Ch 15E- Derivative of Exponential Functions

$\sqrt{-\frac{1}{2}}$

Review of Exponential functions:

- The simplest exponential function is $y = b^x$ where $b \in R$, $b > 0$
- Two types of graphs produced:

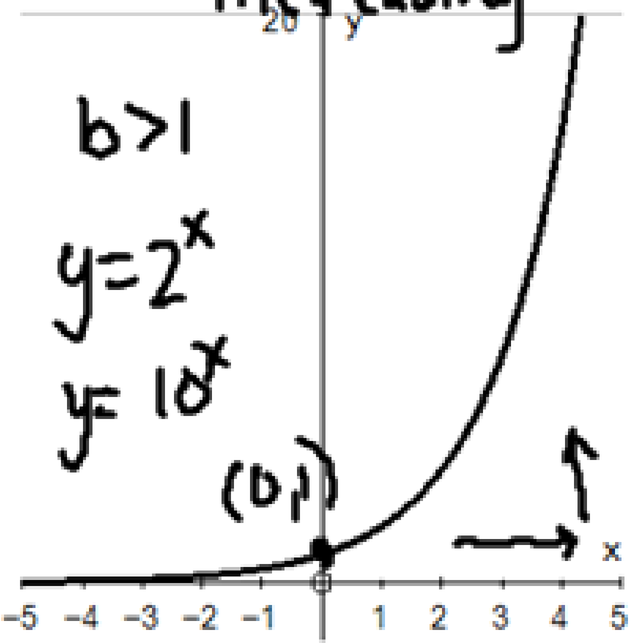
Growth curve

increasing

$b > 1$

$y = 2^x$

$y = 10^x$



Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y > 0\}$

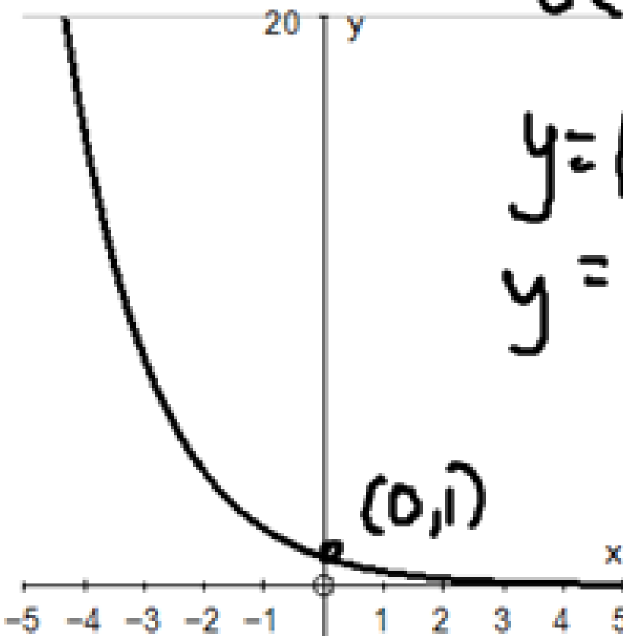
HA: $y = 0$

Decay curve

$0 < b < 1$

$y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{10}\right)^x$



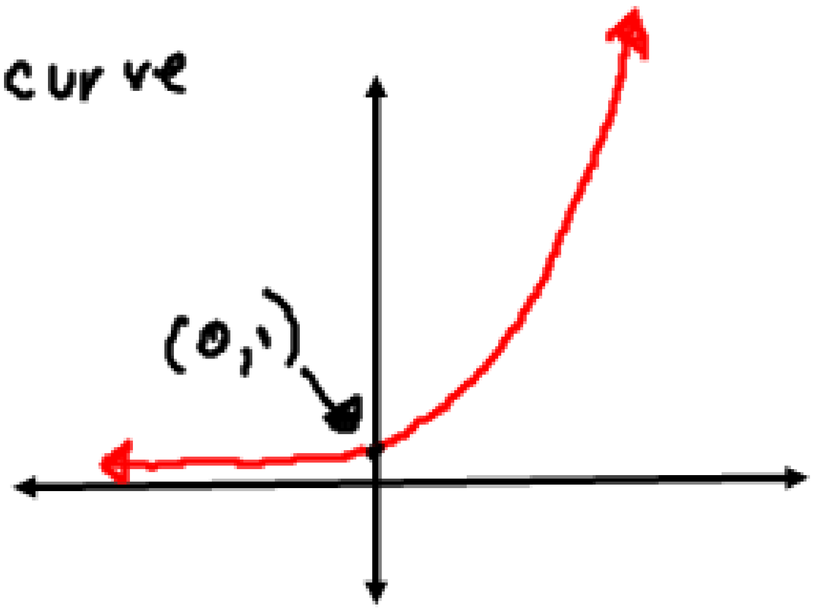
Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y > 0\}$

HA: $y = 0$

- exponential function is $y = e^x$ where e is Euler's Number

-growth curve



$e \approx 2.71$ish

just a number

-irrational

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0\}$

HA: $y = 0$

Find the derivative of $y = e^x$ using the definition of the derivative.

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x(e^h - 1)}{h}$$

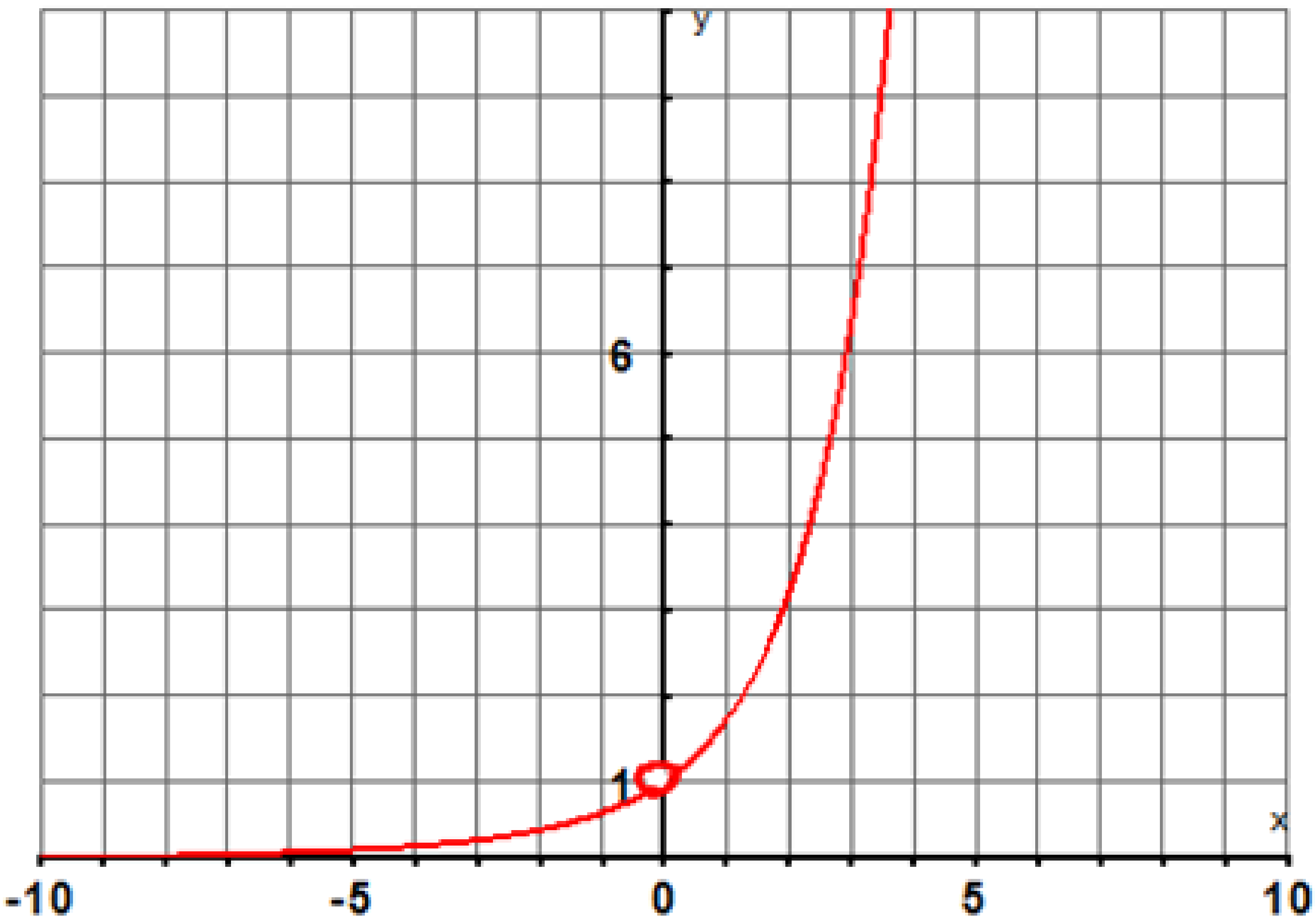
laws of exponents:

$$a^m \cdot a^n = a^{m+n}$$

$$y' = e^x \left(\lim_{h \to 0} \frac{e^h - 1}{h} \right)$$

$$y' = e^x(1)$$

$$\frac{dy}{dx} = e^x$$



$$y = \frac{e^x - 1}{x}$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

What does this mean: It means the slope is the same as the function value (the y-value) for all points on the graph.

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

← derivative of exponent

$y = e^x$

$\frac{dy}{dx} = e^x$

$u = f(x)$

↑ chain Rule

$$\frac{d}{dx}(e^{2x}) = (e^{2x})(2)$$

Example: Find the derivative of :

A) $y = e^{3x^4}$

$u = 3x^4$

$u' = 12x^3$

$y' = (e^{3x^4})(12x^3)$

B) $y = e^{5x}$

$u = 5x$

$u' = 5$

$\frac{dy}{dx} = (e^{5x})(5)$

C) $y = xe^{-x}$

Product Rule :)

$u = x \qquad v = e^{-x}$

$u' = 1 \qquad v' = (e^{-x})(-1)$

$\frac{dy}{dx} = uv' + vu'$

$= (x)(-e^{-x}) + (e^{-x})(1)$

D) $y = \dfrac{e^{-3x+2}}{e^x - x}$

Quotient Rule :)

$u = e^{-3x+2} \qquad v = e^x - x$

$u' = (e^{-3x+2})(-3) \qquad v' = e^x - 1$

$y' = \dfrac{vu' - uv'}{v^2}$

$$y' = \frac{(e^x - x)(-3(e^{-3x+2})) - (e^{-3x+2})(e^x - 1)}{(e^x - x)^2}$$

What about other bases?

$y = a^x$

Since we already know the derivative of $y = e^x$, write a^x with a base of e.

recall from gr 11

Power Rule for logs

$\log_a x^b = b \cdot \log_a x$

$\log_a a^x = x \cdot \log_a a$

$= x(1)$

$\longrightarrow$

$3^{\log_3 9} = 3^{\log_3(3^2)}$

$= 3^{2 \cdot \log_3 3}$

$= 3^2$

$= 9$

$$a = e^{\log_e a}$$ just a number

$$(a^m)^n = a^{m \times n}$$

$$a = e^{\ln a}$$

$$(a)^x = \left(e^{\ln a}\right)^x$$

$$a^x = e^{x \cdot \underline{\ln a}}$$ just a #

$$\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{x \cdot \ln a}\right) = \left(e^{x \ln a}\right)(\ln a)$$

$$= (a^x)(\ln a)$$

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$$\frac{d}{dx}a^x = \ln(a)\cdot a^x$$

$$\frac{d}{dx}a^u = \ln(a)\cdot a^u\frac{du}{dx}$$

Example: Find the derivative of

✓Chain Rule

A) $y = 2^x$

$\frac{dy}{dx} = (2^x)(\ln 2)$

B) $y = 3^{x^2 - 5x}$

$y' = (3^{x^2-5x})(\ln 3)(2x-5)$

Ch 15F – Derivatives of Logarithmic Functions

If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$

Chain Rule: $y = \ln(f(x))$, then $\dfrac{dy}{dx} = \dfrac{1}{f(x)} \times f'(x)$

Example:

A) $f(x)=\ln(x^2)$

Chain Rule

$$f'(x)=\left(\frac{1}{x^2}\right)(2x)$$

$$=\frac{2}{x}$$

B) $f(x)=\ln(3x^4-1)$

$$\frac{d}{dx}f(x)=\left(\frac{1}{3x^4-1}\right)(12x^3)$$

C) $y = [\ln(2x+7)]^3$

Chain Rule x2

$\downarrow \frac{d}{dx}(\ln(2x+7))$

$$y' = 3[\ln(2x+7)]^2 \left[\frac{1}{2x+7}\right][2]$$

D) $f(x) = x^4 \ln(x^2)$

Product Rule

$u = x^4 \quad v = \ln(x^2)$

$u' = 4x^3 \quad v' = \left(\frac{1}{x^2}\right)(2x)$

$$f'(x) = uv' + vu'$$

$$= (x^4)\left(\frac{2}{x}\right) + \ln(x^2)(4x^3)$$

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