

Chapter

14

**Introduction to
differential calculus**

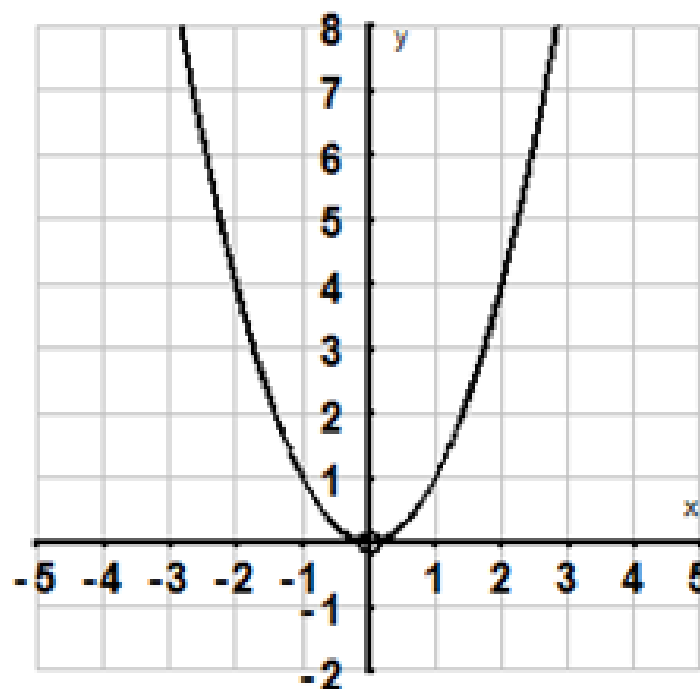
CH 14 A- Limits

A limit is the value that a function, approaches as the input approaches some value.

Notation: $\lim_{x \rightarrow a} f(x) = A$

Reads as : the limit of $f(x)$ as x approaches a is A

Example: $\lim_{x \rightarrow 2} x^2 = 4$



We can make the values of $f(x)$ arbitrarily close to A (as close as we'd like) by taking x to be "really, really" close to a , but not actually equally a .



Example : $\lim_{x \rightarrow 2} f(x)$ $f(x) = \begin{cases} x^2, & x \neq 2 \\ 3, & x = 2 \end{cases}$

Piecewise function

o Point of discontinuity

$f(2) \neq 4$ because $x \neq 2$ for $f(x) = x^2$

$$f(1) = (1)^2 = 1$$

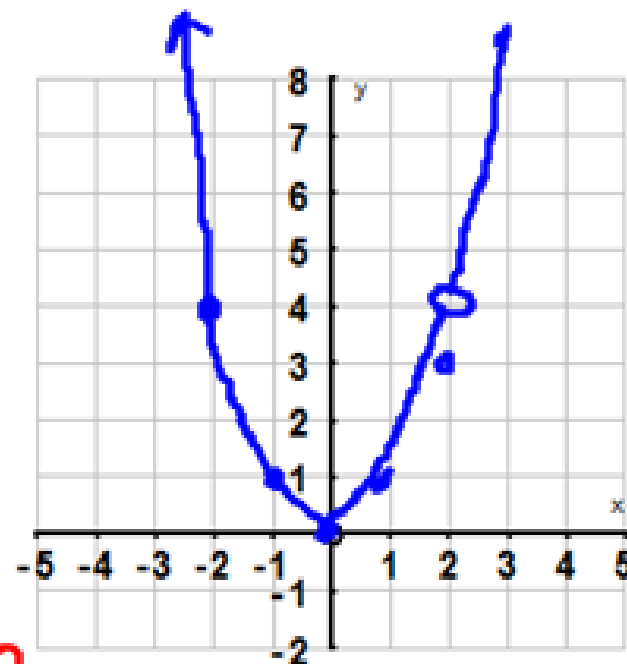
$$f(3) = (3)^2 = 9$$

$$f(2) = 3$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

$$x \rightarrow 2$$

↳ as x gets close to 2
 y gets close to 4



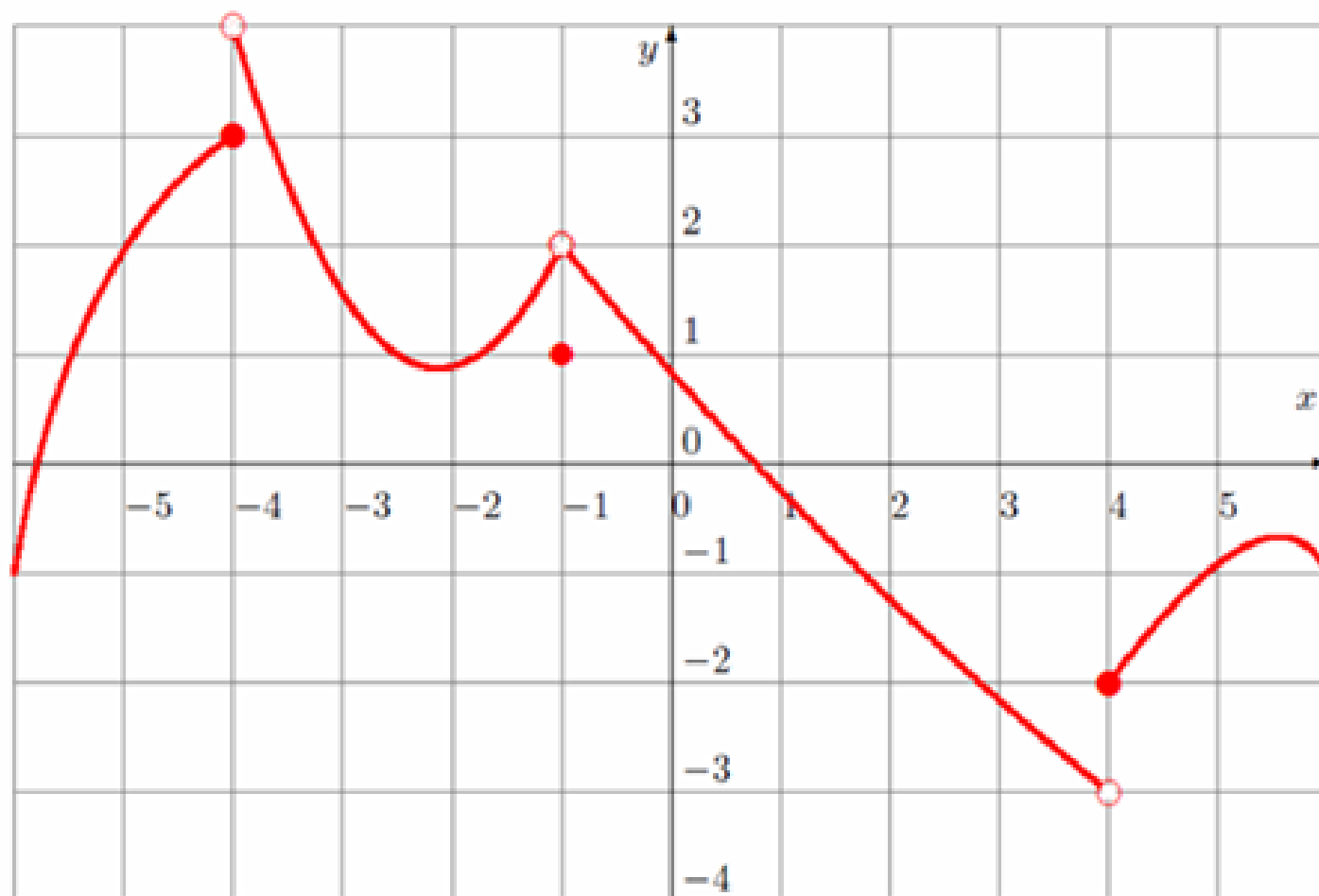
Formal Definition of a limit:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a} f(x) = A$$

↑
as x approaches
"a" from the
left hand side

↑
as x approaches
"a" from the
right hand side

1. Consider the following function defined by its graph:



$$\lim_{x \rightarrow -4^-} f(x) = 3$$

$$\lim_{x \rightarrow -4^+} f(x) = 2$$

$$f(-4) = 2$$

Find the following limits:

a) $\lim_{x \rightarrow -1^-} f(x)$

$$= 2$$

$$f(-1) = 1$$

b) $\lim_{x \rightarrow -1^+} f(x)$

$$= 2$$

c) $\lim_{x \rightarrow -1} f(x)$

$$= 2$$

d) $\lim_{x \rightarrow -4} f(x)$

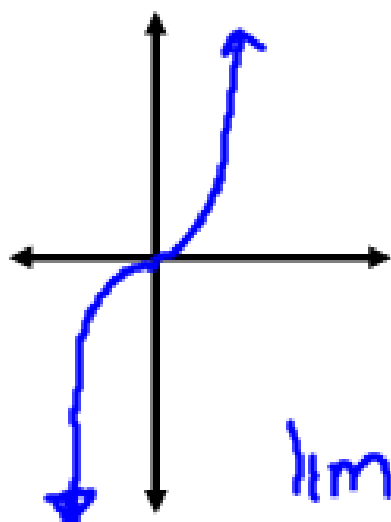
= DNE
does not
exist

e) $\lim_{x \rightarrow 4} f(x)$

= DNE

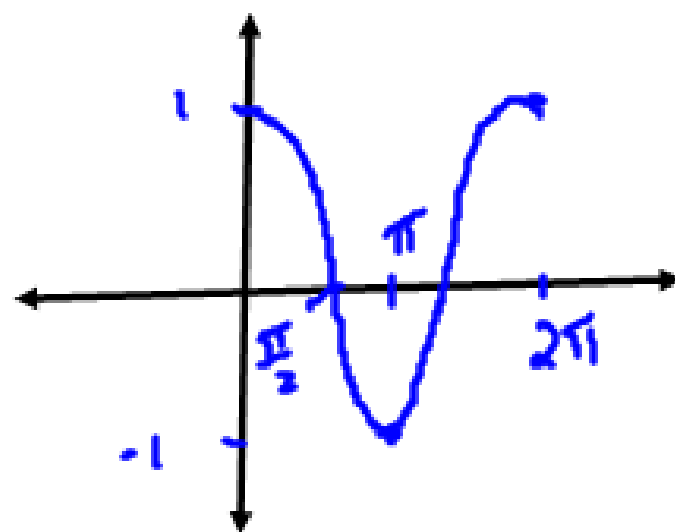
Simple Limit Questions:

A) $\lim_{x \rightarrow 3} 2x^3$ - continuous



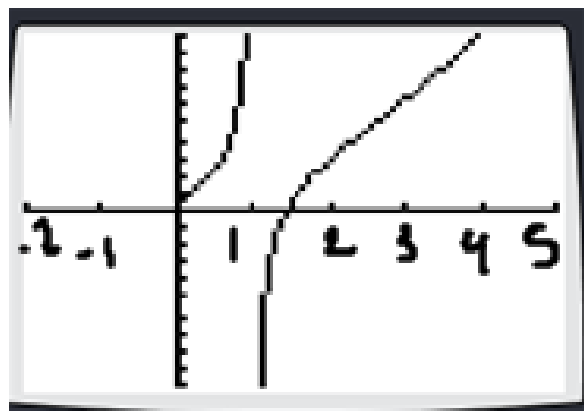
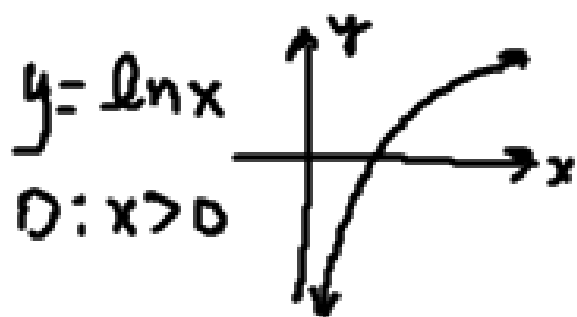
$$\begin{aligned}\lim_{x \rightarrow 3} 2x^3 &= 2(3)^3 \\ &= 2(27) \\ &= 54\end{aligned}$$

B) $\lim_{x \rightarrow \frac{\pi}{4}} \cos x$ - continuous



$$\begin{aligned}\lim_{x \rightarrow \frac{\pi}{4}} \cos x &= \cos\left(\frac{\pi}{4}\right) \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

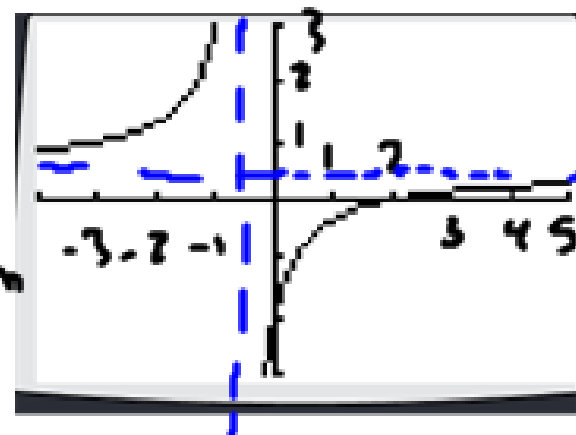
$$C) \lim_{x \rightarrow 3} \frac{x^2 - 2}{\ln x}$$



$$\lim_{x \rightarrow 3} \frac{x^2 - 2}{\ln x} = \frac{(3)^2 - 2}{\ln 3} = \frac{7}{\ln 3}$$

$$D) \lim_{x \rightarrow -2} \frac{x - 2}{2x + 1}$$

- VA come from the denominator
- x-int come from the numerator



- HA: long term behaviour

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{x - 2}{2x + 1} &= \frac{-2 - 2}{2(-2) + 1} \\ &= \frac{-4}{-3} \\ &= \frac{4}{3} \end{aligned}$$

$$E) \text{ Given } g(x) = \begin{cases} x^2, & x \geq 3 \\ x^3, & x < 3 \end{cases}$$

$$i) \lim_{x \rightarrow 5} g(x) = (5)^2 = 25$$

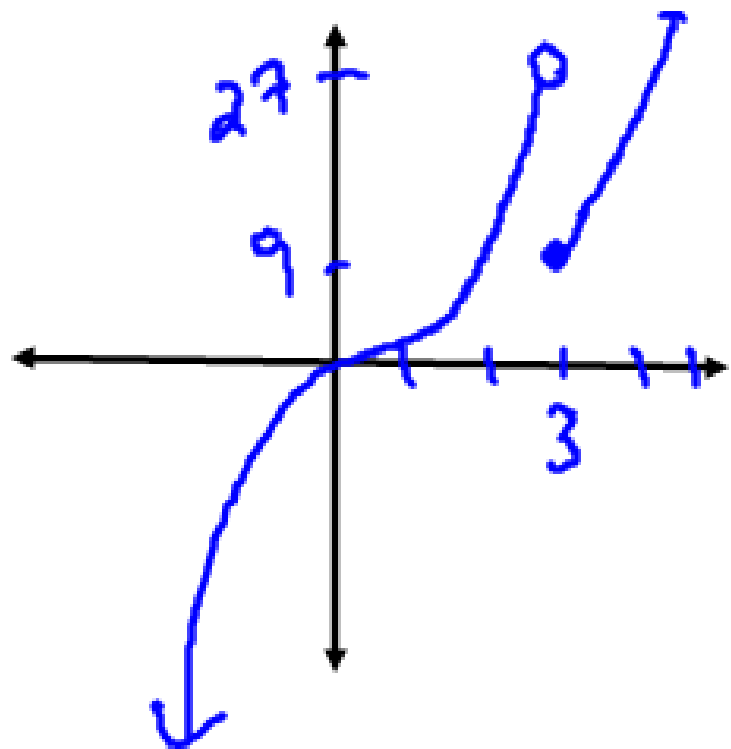
$$ii) \lim_{x \rightarrow -2} g(x) = (-2)^3 = -8$$

iii) $\lim_{x \rightarrow 3} g(x)$ need to look at

$$\lim_{x \rightarrow 3^-} g(x) = (3)^3 = 27$$

$$\lim_{x \rightarrow 3^+} g(x) = (3)^2 = 9$$

$$\lim_{x \rightarrow 3} g(x) = \text{DNE}$$



Harder limit questions:

Points of discontinuity:

$$A) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

If we were to evaluate directly:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \frac{(1)^2 - 1}{1 - 1}$$

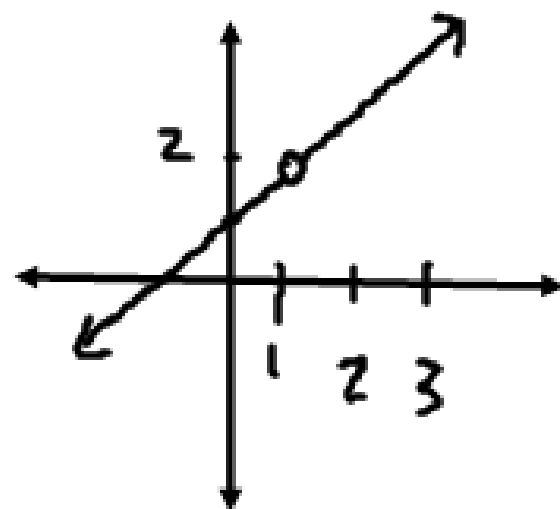
oh no!

Factor!!!

$$\lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{x-1}$$

When factors cancel, we get holes!

$$\begin{aligned} &= \lim_{x \rightarrow 1} x + 1 \\ &= 2 \end{aligned}$$



{f(1) DNE}

X	Y1
.98	1.98
.99	1.99
1	ERROR
1.01	2.01
1.02	2.02
1.03	2.03
1.04	2.04

X=1.04

EXERCISE 14A

1 Evaluate:

a $\lim_{x \rightarrow 3} (x + 4)$

b $\lim_{x \rightarrow -1} (5 - 2x)$

c $\lim_{x \rightarrow 4} (3x - 1)$

d $\lim_{x \rightarrow 2} (5x^2 - 3x + 2)$

e $\lim_{h \rightarrow 0} h^2(1 - h)$

f $\lim_{x \rightarrow 0} (x^2 + 5)$

2 Evaluate:

a $\lim_{x \rightarrow 0} 5$

b $\lim_{h \rightarrow 2} 7$

c $\lim_{x \rightarrow 0} c$, c a constant

3 Evaluate:

a $\lim_{x \rightarrow 1} \frac{x^2 - 3x}{x}$

b $\lim_{h \rightarrow 2} \frac{h^2 + 5h}{h}$

c $\lim_{x \rightarrow 0} \frac{x - 1}{x + 1}$

d $\lim_{x \rightarrow 0} \frac{x}{x}$

4 Evaluate the following limits:

a $\lim_{x \rightarrow 0} \frac{x^2 - 3x}{x} = \lim_{x \rightarrow 0} x - 3$

b $\lim_{x \rightarrow 0} \frac{x^2 + 5x}{x}$

c $\lim_{x \rightarrow 0} \frac{2x^2 - x}{x}$

d $\lim_{h \rightarrow 0} \frac{2h^2 + 6h}{h}$

e $\lim_{h \rightarrow 0} \frac{3h^2 - 4h}{h}$

f $\lim_{h \rightarrow 0} \frac{h^3 - 8h}{h}$

g $\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1}$

h $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x - 2}$

i $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$