close to *A* (as close as we'd like) by taking x to be "really, really" close to *a*, but not actually equally *a*.

Example : $\lim_{x \to 2} f(x) = 4$ $\qquad f(x) = \begin{cases} x^2, & x \neq 2 \\ 3, & x = 2 \end{cases}$

Piecewise function

○ ← Point of discontinuity

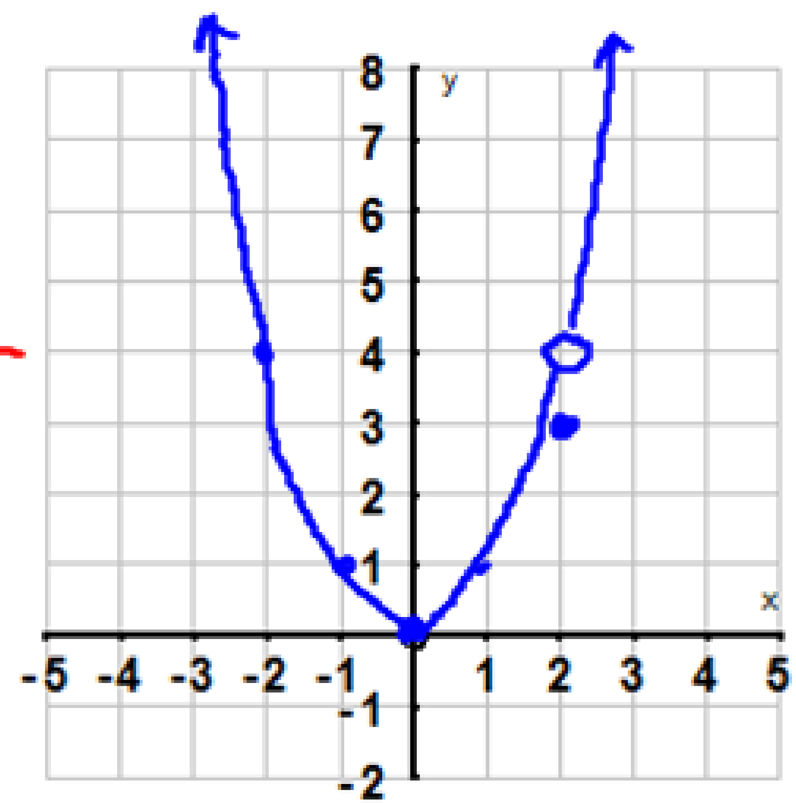
$f(2) \neq 4$ because $x \neq 2$ for $f(x) = x^2$

As x gets close to 2, y gets close to 4.

$f(1) = (1)^2 = 1$

$f(3) = (3)^2 = 9$

$f(2) = 3$



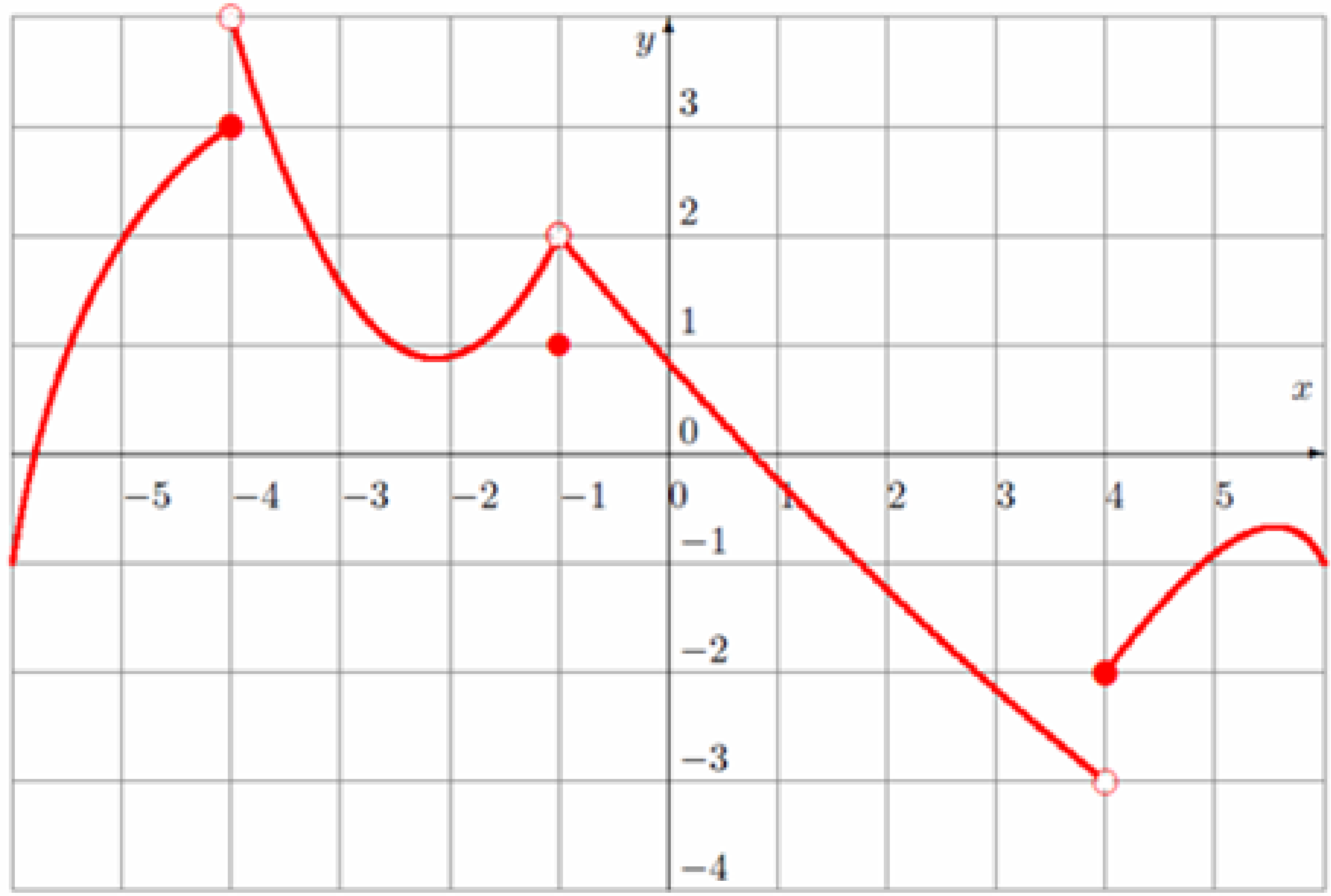
# Formal Definition of a limit:

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x) = A$$

as x approaches a from the Left hand side

as x approaches a from the right hand side

1. Consider the following function defined by its graph:




$\lim\limits_{x\to -4^-} f(x) = 3$

$\lim\limits_{x\to -4^+} f(x) = 4$

$\lim\limits_{x\to 4^-} f(x) = -3$

$\lim\limits_{x\to 4^+} f(x) = -2$

Find the following limits:

a) $\lim\limits_{x\to -1^-} f(x)$ $= 2$

$f(-1) = 1$

b) $\lim\limits_{x\to -1^+} f(x)$ $= 2$

c) $\lim\limits_{x\to -1} f(x)$ $= 2$

d) $\lim\limits_{x\to -4} f(x)$ $=$ DNE

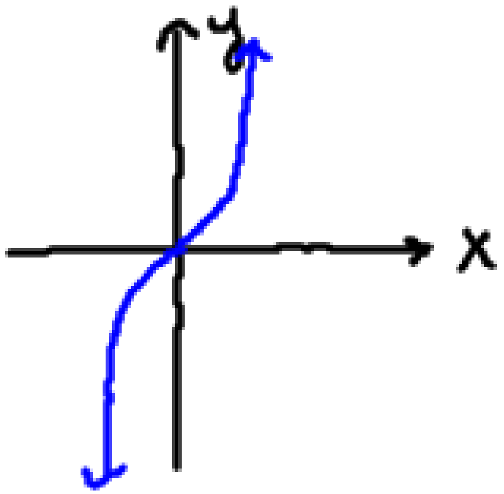
does not exist

e) $\lim\limits_{x\to 4} f(x)$ $=$ DNE

Simple Limit Questions:

A) $\lim_{x \to 3} 2x^3$

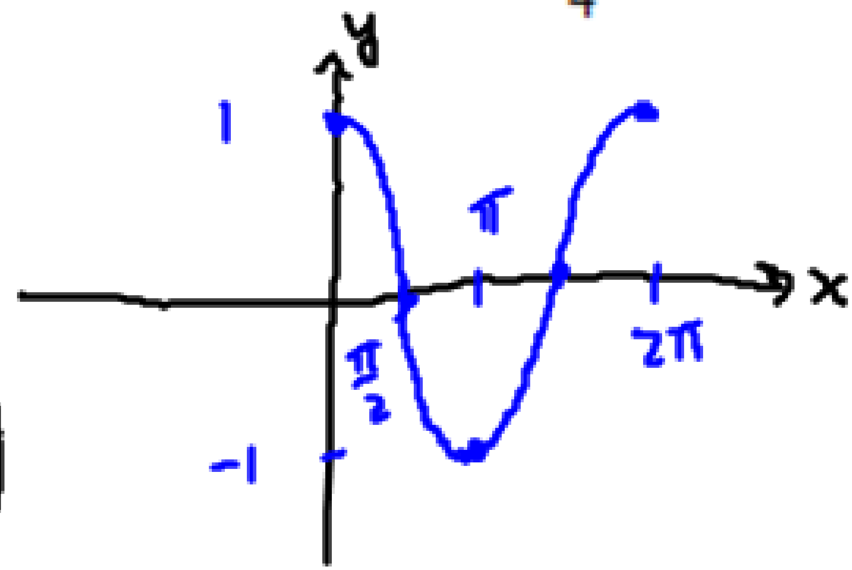
-continous




$$\lim_{x \to 3} 2x^3 = 2(3)^3$$
$$= 2(27)$$
$$= 54$$

B) $\lim_{x \to \frac{\pi}{4}} \cos x$

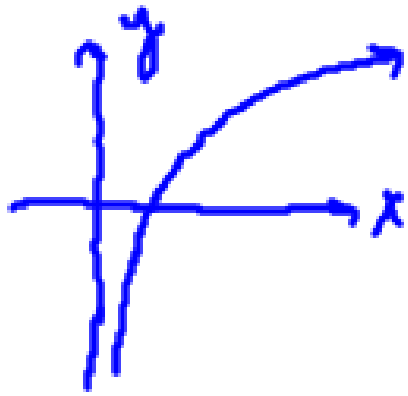
-continuous



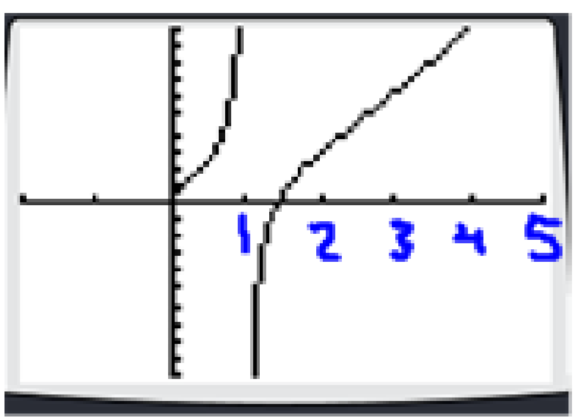

$$\lim_{x \to \frac{\pi}{4}} \cos x = \cos\left(\frac{\pi}{4}\right)$$
$$= \frac{\sqrt{2}}{2}$$

C) $\lim\limits_{x \to 3} \dfrac{x^2 - 2}{\ln x}$

$y = \ln x$



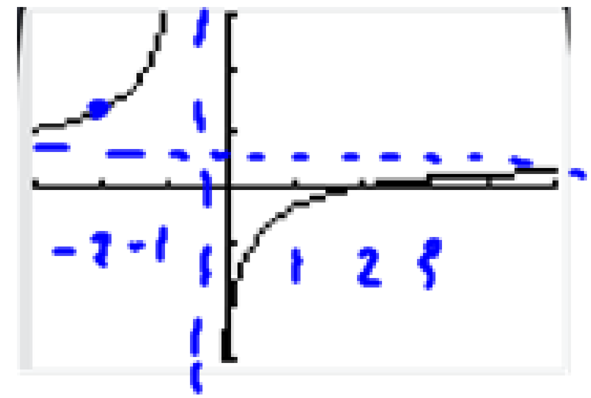

D: $x > 0$




$$\lim_{x \to 3} \frac{x^2-2}{\ln x} = \frac{3^2-2}{\ln 3} = \frac{7}{\ln 3}$$

D) $\lim\limits_{x \to -2} \dfrac{x-2}{2x+1}$

- VA come from denominator
- x-int come from numerator
- HA - long term behaviour.

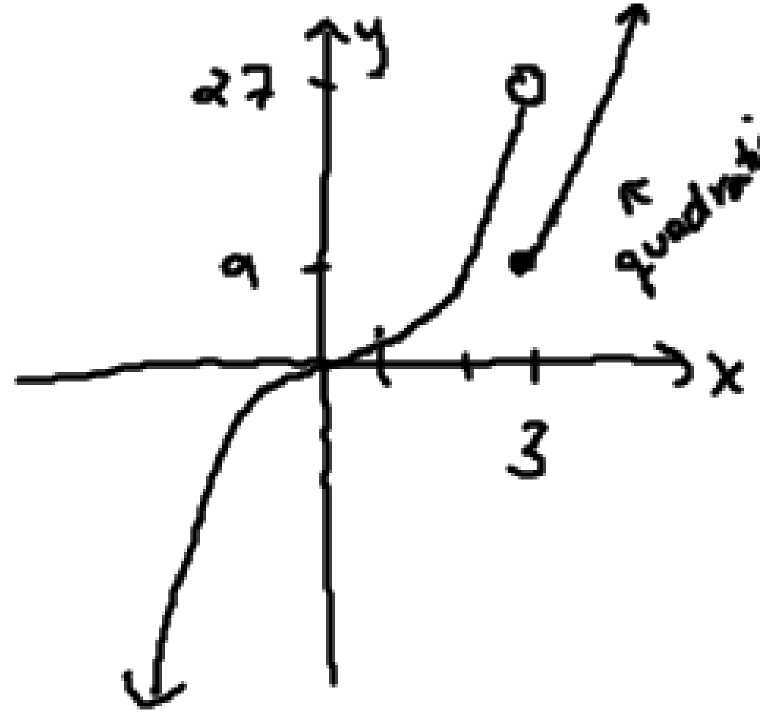



$$\lim_{x \to -2} \frac{x-2}{2x+1} = \frac{-2-2}{2(-2)+1}$$

$$= \frac{-4}{-3}$$

$$= \frac{4}{3}$$

E) Given $g(x)=\begin{cases} x^2, & x \geq 3 \\ x^3, & x < 3 \end{cases}$




i) $\lim\limits_{x \to 5} g(x) = 5^2$

$= 25$

ii) $\lim\limits_{x \to -2} g(x) = (-2)^3$

$= -8$

iii) $\lim\limits_{x \to 3} g(x)$ need to look at

$\lim\limits_{x \to 3^-} g(x) = 3^3$

$= 27$

$\lim\limits_{x \to 3^+} g(x) = 3^2$

$= 9$

Since LHL $\neq$ RHL

$\lim\limits_{x \to 3} g(x) = \text{DNE}$

# Harder limit questions:

# Points of discontinuity:

A) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$ $\rightarrow$ $\lim\limits_{x \to 1} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

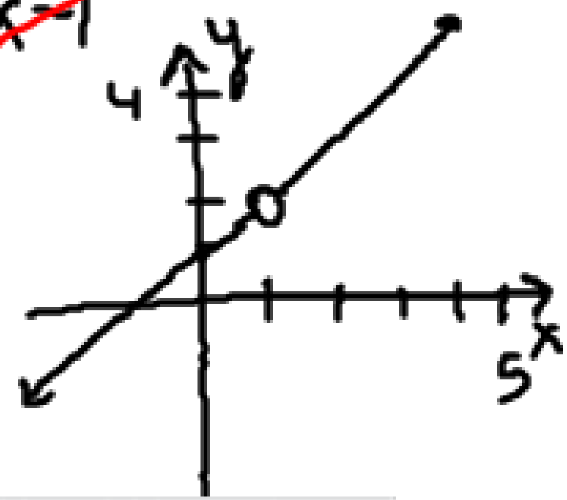
if we were to evaluate directly we would have

$\dfrac{(1)^2 - 1}{1 - 1} = \dfrac{0}{0}$

$\rightarrow$ factor!!

$= \lim\limits_{x \to 1} x + 1$

$= 2$




| X | $Y_1$ |
|---|---|
| .8 | 1.8 |
| .9 | 1.9 |
| 1 | ERROR |
| 1.1 | 2.1 |
| 1.2 | 2.2 |
| 1.3 | 2.3 |
| 1.4 | [illegible] |

$Y_1 = 2.4$

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## EXERCISE 14A

**1** Evaluate:

a $\lim_{x \to 3} (x+4)$ b $\lim_{x \to -1} (5-2x)$ c $\lim_{x \to 4} (3x-1)$

d $\lim_{x \to 2} (5x^2-3x+2)$ e $\lim_{h \to 0} h^2(1-h)$ f $\lim_{x \to 0} (x^2+5)$

**2** Evaluate:

a $\lim_{x \to 0} 5$ b $\lim_{h \to 2} 7$ c $\lim_{x \to 0} c$, $c$ a constant

**3** Evaluate:

a $\lim_{x \to 1} \frac{x^2-3x}{x}$ b $\lim_{h \to 2} \frac{h^2+5h}{h}$ c $\lim_{x \to 0} \frac{x-1}{x+1}$ d $\lim_{x \to 0} \frac{x}{x}$

**4** Evaluate the following limits:

a $\lim_{x \to 0} \frac{x^2-3x}{x}$ b $\lim_{x \to 0} \frac{x^2+5x}{x}$ c $\lim_{x \to 0} \frac{2x^2-x}{x}$

d $\lim_{h \to 0} \frac{2h^2+6h}{h}$ e $\lim_{h \to 0} \frac{3h^2-4h}{h}$ f $\lim_{h \to 0} \frac{h^3-8h}{h}$

g $\lim_{x \to 1} \frac{x^2-x}{x-1}$ h $\lim_{x \to 2} \frac{x^2-2x}{x-2}$ i $\lim_{x \to 3} \frac{x^2-x-6}{x-3}$