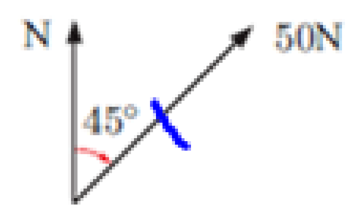
Ch 12 A – Vectors and Scalars

Vector: A quantity with a magnitude and a direction
Ex: velocity, force, displacement

Scalar : A quantity that has only a magnitude only
Ex: speed

We can represent a vector quantity using a directed line segment

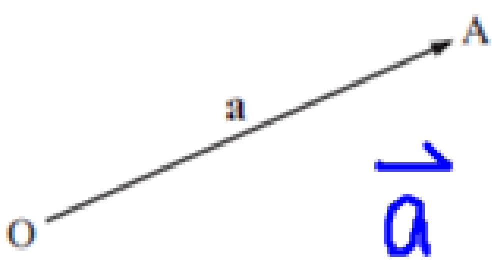
For example, if farmer Giles pushes the post with a force of 50 Newtons (N) to the north-east, we can draw a scale diagram of the force relative to the north line.



Scale: 1 cm represents 25 N

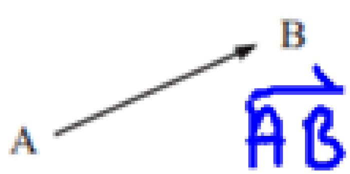
Vector Notation:

Position vector: from the origin to point A

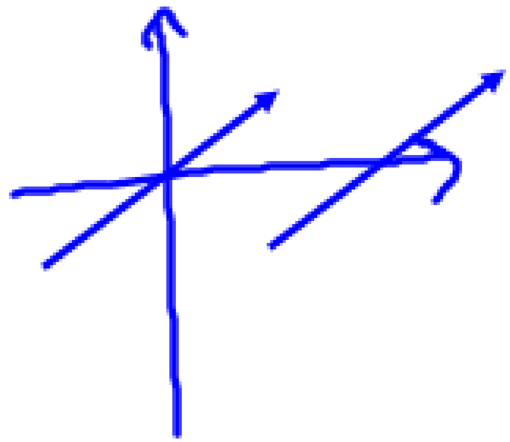


- This position vector could be represented by
  $\overrightarrow{OA}$ or $\mathbf{a}$ (bold used in text books) or $\bar{a}$ or $\vec{a}$ (used by students)
- The magnitude or length could be represented by
  $|\overrightarrow{OA}|$ or OA or $|\mathbf{a}|$ or $|\bar{a}|$ or $|\vec{a}|$

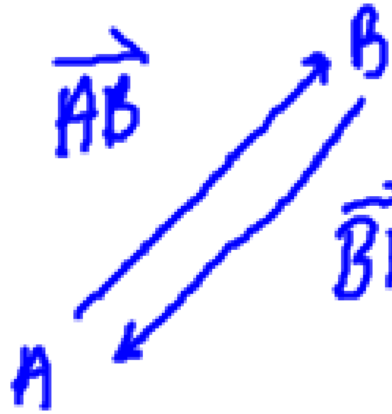
If we have a vector that STARTS at point A and ENDS at point B, we say that it is the position vector of B relative to A



Two vectors are equal if the have the same direction and magnitude



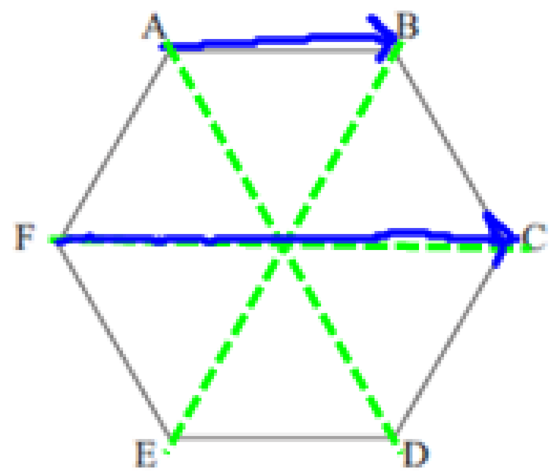
Geometric Negative vectors:



$\overrightarrow{BA} = -\overrightarrow{AB}$

Example: pg 279

3



ABCDEF is a regular hexagon.

a Write down the vector which:

- **i** originates at B and terminates at C
- **ii** is equal to $\overrightarrow{AB}$.

b Write down *all* vectors which:

- **i** are the negative of $\overrightarrow{EF}$
- **ii** have the same length as $\overrightarrow{ED}$.

c Write down a vector which is parallel to $\overrightarrow{AB}$ and twice its length.

A) i) $\overrightarrow{BC}$

ii) $\overrightarrow{ED}$

B) i) $\overrightarrow{BC}$

$\overrightarrow{FE}$

ii) $\overrightarrow{AB}, \overrightarrow{BA}$ | $\overrightarrow{EF}, \overrightarrow{FE}$

$\overrightarrow{BC}, \overrightarrow{CB}$ | $\overrightarrow{AF}, \overrightarrow{FA}$

$\overrightarrow{CD}, \overrightarrow{DC}$

$\overrightarrow{DE}$

c) $\overrightarrow{FC}, \overrightarrow{CF}$

Ch 12 B – Geometric Operations with Vectors

Zero Vector: a vector of length 0

Vector Addition:

To construct **a** + **b**:

Step 1: Draw **a**.

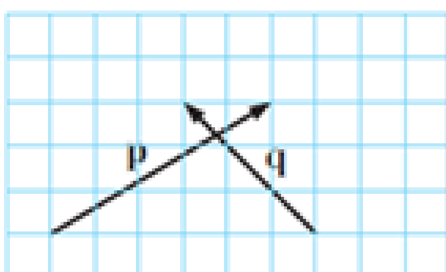
Step 2: At the arrowhead end of **a**, draw **b**.

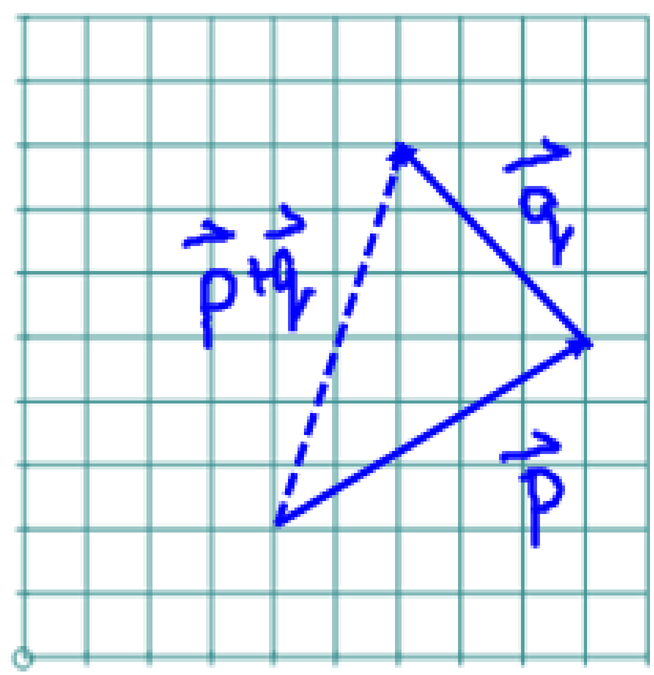
Step 3: Join the beginning of **a** to the arrowhead end of **b**. This is vector **a** + **b**.

$\vec{p}+\vec{q}$

Example pg 281 #1f

f





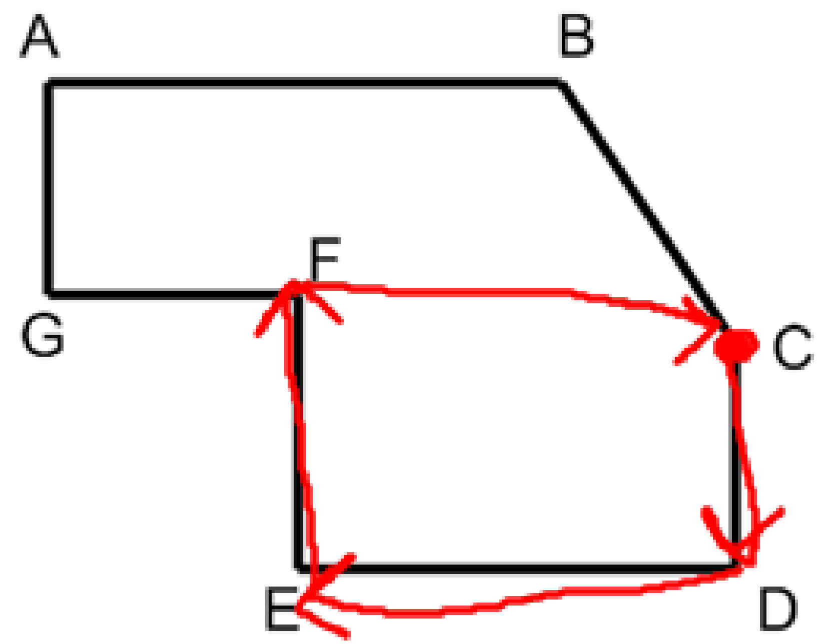
Find a single vector which is equal to :

$\overrightarrow{AB} + \overline{BF} = \overrightarrow{AF}$

$\overline{GF} + \overline{FE} = \overrightarrow{GE}$

$\overline{AB} + \overline{BC} + \overline{CD} = \overrightarrow{AD}$

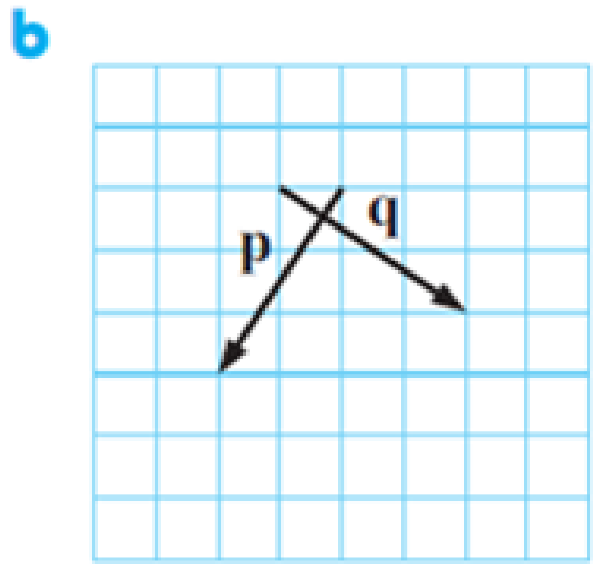
$\overline{CD} + \overline{DE} + \overline{EF} + \overline{FC} = \overrightarrow{CC} = \vec{0}$ ← zero vector

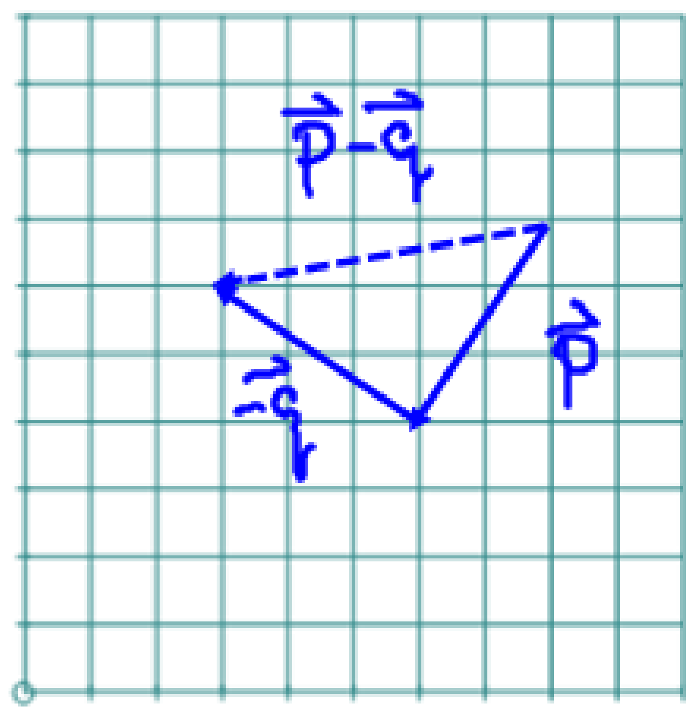


Vector Subtraction: Add its negative

$\vec{p} - \vec{q} = \vec{p} + (-\vec{q})$

Ex: pg 282 #1b



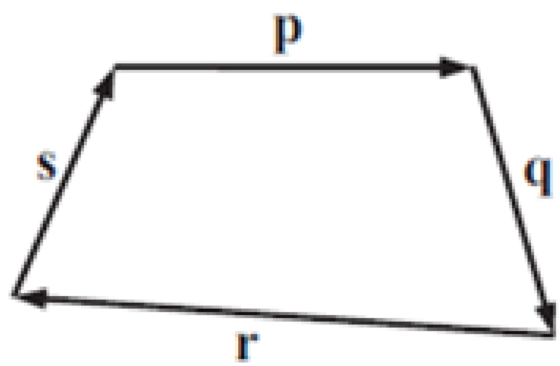


Vector Equations

EXERCISE 12B.3

1 Construct vector equations for:

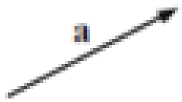
c

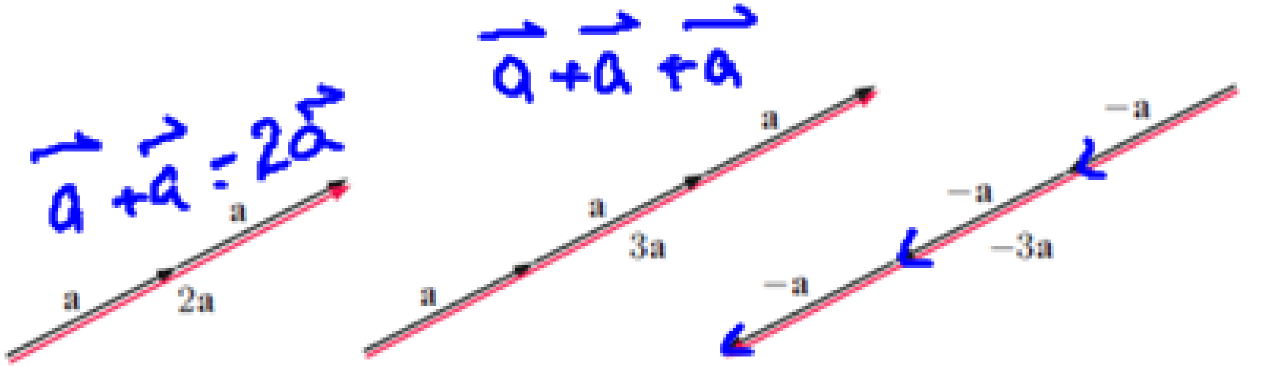


$\vec{r} = -\vec{s} - \vec{p} - \vec{q}$

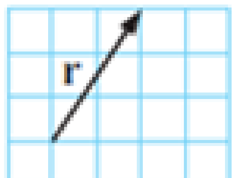
$\vec{s} + \vec{p} + \vec{q} = -\vec{r}$

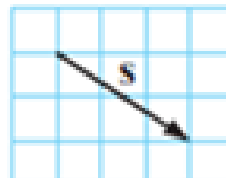
Geometric Scalar Multiplication

If a is 

then 

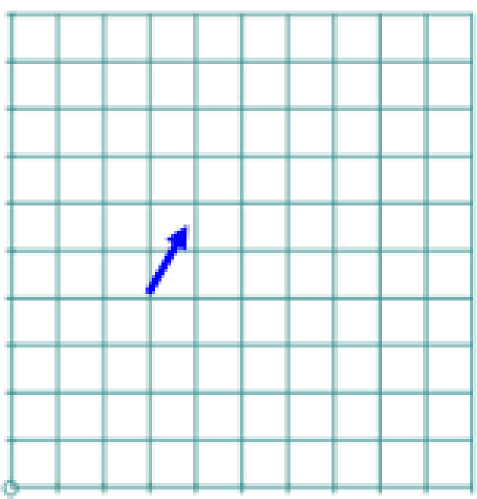
EXERCISE 12B.4

1 Given vectors 

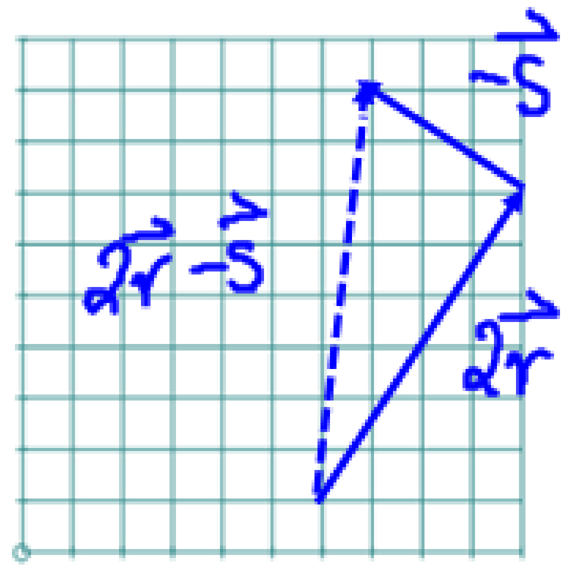
and 

, construct geometrically:

c $\frac{1}{2}$r



e 2r $-$ s



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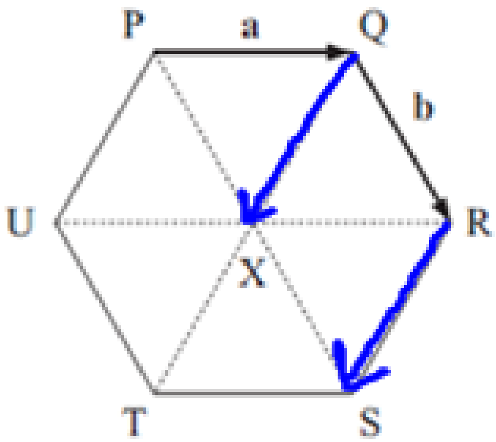


PQRSTU is a regular hexagon.

If $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$, find in terms of $\mathbf{a}$ and $\mathbf{b}$:

a $\overrightarrow{PX}$

$\vec{b}$

b $\overrightarrow{PS}$

$2\vec{b}$

c $\overrightarrow{QX}$

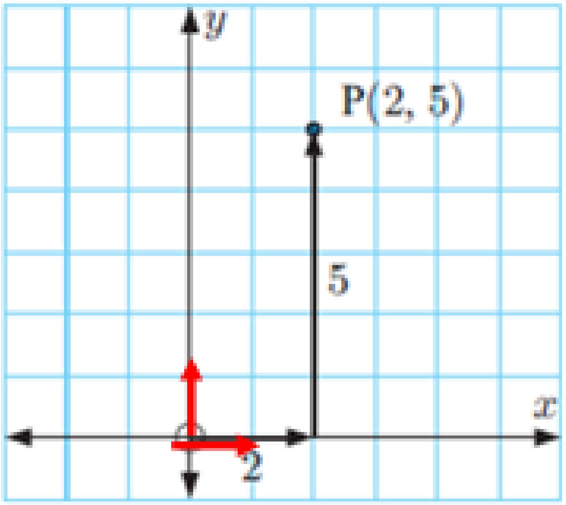
$\vec{b} - \vec{a}$

d $\overrightarrow{RS}$

$\vec{b} - \vec{a}$

parallel to $\overrightarrow{QX}$

CH 12 C – Vectors in the Plane



The coordinate point is (2,5). The vector is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ 2 units in the x-direction and 5 units in the y-direction

$\vec{i}$ $\quad$ $i=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\leftarrow$ unit vector
1 unit in the x direction
0 units in the y-direction

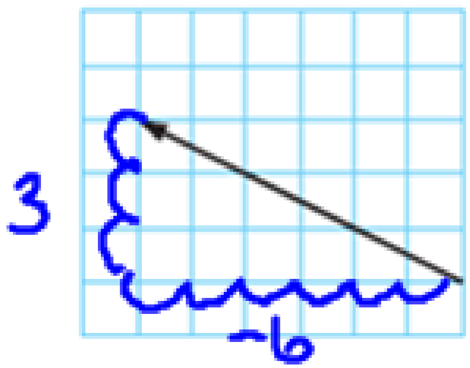
$\vec{j}$ $\quad$ $j=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\leftarrow$ unit vector
0 units in x-direction
1 y

EXERCISE 12C

1 Write the illustrated vectors in component form and in unit vector form:

e

comp. form $\begin{pmatrix}-6\\3\end{pmatrix}$

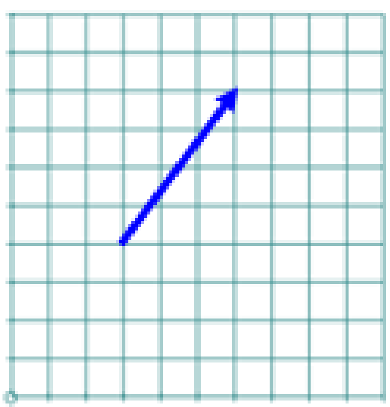


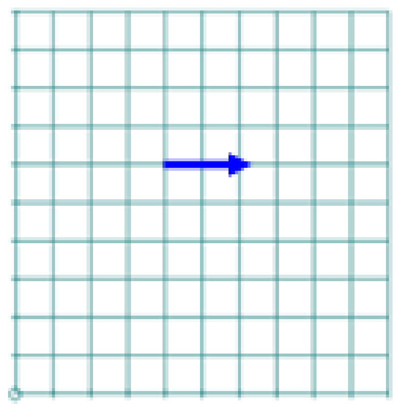
unit vector form $-6\vec{i}+3\vec{j}$

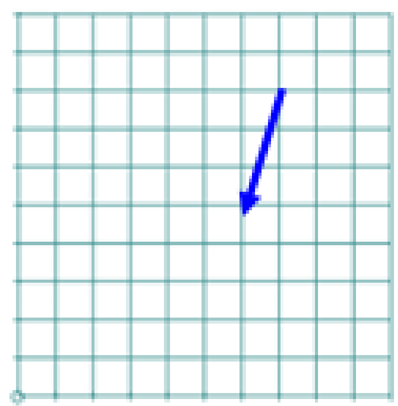
2 Write each vector in unit vector form, and illustrate it using an arrow diagram:

a $\begin{pmatrix}3\\4\end{pmatrix}=3\vec{i}+4\vec{j}$ b $\begin{pmatrix}2\\0\end{pmatrix}=2\vec{i}+0\vec{j}$ c $\begin{pmatrix}2\\-5\end{pmatrix}$ d $\begin{pmatrix}-1\\-3\end{pmatrix}$

$=-1\vec{i}-3\vec{j}$







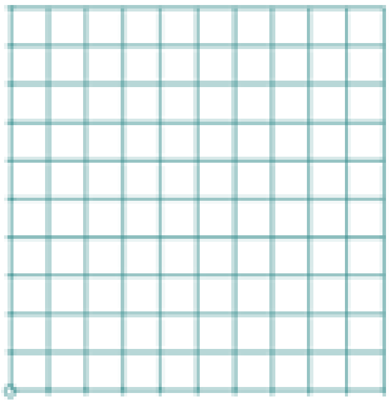
4 Write in component form and illustrate using a directed line segment:

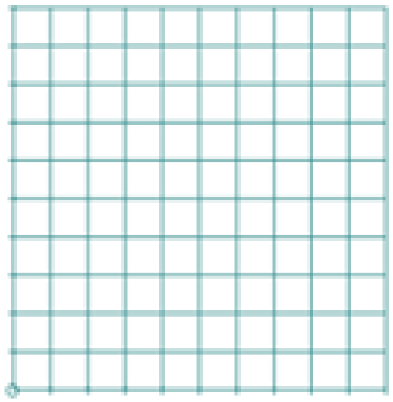
a $\mathbf{i} + 2\mathbf{j}$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

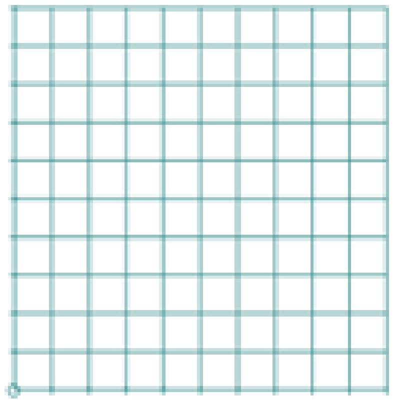
b $-\mathbf{i} + 3\mathbf{j}$ $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

c $-5\mathbf{j}$ $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

d $4\mathbf{i} - 2\mathbf{j}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$







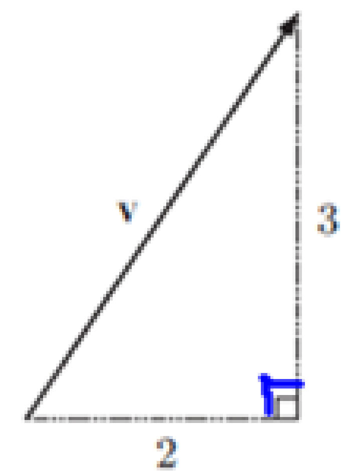
Ch 12 D – The Magnitude of a Vector

Consider vector $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2\mathbf{i} + 3\mathbf{j}$.

The magnitude or length of $\mathbf{v}$ is represented by $|\mathbf{v}|$.

By Pythagoras, $|\mathbf{v}|^2 = 2^2 + 3^2 = 4 + 9 = 13$

$\therefore \quad |\mathbf{v}| = \sqrt{13}$ units $\quad \{$since $\quad |\mathbf{v}| > 0\}$



If $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j}$, the magnitude or length of $\mathbf{v}$ is $|\mathbf{v}| = \sqrt{{v_1}^2 + {v_2}^2}$.

EXERCISE 12D

1 Find the magnitude of:

a $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$|v|^2 = 3^2 + 4^2$

$= 9 + 16$

$= 25$

$|v| = 5$

b $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$|v| = \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16+9}$

$= 5$

2 Find the length of:

a $i + j = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|v| = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

b $5i - 12j$

$\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

$|v| = \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

Pythagorean Triples

3-4-5, 5-12-13,

3 Which of the following are unit vectors?

a $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ b $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ c $\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$

length of 1 unit

$|\vec{v}| = \sqrt{0^2+(-1)^2}$

$= \sqrt{1}$

$= 1$

Yes!

$|\vec{v}| = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2}+\frac{1}{2}}$

$= \sqrt{1}$

Yes!

$|\vec{v}| = \sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)^2}$

$= \sqrt{\frac{4}{9}+\frac{1}{9}}$

$= \sqrt{\frac{5}{9}}$

$= \frac{\sqrt{5}}{3}$

No!