9.2

Analysing Rational Functions

Focus on...

- graphing, analysing, and comparing rational functions
- determining whether graphs of rational functions have an asymptote or a point of discontinuity for a non-permissible value

Points of Discontinuities (holes) (POD)

$$f(x)=\frac{x^2+2x-3}{x-1} = \frac{\cancel{(x-1)}(x+3)}{\cancel{x-1}}$$

$f(x) = x+3$

linear function

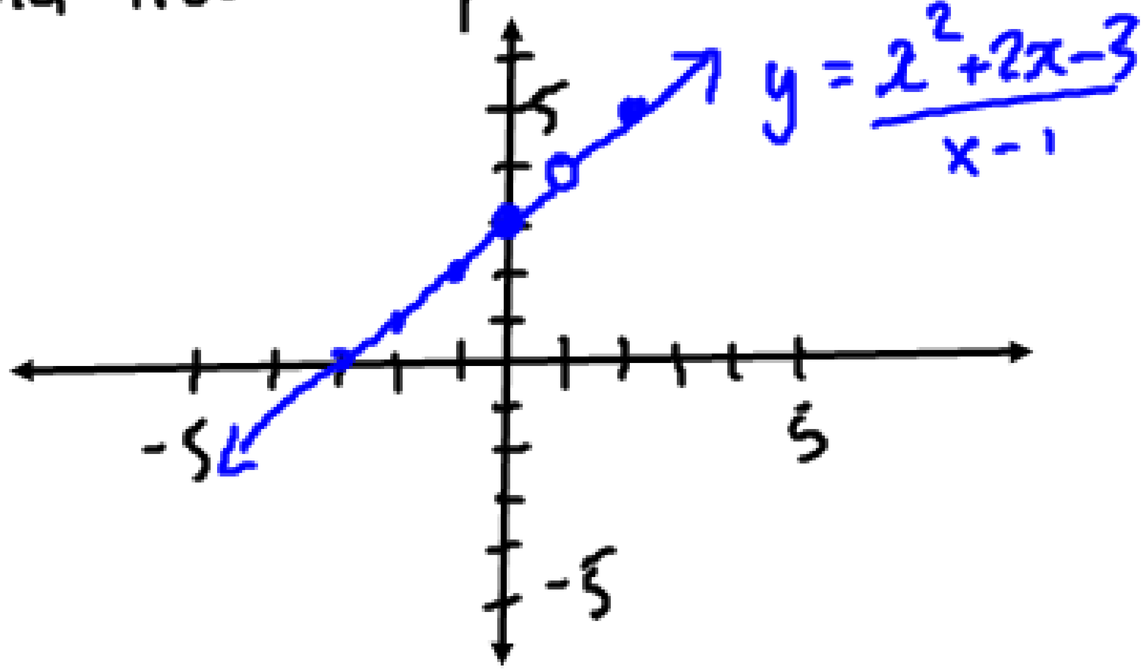
The function is undefined when x=1 but the FACTOR can be removed and simplified to $f(x)=x+3$. We say that at $x=1$ there is a hole.

Always find from simplified function

Find $f(1) = 1+3$

$f(1) = 4$

POD at (1,4)



The hole is at (1, 4). Points of discontinuity (POD) occur when a factor in the numerator "cancels" with a factor in the denominator.

Sketch $f(x)=\dfrac{(x-1)(x+3)}{x-1}$

Example: Find the following for the given function and sketch

$$f(x)=\frac{(2x-1)(x+4)}{(6x-3)(x-4)} = \frac{\cancel{(2x-1)}(x+4)}{3\cancel{(2x-1)}(x-4)}$$

HOLE at $2x-1=0$

$x=\frac{1}{2}$

$$f(x)=\frac{x+4}{3(x-4)}$$

$$f\left(\tfrac{1}{2}\right)=\frac{\frac{1}{2}+4}{3\left(\frac{1}{2}-4\right)}=-0.42$$

A) POD

$\left(\frac{1}{2},-0.42\right)$

B) x-int $(y=0)$

$x+4=0$

$x=-4$

$(-4,0)$

C) y-int $(x=0)$

$$y=\frac{0+4}{3(0-4)}$$

$$=\frac{4}{-12}=\frac{-1}{3}$$

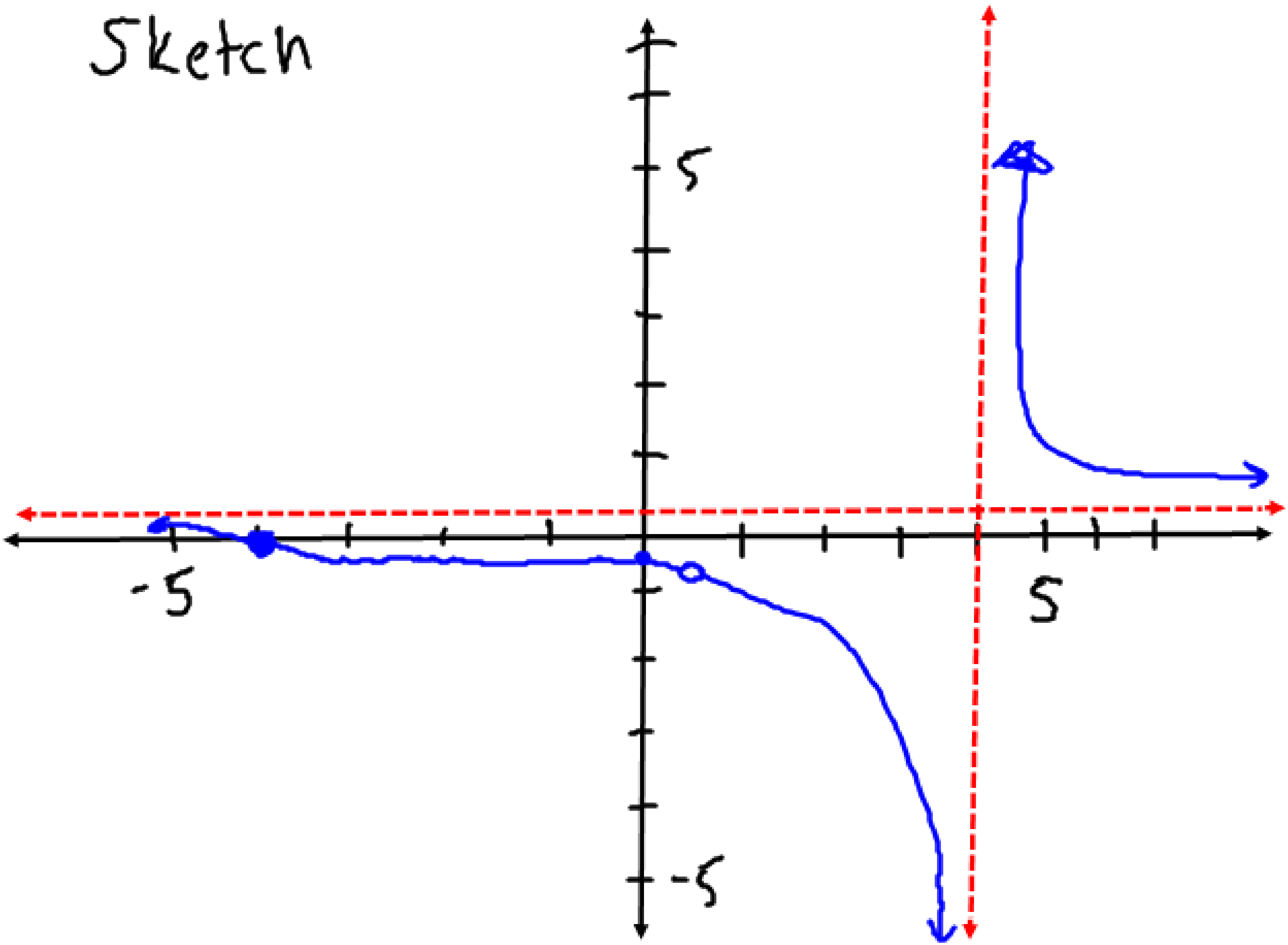
$\left(0,\frac{-1}{3}\right)$

D) Asymptotes

VA: $x=4$

HA: $y=\frac{1}{3}$

Sketch



Example: Find the following for the given function and sketch

$$f(x) = \frac{x^2 - 9}{x^2 - 6x + 9} = \frac{(x+3)\cancel{(x-3)}}{(x-3)\cancel{(x-3)}}$$

$$f(x) = \frac{x+3}{x-3}$$

→ Since the common factor simplifies, normally there would be a POD, however, the VA takes precedence

A) POD

NONE

D) Asymptotes

VA: $x = 3$

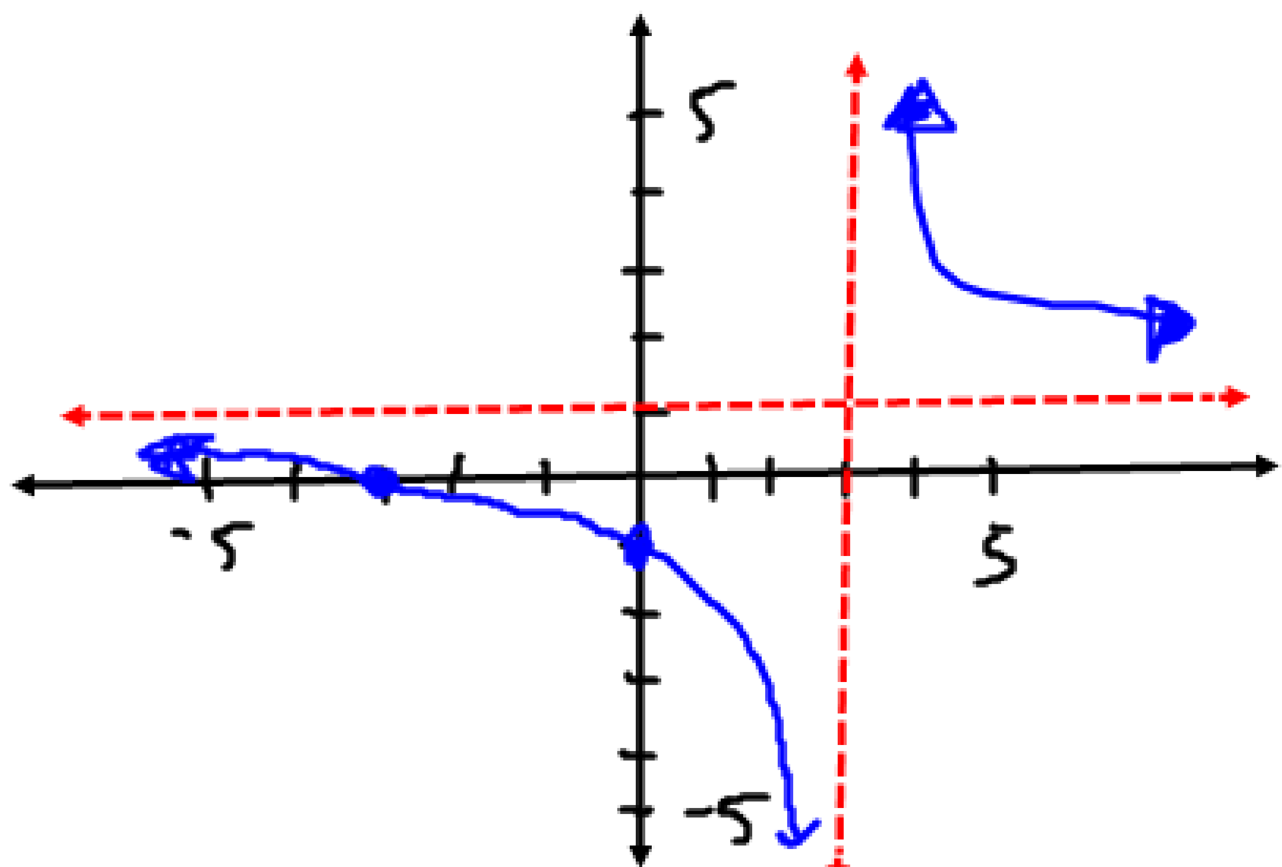
HA: $y = 1$

B) x-int

$(-3, 0)$

C) y-int

$(0, -1)$



Example: Find the following for the given function

$$f(x) = \frac{x^2+9}{x^2-6x+9} = \frac{x^2+9}{(x-3)(x-3)}$$

$(x-3)^2$

A) POD

NONE

D) Asymptotes

VA: $x=3$ multiplicity 2

HA: $y=1$

B) x-int ($y=0$)

$x^2+9=0$

$x^2=-9$

NO SOL'N

no x-int

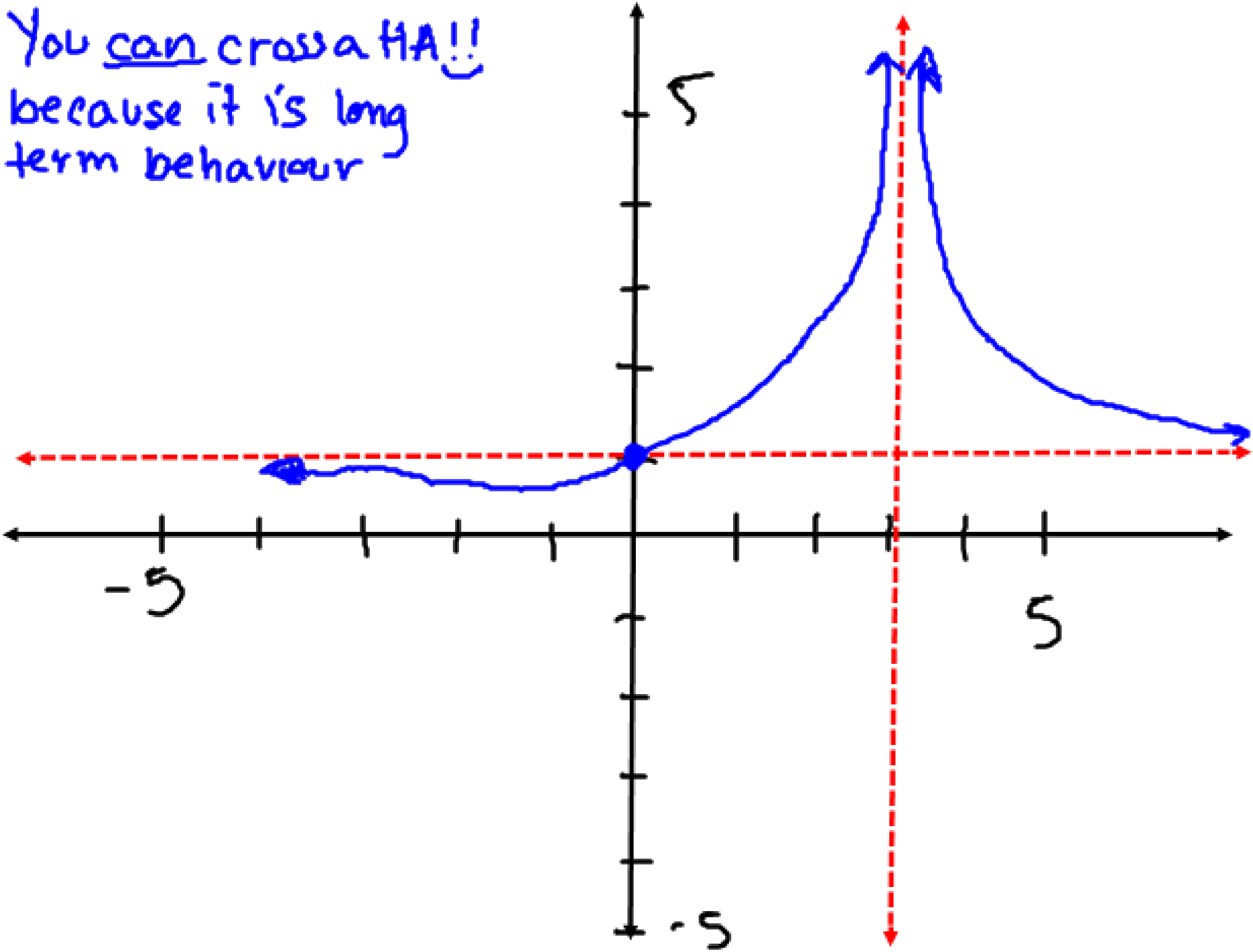
C) y-int ($x=0$)

$$y=\frac{0+9}{0-0+9}$$

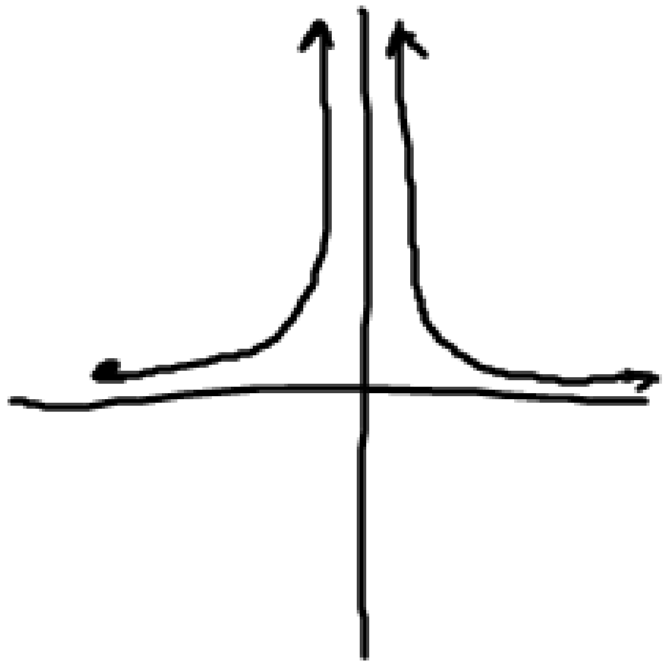
$y=1$

$(0,1)$

You can cross a HA!! :)
because it's long term behaviour



$$y = \frac{1}{x^2}$$

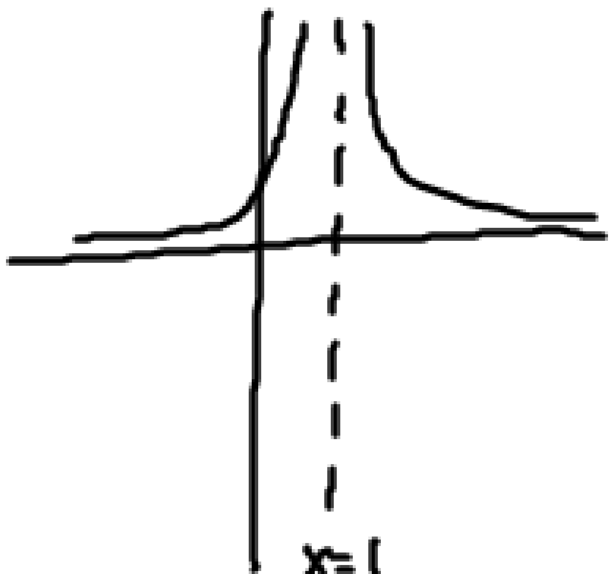


mult of 2

$$y = \frac{1}{(x-1)^2}$$

HT +1

$$y = \frac{1}{x^2-2x+1}$$



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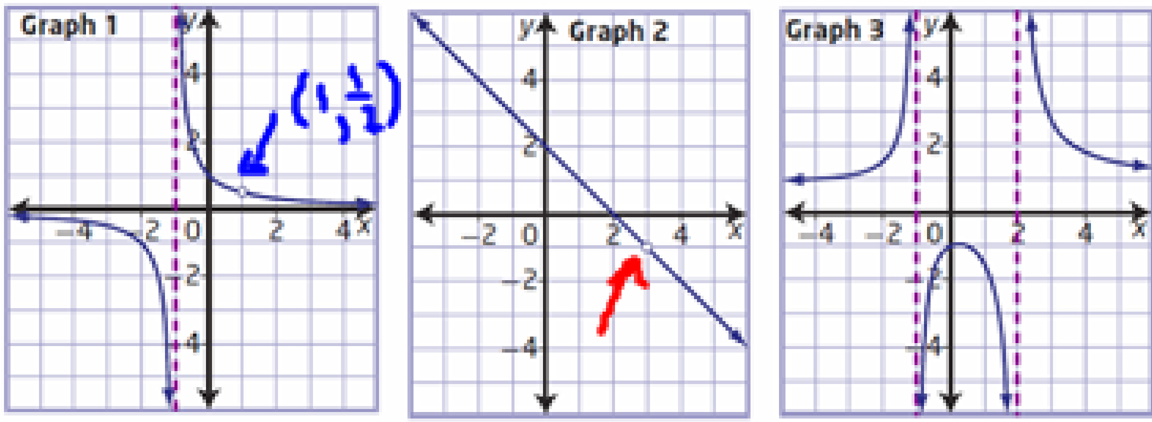
Your Turn

Match the equation of each rational function with the most appropriate graph. Explain your reasoning.

$K(x) = \dfrac{x^2 + 2}{x^2 - x - 2}$ $\qquad L(x) = \dfrac{x - 1}{x^2 - 1}$ $\qquad M(x) = \dfrac{x^2 - 5x + 6}{3 - x}$



$$K(x) = \frac{x^2+2}{(x-2)(x+1)}$$

→ NO HOLE !

→ VA: $x=2$, $x=-1$

→ Graph 3

→ NO x-int

- y-int $(0,-1)$

$$L(x) = \frac{x-1}{x^2-1}$$ Graph 1

VA: $x=-1$

HA: $y=0$

$$= \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{x+1}$$

HOLE at $x=1, y=\frac{1}{2}$

$$m(x) = \frac{x^2-5x+6}{-(x-3)}$$

$$= \frac{(\cancel{x-3})(x-2)}{-(\cancel{x-3})}$$

HOLE at $x=3$ $y=-(x-2)$

Slant/Oblique Asymptotes

An oblique line is a line with slope other than zero or undefined; it is neither vertical nor horizontal.

An oblique asymptote is the long term behaviour of the function. If we were to zoom out on a function, it would look like a line with a slope.

Happens with rational functions, with no common factors, in which the degree of the numerator is one more than that of the denominator.

To find the slant asymptote, divide using long or synthetic division

Example:

NO HOLES

$$f(x)=\frac{x^2+7x+10}{x-3}=\frac{(x+5)(x+2)}{x-3}$$

$$\begin{array}{r} x+10 \\ x-3\,\overline{)\,x^2+7x+10} \\ -(x^2-3x) \\ \hline 0+10x+10 \\ -(10x-30) \\ \hline 0+40 \end{array}$$

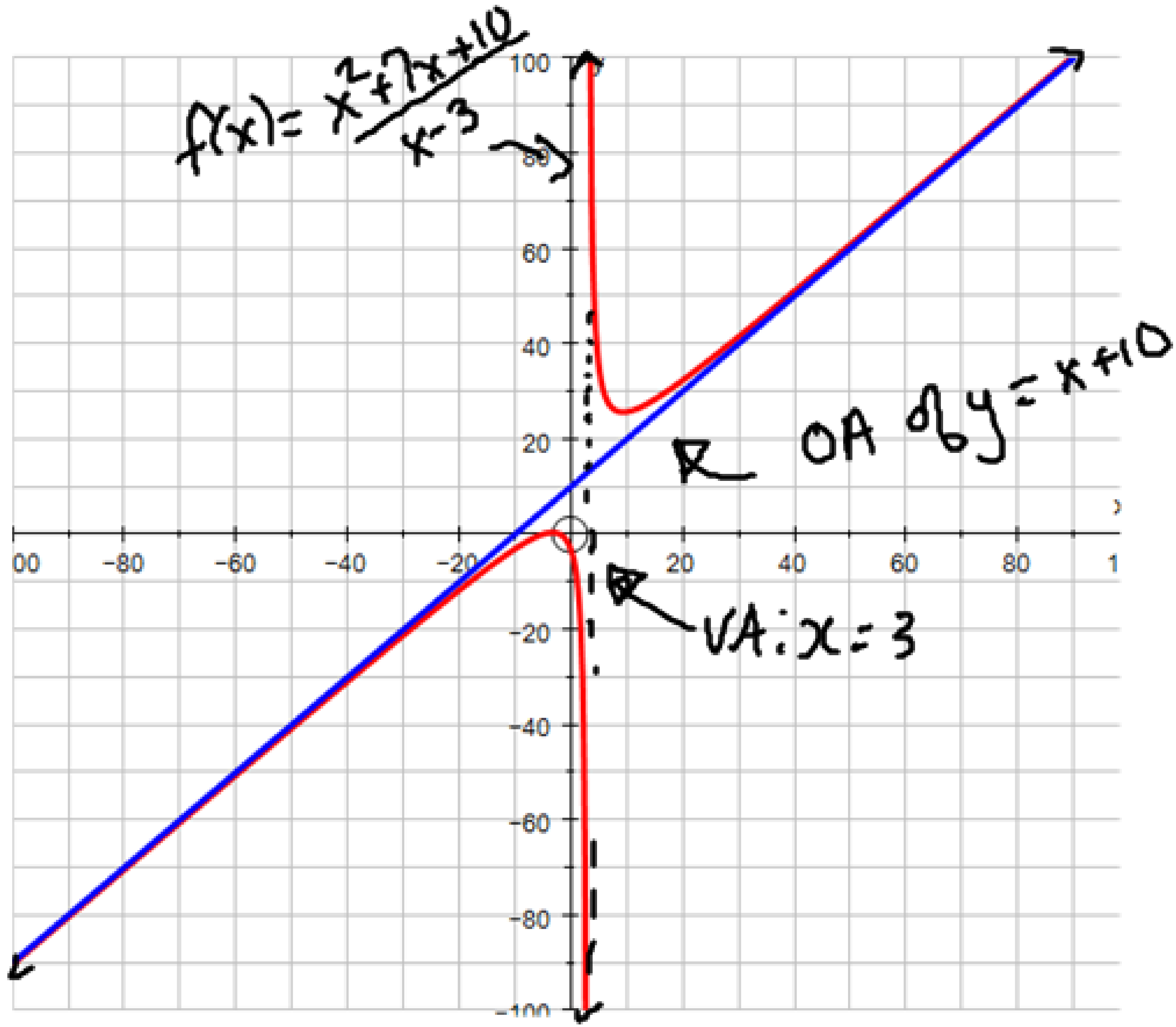
$\leftarrow$ remainder

$f(x)=x+10+\frac{40}{x-3}$

as $x\to\pm\infty$, $\frac{40}{x-3}\to 0$

long term, graph will look linear $y=x+10$

OA: $y=x+10$


$f(x) = \frac{x^2+7x+10}{x-3}$
OA $y = x+10$
VA: $x = 3$
100
60
40
20
-20
-40
-60
-80
-100
-80
-60
-40
-20
20
40
60
80

Example: Sketch the following graph, identifying the intercepts and the asymptotes.

$$f(x) = \frac{x^2+5x+6}{x+4} = \frac{(x+2)(x+3)}{x+4}$$

PODS NONE

VA $x=-4$

HA NONE

OA $y=x+1$

x-int $(-2,0)$ $(-3,0)$

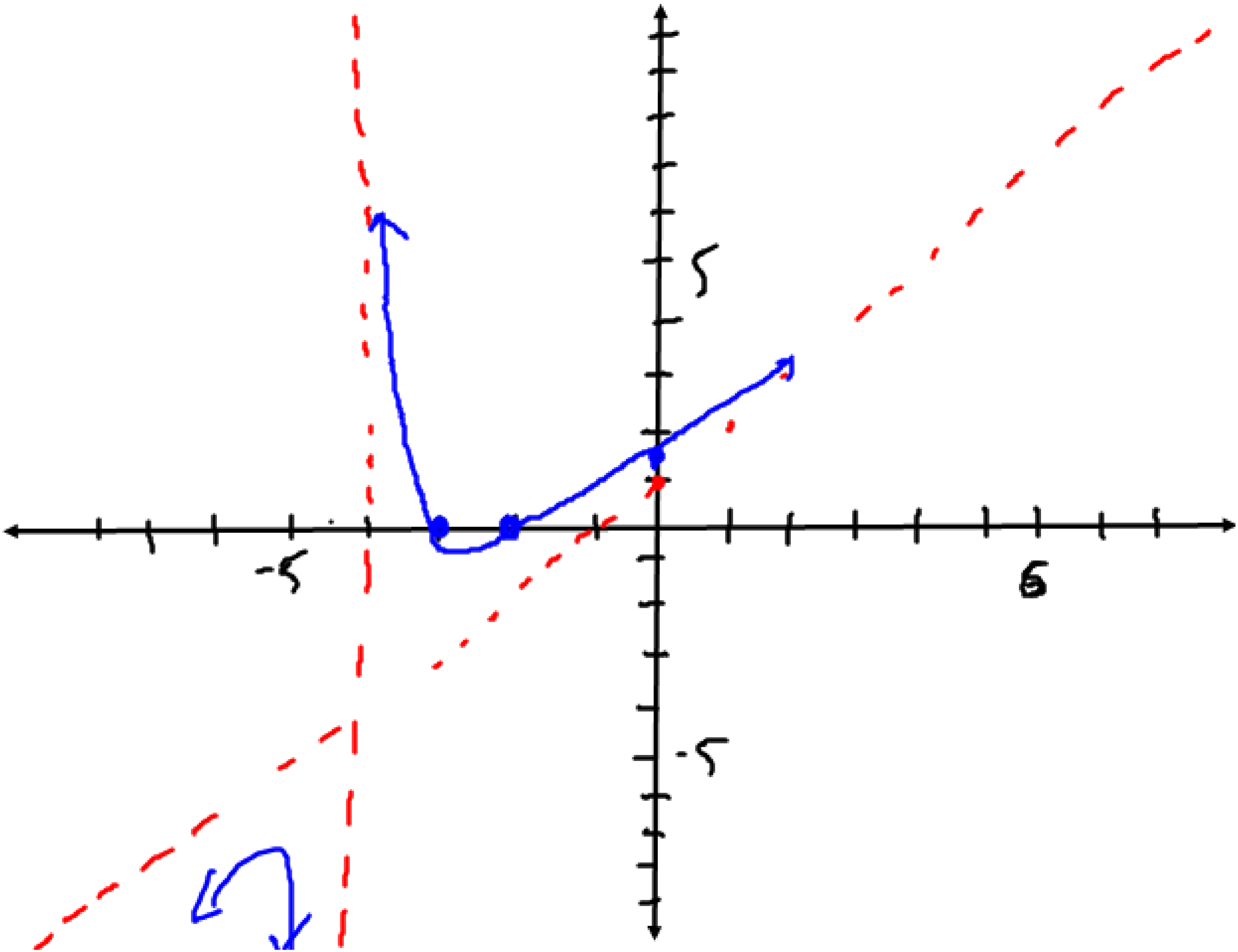
y-int $\left(0, \frac{6}{4}\right)$

$$x+4 \overline{)\, x^2+5x+6}$$

$$\begin{array}{r|rrr} -4 & 1 & 5 & 6 \\ & \downarrow & -4 & -4 \\ \hline x & 1 & 1 & 2 \end{array}$$

$$f(x) = \underbrace{x+1}_{\text{OA}} + \frac{2}{x+4}$$

Sketch


5
-5
6
-5

HW: pg 454 #5-8, 10,
16 & sheet