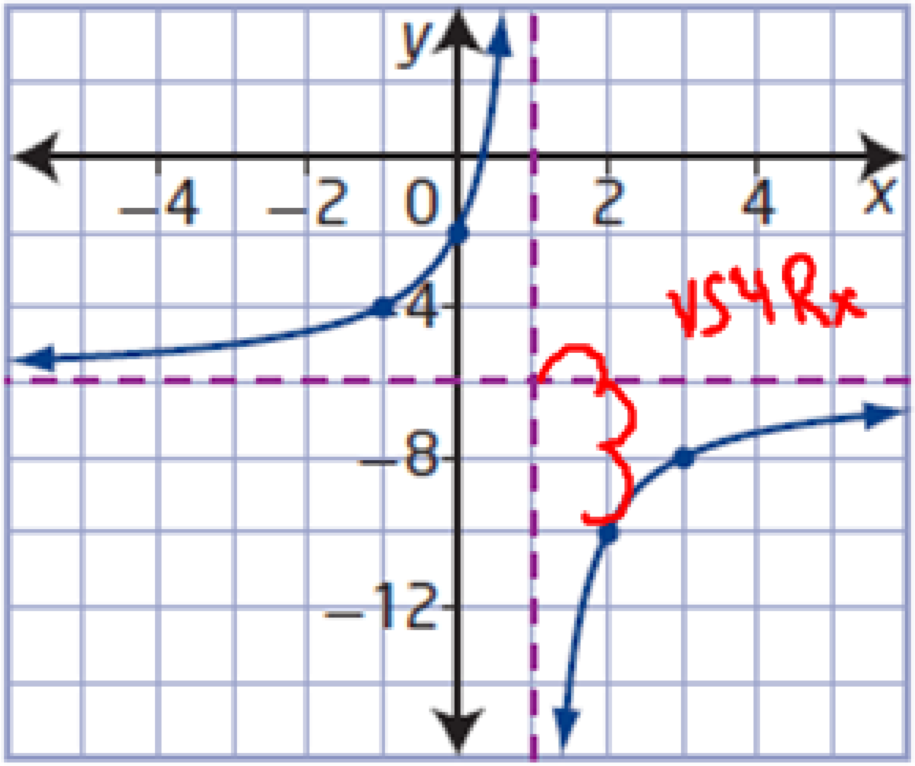
Apply

7. Write the equation of each function in the form $y = \frac{a}{x - h} + k$.

d)



VA: $x = 1$ ∴ HT+1

HA: $y = -6$ ∴ VT−6

$$y = \frac{a}{x-1} - 6$$

$$y = \frac{-4}{x-1} - 6$$

use coord pt $(0,-2)$

$$-2 = \frac{a}{-1} - 6$$

$4 = -a$ $a = -4$

More Complex Rational Functions $y = \frac{a}{x-h} + k$

Example: Find the following for the given function: $y = \frac{x+3}{x-1}$

A) x-intercept $(y=0)$

$0 = \frac{x+3}{x-1}$

$(0)(x-1) = \left(\frac{x+3}{\cancel{x-1}}\right)\cancel{(x-1)}$

$0 = x+3$

$x = -3$

$(-3, 0)$

x-int come from the numerator

B) y-intercept $(x=0)$

$y = \frac{0+3}{0-1}$

$y = -3$

$(0, -3)$

C) Vertical asymptote

(HT)

Set the denominator to zero

$x - 1 = 0$

$x = 1$

D) Horizontal asymptote (Long term behaviour)
(VT)
end behaviour when $x \to \infty$
$x \to -\infty$

$y = \frac{x+3}{x-1}$ make it look like $y = \frac{a}{x-h} + k$

$y = \frac{x+1-1+3}{x-1}$ ← add a fancy zero

$y = \frac{(x-1)+4}{x-1}$ ← Break up

$y = \frac{x-1}{x-1} + \frac{4}{x-1}$

$y = 1 + \frac{4}{x-1}$

VT: 1
HA: $y = 1$

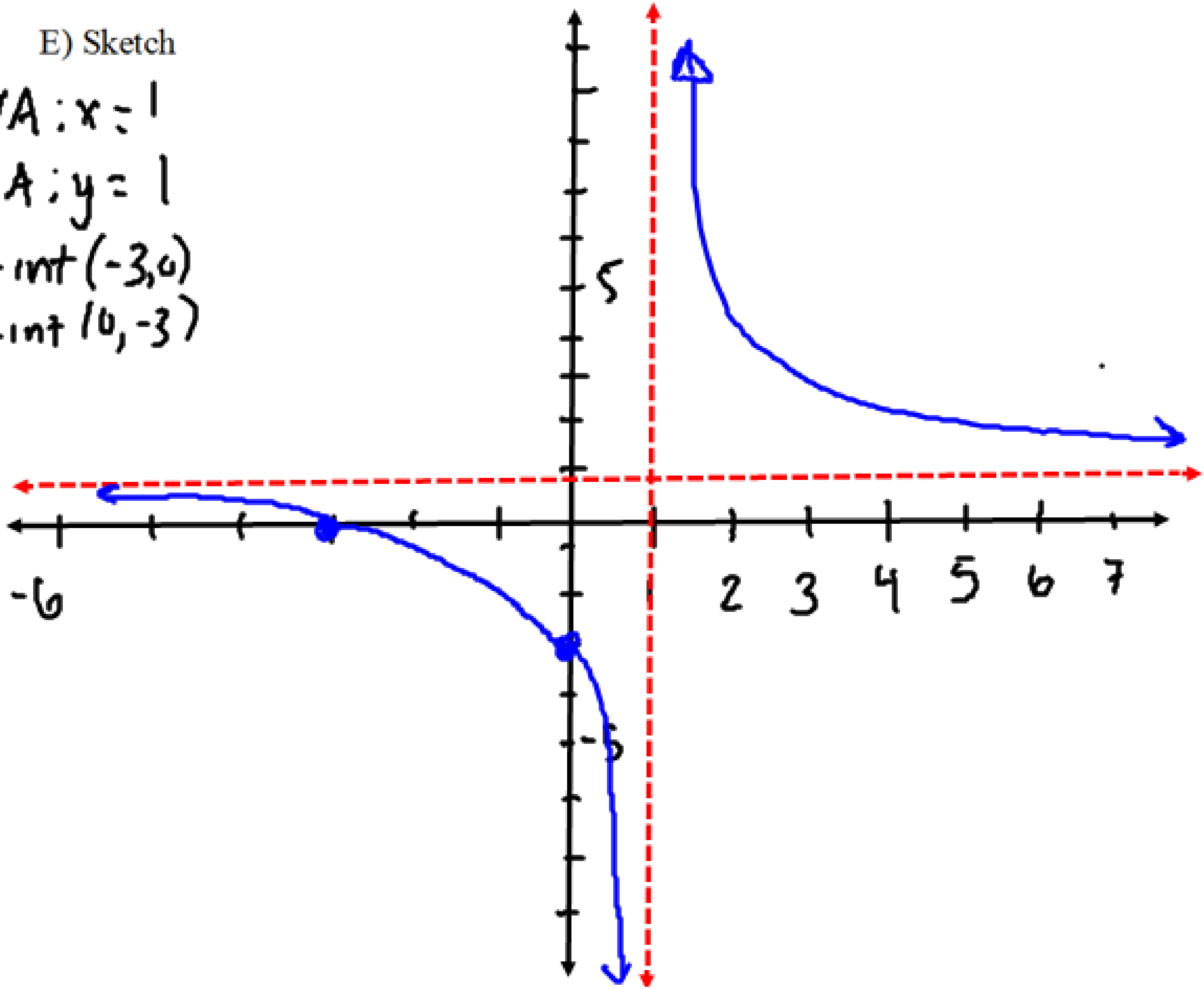
E) Sketch

VA: $x = 1$

HA: $y = 1$

x-int $(-3,0)$

y-int $(0,-3)$



Example: Find the following for the given function:

$$y=\frac{2x-4}{x+3}$$

A) x-intercept (y=0)

numerator

$0=2x-4$

$2x=4$ $(2,0)$

$x=2$

B)y-intercept (x=0)

$y=\frac{0-4}{0+3}=\frac{-4}{3}$

$(0,\frac{-4}{3})$

C) Vertical asymptote

denominator

$0=x+3$ $x=-3$

D) Horizontal asymptote

look for multiples of (x+3) in the numerator

$$y=\frac{2x+6-6-4}{x+3}$$

$$y=\frac{(2x+6)-10}{x+3}$$

$$y=\frac{2x+6}{x+3}-\frac{10}{x+3}$$

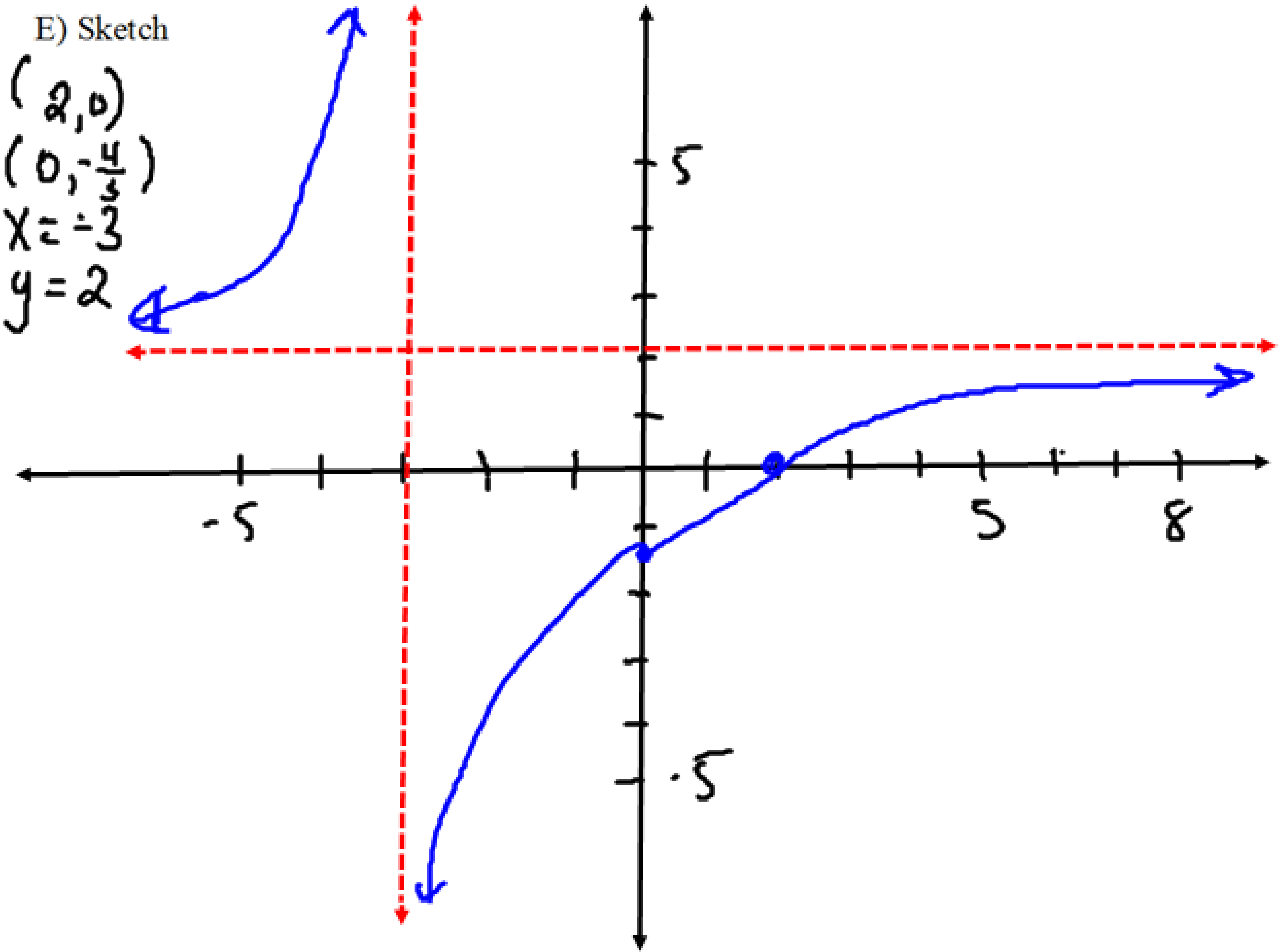
$$y=\frac{2(x+3)}{x+3}-\frac{10}{x+3}$$

$$y=2-\frac{10}{x+3}$$

HA: $y=2$

E) Sketch

(2,0)
$(0,-\frac{4}{3})$
$X=-3$
$y=2$



A faster way to determine Horizontal asymptotes:

- Recall: Horizontal asymptotes are the long term behaviour of the function. They are horizontal lines which the graph approaches but doesn't actually reach.
- These can be found by making $x \to \infty$ and $x \to -\infty$

Finding Horizontal Asymptotes

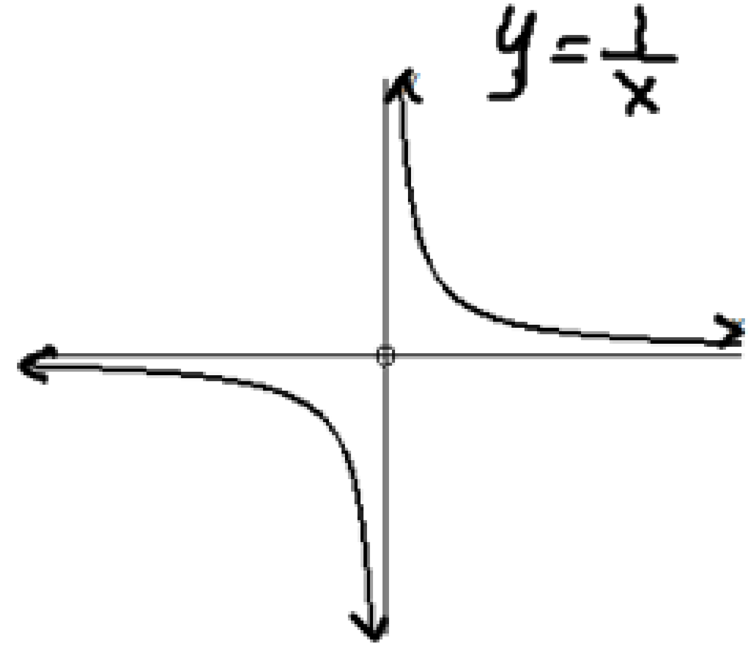
Things to know:

As $x \to \infty$, $\frac{1}{x} \to 0$

$y \to 0$

As $x \to -\infty$, $\frac{1}{x} \to 0$

approaches



A- Both polynomials are the same degree

$$f(x) = \frac{2x-3}{1x+2}$$

-common factor x

$$f(x) = \frac{x\left(2 - \frac{3}{x}\right)}{x\left(1 + \frac{2}{x}\right)}$$

as $x \to \pm\infty$, $\frac{3}{x} \to 0$

, $\frac{2}{x} \to 0$

$$f(x) = \frac{2 - \frac{3}{x}}{1 + \frac{2}{x}}$$

as $x \to \pm\infty$, $f(x) \to \frac{2-0}{1+0}$

$\underbrace{f(x) \to 2}_{\text{long term behaviour}}$

HA: $y = 2$

If the degrees are the same the horizontal asymptote is

$$y = \frac{\text{coefficient of numerator}}{\text{coefficient of denominator}}$$

leading coefficients

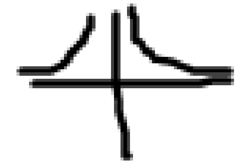
$$y = \frac{6x^2 - 9}{3x^2 + 2x} \qquad y = \frac{9 - 15x^4}{x^4 + 7} \qquad y = \frac{3x - 7}{12x - 5}$$

HA: $y = \frac{6}{3}$ $\qquad y = \frac{-15}{1}$ $\qquad y = \frac{3}{12}$

$y = 2$ $\qquad y = -15$ $\qquad y = \frac{1}{4}$

B- Higher degree in the denominator

$\frac{1}{x}$ 

$\frac{1}{x^2}$ 

$$f(x)=\frac{3x-9}{x^2-x-2} \quad \begin{matrix} \leftarrow \text{factor } x \\ \leftarrow \text{factor } x^2 \end{matrix}$$

$$f(x)=\frac{x\left(3-\frac{9}{x}\right)}{x^2\left(1-\frac{1}{x}-\frac{2}{x^2}\right)}$$

$$=\left(\frac{1}{x}\right)\left(\frac{3-\frac{9}{x}}{1-\frac{1}{x}-\frac{2}{x^2}}\right)$$

as $x\to\pm\infty$, $\frac{9}{x}\to 0$

$\frac{1}{x}\to 0$

$\frac{2}{x^2}\to 0$

as $x\to\pm\infty$, $f(x)\to(0)\left(\frac{3-0}{1-0-0}\right)$

$f(x)\to 0$

HA: $y=0$

If the degree is higher in the denominator H.A.: y=0

Example: Find the following for the given function:

$$y = \frac{3x-1}{x-1}$$

A) x-intercept

$3x-1=0$

$x=\frac{1}{3}$

$(\frac{1}{3},0)$

B) y-intercept

$y=\frac{0-1}{0-1}$

$y=1$

$(0,1)$

C) Vertical asymptote

$x-1=0$

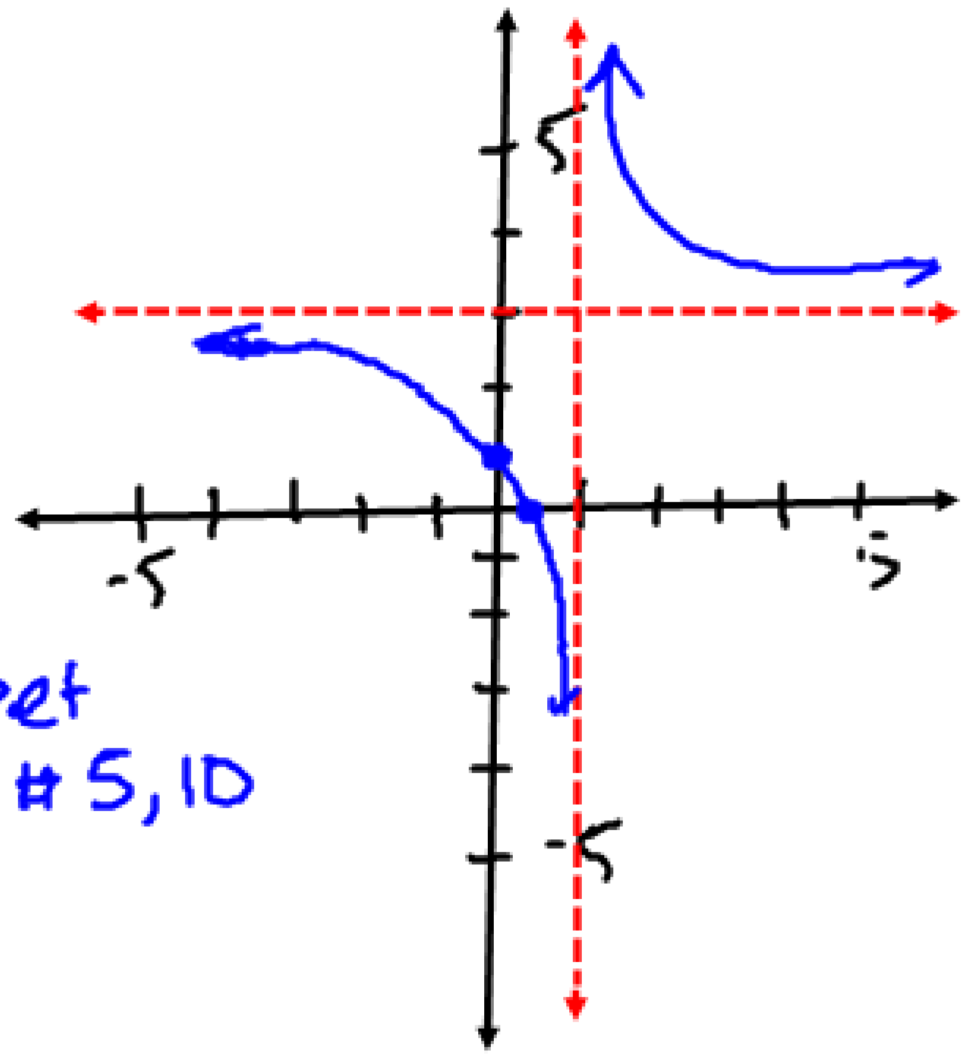
$x=1$

D) Horizontal asymptote

$y=\frac{3}{1}$

$y=3$

E) Sketch



HW: Sheet
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