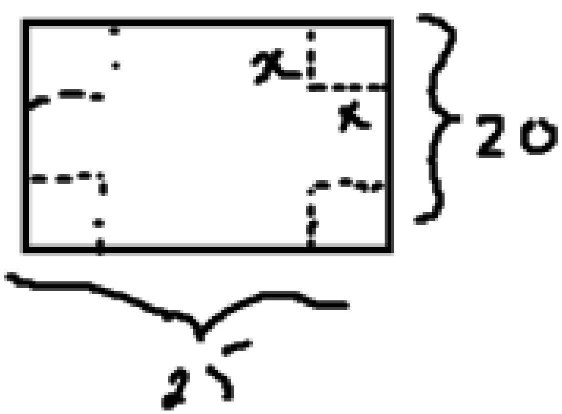
Ch 17C - Optimization Day 2

cm cm

Example- An open top box is to be made by cutting congruent squares of side length x from the corners of 20 by 25 sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting maximum volume?

1 want max volume. → x represents the height of the box

2

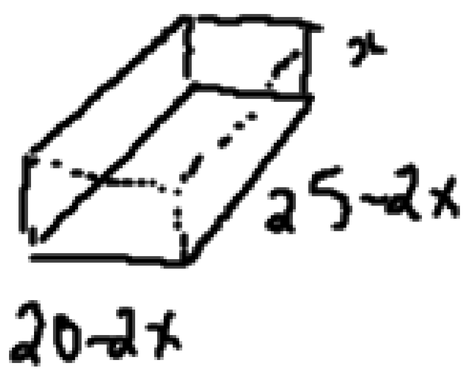


→ $V = l \times w \times h$

$V = (25-2x)(20-2x)(x)$

$V = (25-2x)(20x-2x^2)$

$V = 4x^3 - 90x^2 + 500x$



$$V = 4x^3 - 90x^2 + 500x$$

$$\frac{dV}{dx} = 12x^2 - 180x + 500$$

$$0 = 12x^2 - 180x + 500$$

$$0 = 6x^2 - 90x + 250$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+90 \pm \sqrt{(-90)^2 - 4(6)(250)}}{2(6)}$$

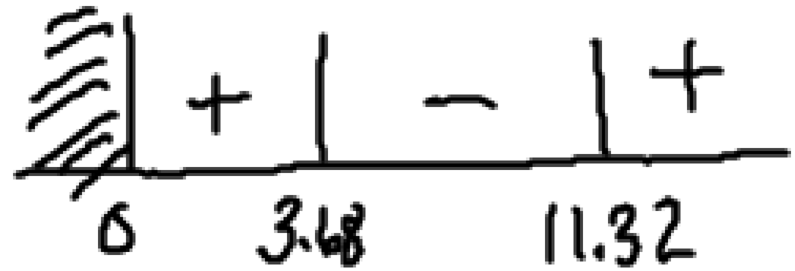
$$x = \frac{90 \pm \sqrt{2100}}{12}$$

$$x = \frac{90 \pm 10\sqrt{21}}{12}$$

$$x = \frac{45 \pm 5\sqrt{21}}{6}$$

$x = 11.32$ or

$x = 3.68$

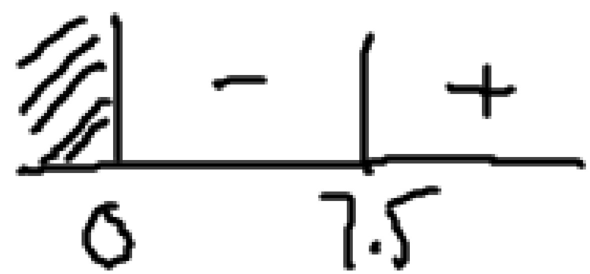


local maximum at $x = 3.68$

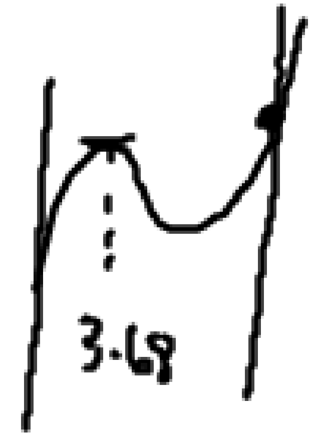
$$V'' = 24x - 180$$

$$0 = 24x - 180$$

$$x = 7.5$$



$x = 3.68$ is our global max.



$0 \le x \le 10$

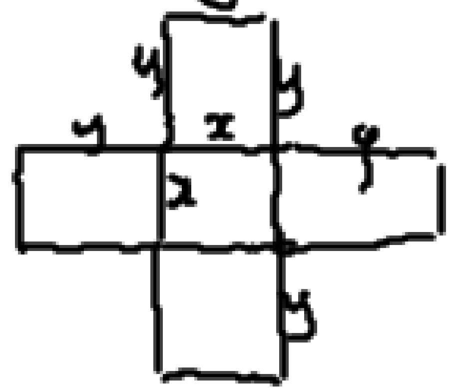
$V(0) = 0$ $\quad$ $V(3.68) = 820.53$ $\quad$ $V(10) = 0$

The max volume occurs when you cut out squares $3.68\text{cm} \times 3.68\text{cm}$ $\quad$ $V = 820.53\text{cm}^3$

Example- If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Box

Net diagram



$SA = 4xy + x^2$

$1200 = 4xy + x^2$

$\frac{4xy}{4x} = \frac{1200 - x^2}{4x}$

$y = \frac{1200 - x^2}{4x}$

$V = l \times w \times h$

$V = (x)(x)(y)$

$V = x^2 y$

$V = x^2 \left(\frac{1200 - x^2}{4x} \right)$

$V = 300x - \frac{1}{4}x^3$

$V = -\frac{1}{4}x^3 + 300x$

$V = -\frac{1}{4}x^3 + 300x$

$\frac{dv}{dx} = \frac{-3}{4}x^2 + 300$

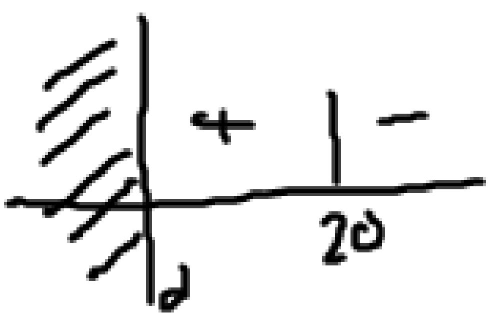
$0 = \frac{-3}{4}x^2 + 300$

$0 = -3x^2 + 1200$

$3x^2 = 1200$

$x^2 = 400$

$x = \pm 20$



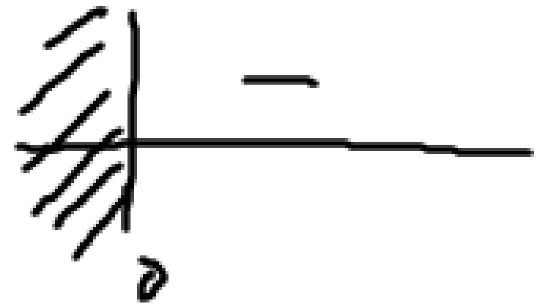
$\rightarrow$ $x = 20$ is a local max

$V'' = \frac{-3}{4}(2x) + 0$

$V'' = \frac{-3}{2}x$

$0 = \frac{-3}{2}x$

$x = 0$



CD everywhere
$x \in (0, \infty)$

Which means $x = 20$ is global max.

$V(20) = \frac{-1}{4}(20)^3 + 300(20)$

$= 4000 \text{ cm}^3$

max volume.