Ch 15E- Derivative of Exponential Functions

$\sqrt{-\frac{1}{2}}$ $\left(-\frac{1}{2}\right)^x$

Review of Exponential functions:

- The simplest exponential function is $y = b^x$ where $b \in R$, $b > 0$
- Two types of graphs produced:

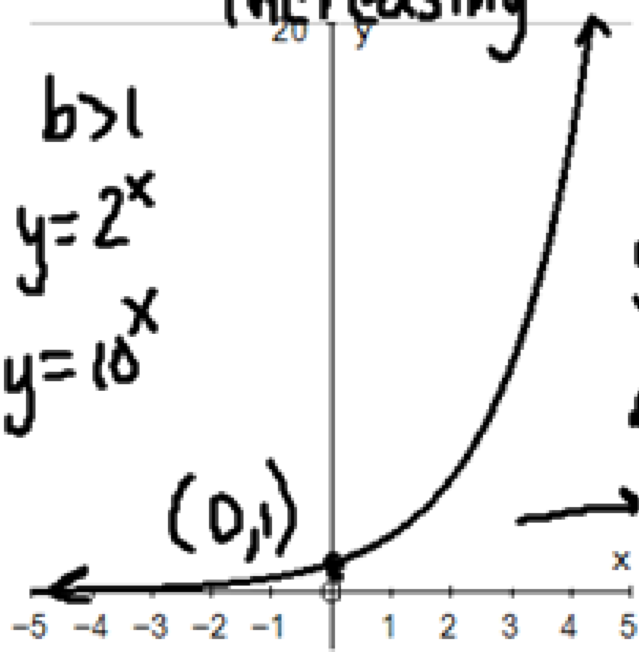
Growth curve

increasing

$b > 1$

$y = 2^x$

$y = 10^x$



$y = 2^{-1000}$

$= \frac{1}{2^{1000}}$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y > 0\}$

HA:

$y = 0$

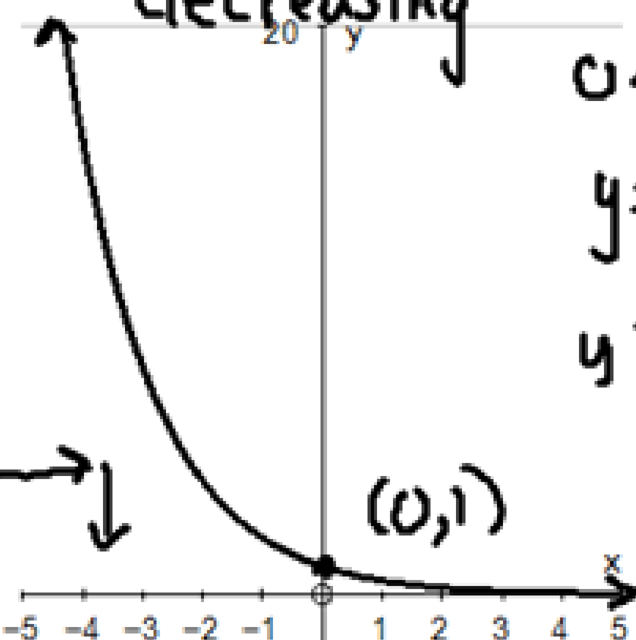
Decay curve

decreasing

$0 < b < 1$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{10}\right)^x$



Domain: $\{x \mid x \in R\}$

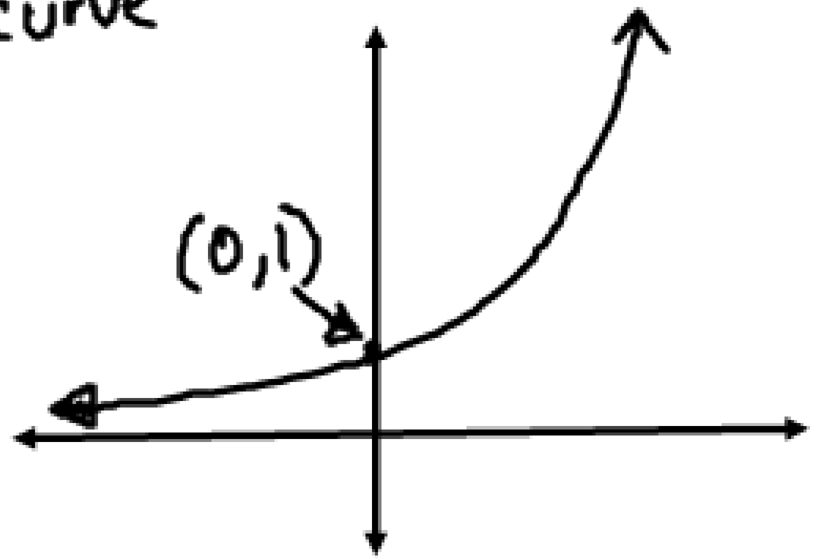
Range: $\{y \mid y > 0\}$

HA:

$y = 0$

- exponential function is $y = e^x$ where e is Euler's Number

-growthcurve



- $e \approx 2.71$ ish
- Irrational #
- just a number

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0\}$

HA: $y = 0$

Find the derivative of $y = e^x$ using the definition of the derivative.

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{(e^x)(e^h) - e^x}{h}$$

$$= \lim_{h \to 0} e^x \left(\frac{e^h - 1}{h} \right)$$

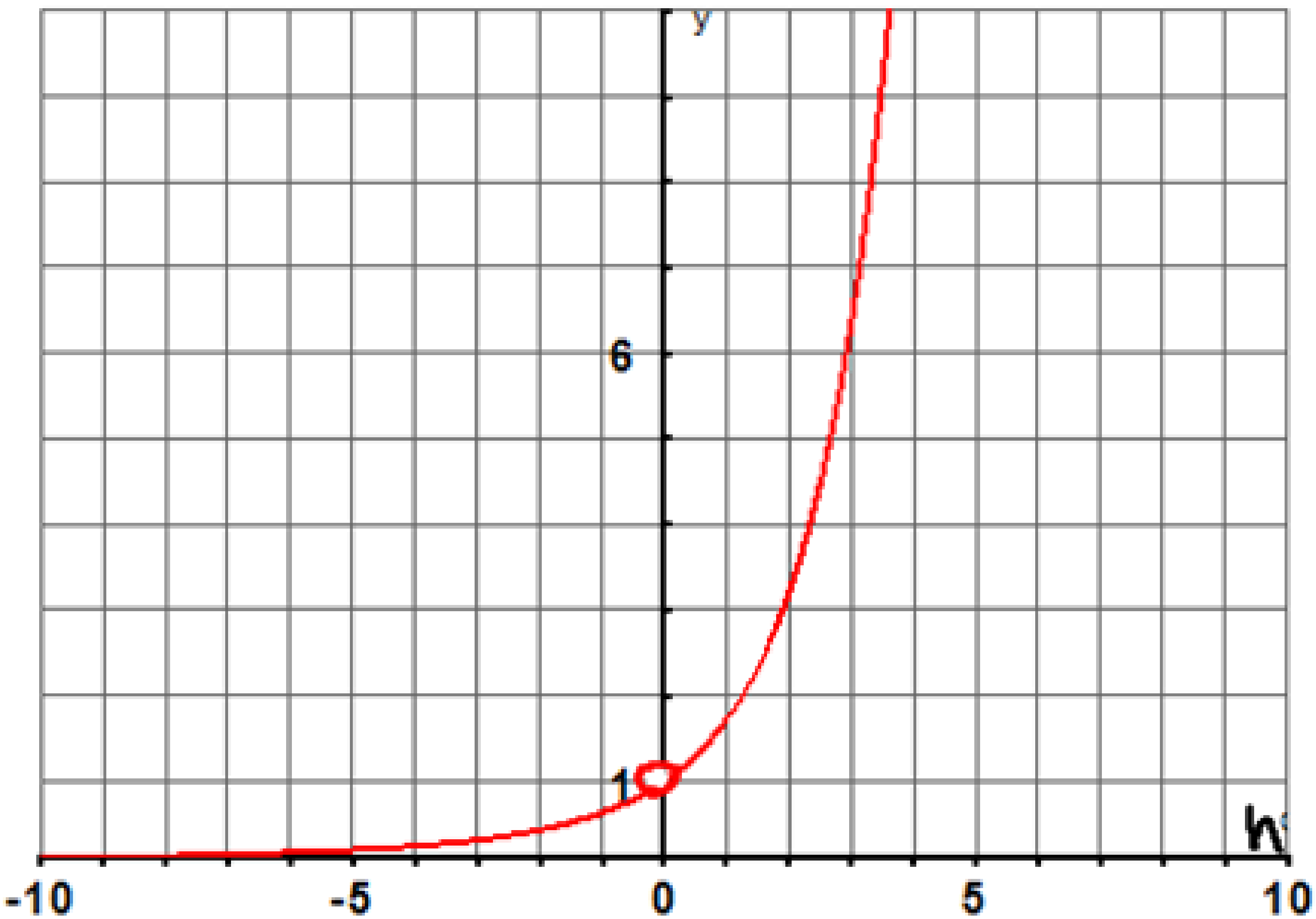
laws of exponents

$$a^m \cdot a^n = a^{m+n}$$

$$y' = e^x \cdot \left(\lim_{h \to 0} \frac{e^h - 1}{h} \right)$$

$$y' = e^x \cdot (1)$$

$$\frac{dy}{dx} = e^x$$



$$y = \frac{e^h - 1}{h}$$

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

What does this mean: It means the slope is the same as the function value (the y-value) for all points on the graph.

Where $u = f(x)$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^u = e^u \frac{du}{dx}$$

← derivative of the exponent

→ Chain Rule.

Example: Find the derivative of :

A) $y = e^{3x^4}$

$u = 3x^4$

$u' = 12x^3$

$y' = (e^{3x^4})(12x^3)$

B) $y = e^{5x}$

$\frac{dy}{dx} = (e^{5x})(5)$

C) $y = xe^{-x}$

Product Rule

$u = x$ $\quad v = e^{-x}$

$u' = 1$ $\quad v' = (e^{-x})(-1)$

$y' = uv' + vu'$

$= (x)(-e^{-x}) + (e^{-x})(1)$

D) $y = \dfrac{e^{-3x+2}}{e^x - x}$

Quotient Rule

$u = e^{-3x+2}$ $\quad v = e^x - x$

$u' = (e^{-3x+2})(-3)$ $\quad v' = e^x - 1$

$y' = \dfrac{vu' - uv'}{v^2}$

$$y' = \frac{(e^x - x)(-3e^{-3x+2}) - (e^{-3x+2})(e^x - 1)}{(e^x - x)^2}$$

Don't simplify

What about other bases? Similar to pg375 # 5

$y = a^x$

Since we already know the derivative of $y = e^x$, write a^x with a base of e.

recall from gr11

Power Rule for logs

$$\log_a x^b = b \cdot \log_a x$$

$$\log_a a^x = x \cdot \log_a a$$
$$= x \cdot (1)$$

$\longrightarrow$

$$3^{\log_3 9} = 3^{\log_3 (3^2)}$$
$$= 3^{2 \cdot \log_3 3}$$
$$= 3^2$$
$$= 9$$

$$a = e^{\log_e a}$$

$$a = e^{\ln a}$$

$$(a)^x = (e^{\ln a})^x$$

$$a^x = e^{x \cdot \ln a}$$

a just a #

$$\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{x \cdot \ln a})$$

Chain Rule

$$= (e^{x \cdot \ln a})(\ln a)$$

$$= (a^x)(\ln a)$$

$$\frac{d}{dx}a^x = \ln(a)\cdot a^x$$

$$\frac{d}{dx}a^u = \ln(a)\cdot a^u\,\frac{du}{dx}$$

$u = f(x)$

chain Rule

Example: Find the derivative of

A) $y = 2^x$

$y' = (\ln 2) \cdot 2^x$

B) $y = 3^{x^2 - 5x}$

$y' = \ln(3)\left(3^{x^2-5x}\right)(2x-5)$

* 5 pg
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Ch 15F – Derivatives of Logarithmic Functions

If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$

Chain Rule: $y = \ln(f(x))$, then $\dfrac{dy}{dx} = \dfrac{1}{f(x)} \times f'(x)$

Example:

A) $f(x) = \ln(x^2)$

Chain Rule

$$f'(x) = \left(\frac{1}{x^2}\right)(2x)$$

$$= \frac{2}{x}$$

B) $f(x) = \ln(3x^4 - 1)$

$u = 3x^4 - 1$

$u' = 12x^3$

$$\frac{d}{dx} f(x) = \left(\frac{1}{3x^4 - 1}\right)\left(12x^3\right)$$

C) $y = [\ln(2x+7)]^3$

Chain Rule x2

let $u = \ln(2x+7)$

$y = u^3$

$y' = 3u^2 \cdot \frac{du}{dx}$

$u' = \left(\frac{1}{2x+7}\right)(2)$

$y' = 3\left(\ln(2x+7)\right)^2\left(2\left(\frac{1}{2x+7}\right)\right)$

D) $f(x) = x^4 \ln(x^2)$

Product Rule

$u = x^4 \qquad v = \ln(x^2)$

$u' = 4x^3 \qquad v' = \frac{1}{x^2}(2x) = \frac{2}{x}$

$f'(x) = uv' + vu'$

$= (x^4)\left(\frac{2}{x}\right) + (\ln(x^2))(4x^3)$

HW: pg 375 #1-6

pg 377 # 1-3