Ch 12 A – Vectors and Scalars

Vector: A quantity with a magnitude and a direction

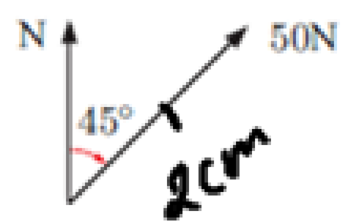
Ex: velocity, force, displacement

Scalar : A quantity that has only a magnitude only

Ex: speed

We can represent a vector quantity using a directed line segment

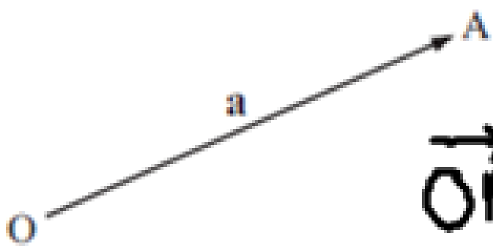
For example, if farmer Giles pushes the post with a force of 50 Newtons (N) to the north-east, we can draw a scale diagram of the force relative to the north line.



Scale: 1 cm represents 25 N

Vector Notation:

Position vector: from the origin to point A



$\overrightarrow{OA}$

- This position vector could be represented by

  $\overrightarrow{OA}$ or $\mathbf{a}$ or $\tilde{a}$ or $\vec{a}$

  bold used in text books   used by students

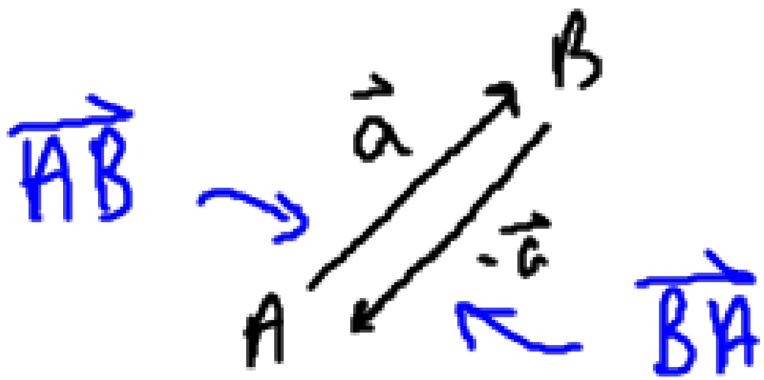
- The magnitude or length could be represented by

  $|\overrightarrow{OA}|$ or OA or $|\mathbf{a}|$ or $|\tilde{a}|$ or $|\vec{a}|$

If we have a vector that STARTS at point A and ENDS at point B, we say that it is the position vector of B relative to A

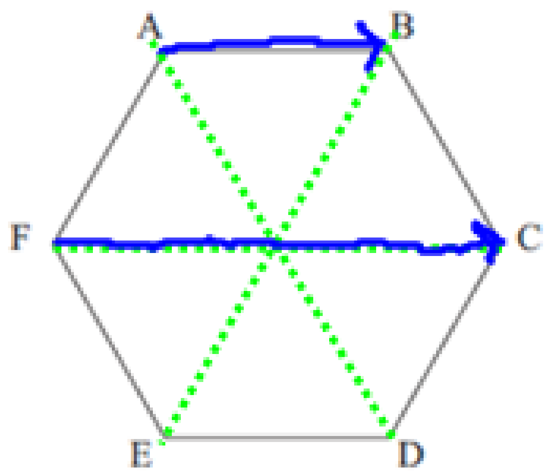
Two vectors are equal if the have they same direction and magnitude.

Geometric Negative vectors:



Example: pg 279

3



ABCDEF is a regular hexagon.

a Write down the vector which:

- **i** originates at B and terminates at C
- **ii** is equal to $\overrightarrow{AB}$.

b Write down *all* vectors which:

- **i** are the negative of $\overrightarrow{EF}$
- **ii** have the same length as $\overrightarrow{ED}$.

c Write down a vector which is parallel to $\overrightarrow{AB}$ and twice its length.

A) i) $\overrightarrow{BC}$

ii) $\overrightarrow{ED}$

B) i) $\overrightarrow{FE}$, $\overrightarrow{BC}$

ii) $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{DC}$, $\overrightarrow{DE}$ | $\overrightarrow{EF}$, $\overrightarrow{FE}$, $\overrightarrow{AF}$, $\overrightarrow{FA}$

C) $\overrightarrow{FC}$, $\overrightarrow{CF}$

Ch 12 B – Geometric Operations with Vectors

Zero Vector: a vector of length 0

Vector Addition:

To construct **a** + **b**:

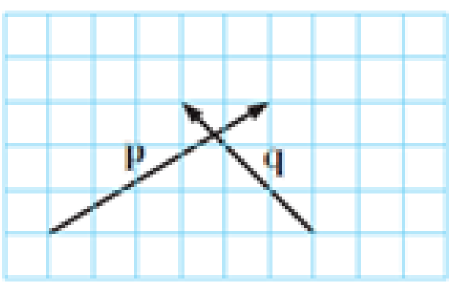
Step 1: Draw **a**.

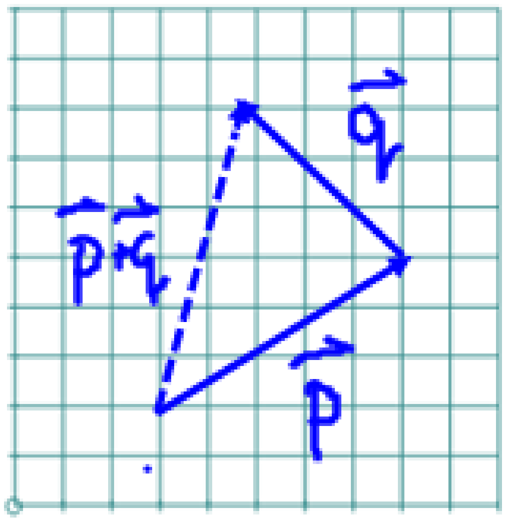
Step 2: At the arrowhead end of **a**, draw **b**.

Step 3: Join the beginning of **a** to the arrowhead end of **b**. This is vector **a** + **b**.

Example pg 281 #1f

f





Find a single vector which is equal to :

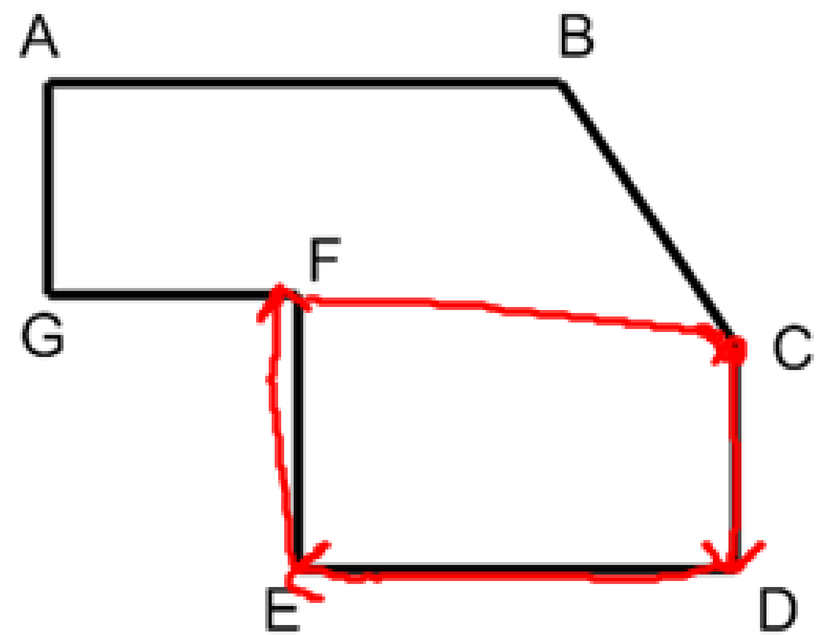
$\overline{AB}+\overline{BF}$ = $\overrightarrow{AF}$

$\overline{GF}+\overline{FE}$ = $\overrightarrow{GE}$

$\overline{AB}+\overline{BC}+\overline{CD}$ = $\overrightarrow{AD}$

$\overline{CD}+\overline{DE}+\overline{EF}+\overline{FC}$ = $\overrightarrow{CC}$ = $\vec{0}$

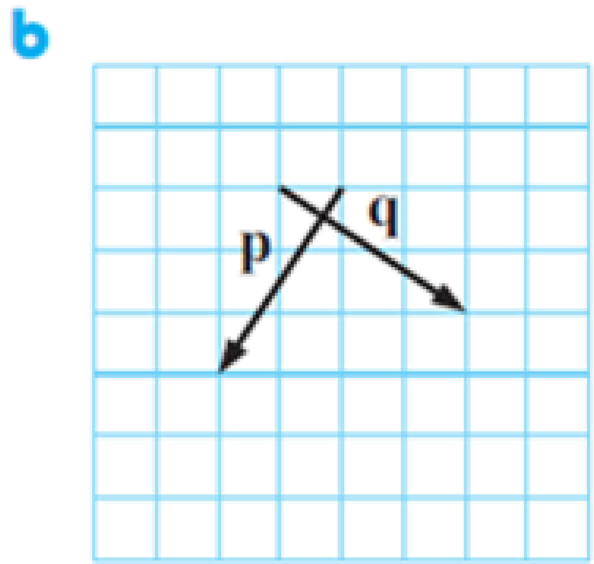
zero vector

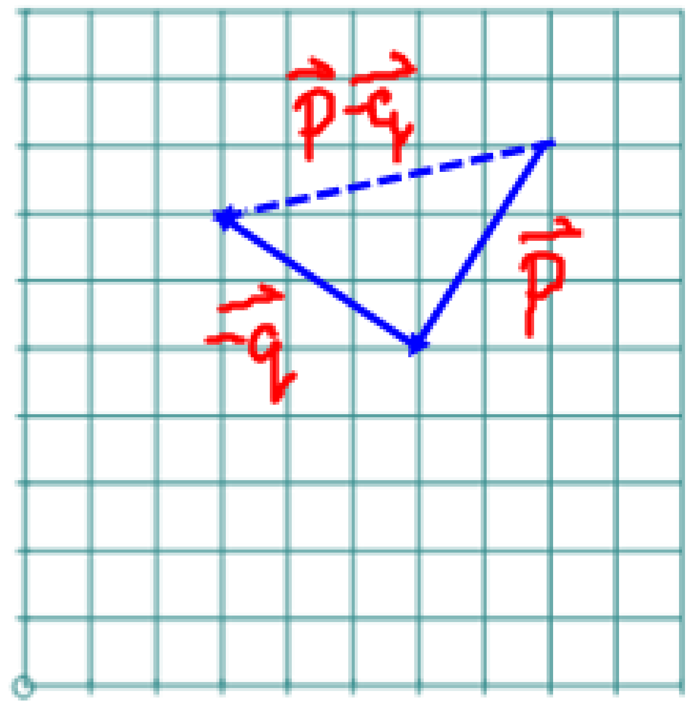


Vector Subtraction: Add its negative

Ex: pg 282 #1b

$\vec{p} - \vec{q} = \vec{p} + (\vec{-q})$



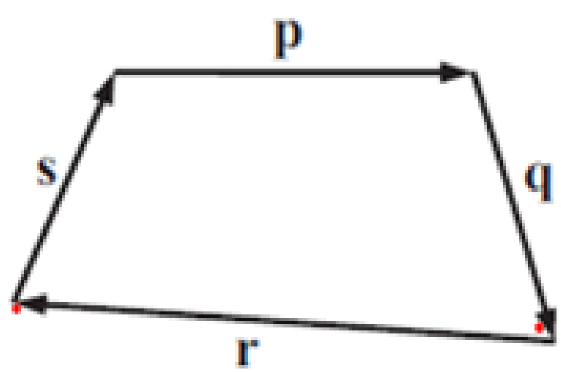


Vector Equations

EXERCISE 12B.3

1 Construct vector equations for:

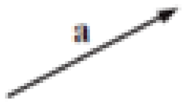
c

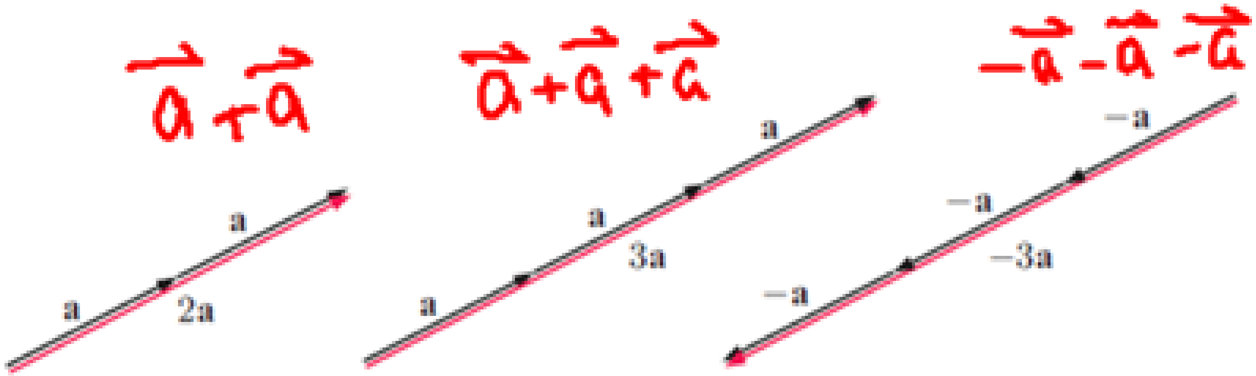


$\vec{r} = -\vec{s} - \vec{p} - \vec{q}$

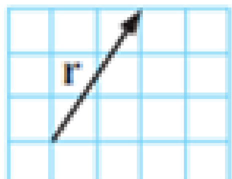
$\vec{s} + \vec{p} + \vec{q} = -\vec{r}$

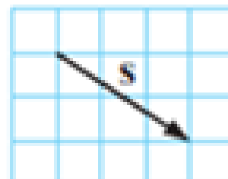
Geometric Scalar Multiplication

If a is 

then 

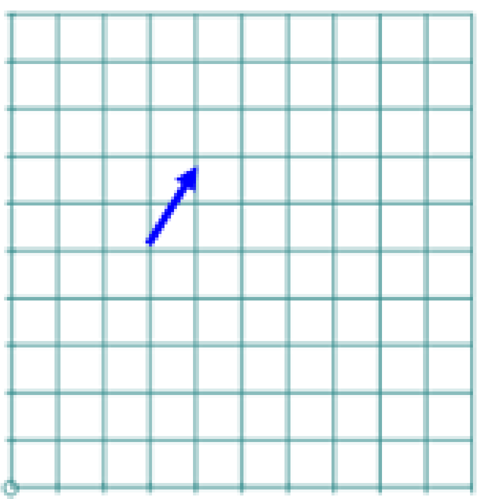
EXERCISE 12B.4

1 Given vectors 

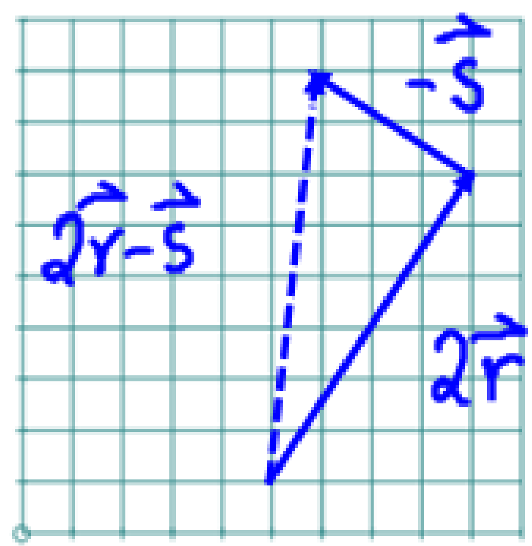
and 

, construct geometrically:

c $\frac{1}{2}$r



e 2r − s



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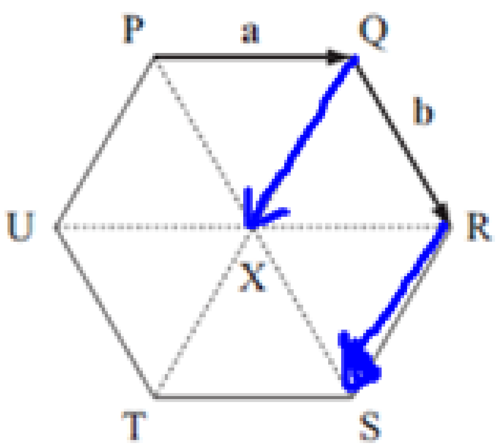


PQRSTU is a regular hexagon.

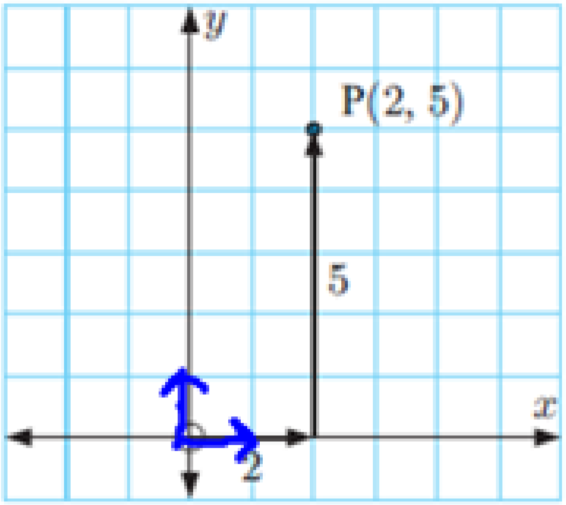
If $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$, find in terms of $\mathbf{a}$ and $\mathbf{b}$:

a $\overrightarrow{PX}$ b $\overrightarrow{PS}$ c $\overrightarrow{QX}$ d $\overrightarrow{RS}$

$\vec{b}$ $2\vec{b}$ $\vec{b}-\vec{a}$ $\vec{b}-\vec{a}$

parallel to $\overrightarrow{QX}$

CH 12 C – Vectors in the Plane



The coordinate point is (2,5). The vector is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

2 units in the x-direction and

5 units in the y-direction

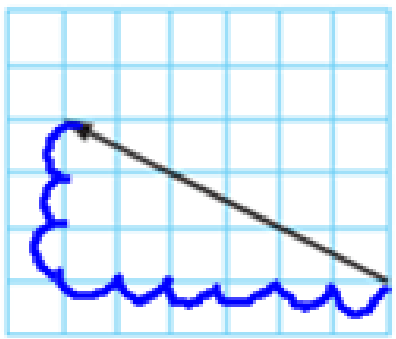
$\mathbf{i}=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ - unit vector 1 unit in the x-direction
0 unit in the y-direct

$\mathbf{j}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ - unit vect 0 unit in the x direction
1 unit in the y direction

EXERCISE 12C

1 Write the illustrated vectors in component form and in unit vector form:

e



$\begin{pmatrix}-6\\3\end{pmatrix}$ ← component form

unit vector form: $-6\vec{i}+3\vec{j}$

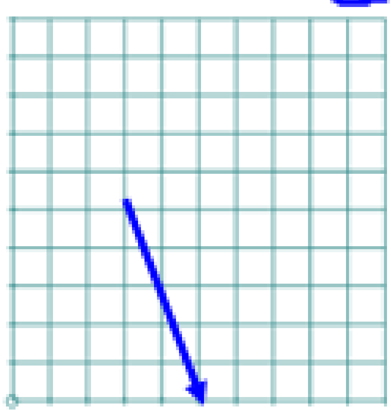
$-6\begin{pmatrix}1\\0\end{pmatrix}+3\begin{pmatrix}0\\1\end{pmatrix}$

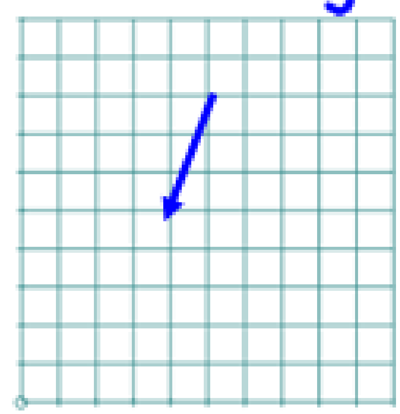
2 Write each vector in unit vector form, and illustrate it using an arrow diagram:

a $\begin{pmatrix}3\\4\end{pmatrix}$ b $\begin{pmatrix}2\\0\end{pmatrix}$ c $\begin{pmatrix}2\\-5\end{pmatrix}$ d $\begin{pmatrix}-1\\-3\end{pmatrix}$

$2\vec{i}-5\vec{j}$

$-\vec{i}-3\vec{j}$





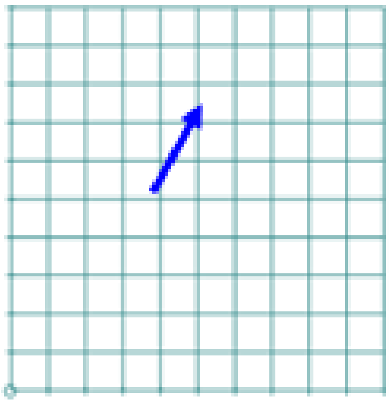
4 Write in component form and illustrate using a directed line segment:

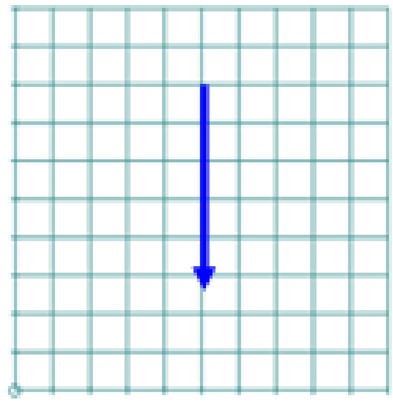
a $\mathbf{i} + 2\mathbf{j}$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

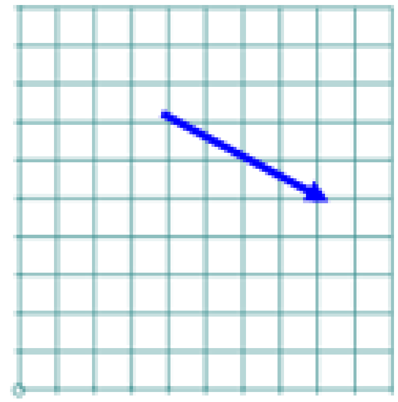
b $-\mathbf{i} + 3\mathbf{j}$

c $-5\mathbf{j}$ $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

d $4\mathbf{i} - 2\mathbf{j}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$







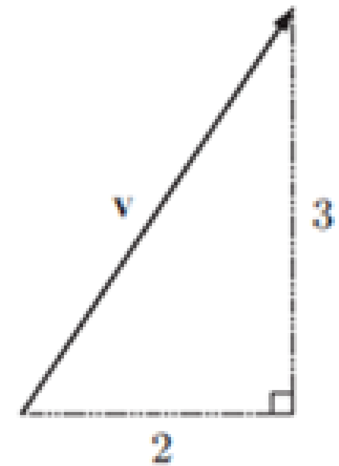
Ch 12 D – The Magnitude of a Vector

Consider vector $\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2\mathbf{i} + 3\mathbf{j}$.

The magnitude or length of $\mathbf{v}$ is represented by $|\mathbf{v}|$.

By Pythagoras, $|\mathbf{v}|^2 = 2^2 + 3^2 = 4 + 9 = 13$

$\therefore \quad |\mathbf{v}| = \sqrt{13}$ units $\quad \{$since $|\mathbf{v}| > 0\}$



If $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = v_1\mathbf{i} + v_2\mathbf{j}$, the magnitude or length of $\mathbf{v}$ is $|\mathbf{v}| = \sqrt{{v_1}^2 + {v_2}^2}$.

$|\vec{v}|$

EXERCISE 12D

1 Find the magnitude of:

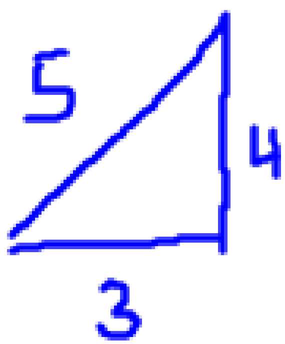
a $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{matrix} \leftarrow x \\ \leftarrow y \end{matrix}$ b $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$|\vec{v}|^2 = (3)^2 + (4)^2$

$= 9 + 16$

$= 25$

$|\vec{v}| = 5$



2 Find the length of:

a $\mathbf{i} + \mathbf{j}$ b $5\mathbf{i} - 12\mathbf{j}$

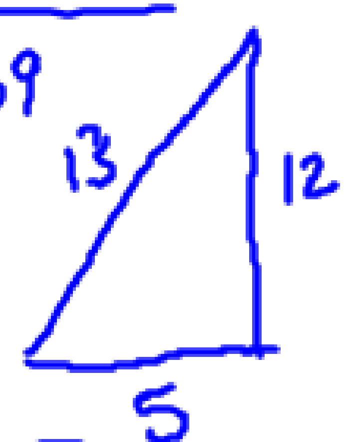
$|\vec{v}| = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

$|\vec{v}| = \sqrt{(5)^2 + (-12)^2}$

$= \sqrt{169}$

$= 13$



Pythagorean Triples

length of 1 unit

3 Which of the following are unit vectors?

a $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ **c** $\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$

$|\vec{v}| = \sqrt{0^2 + (-1)^2}$

$= 1$

Yes!

$|\vec{v}| = \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2} + \frac{1}{2}}$

$= \sqrt{1}$

$= 1$

Yes!

$|\vec{v}| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$

$= \sqrt{\frac{4}{9} + \frac{1}{9}}$

$= \sqrt{\frac{5}{9}}$

No!