

Solve for x:

$$\ln(x-4) + \ln(2x-3) = \ln(2x^2-5x)$$

$$\log_5 x = \log_5 12$$

Product
Law

$$x = 12$$

$$\ln[(x-4)(2x-3)] = \ln(2x^2-5x)$$

$$(x-4)(2x-3) = 2x^2-5x$$

$$2x^2-11x+12 = 2x^2-5x$$

$$0 = \cancel{2x^2} - 5x - \cancel{2x^2} + 11x - 12$$

$$0 = 6x - 12$$

$$12 = 6x$$

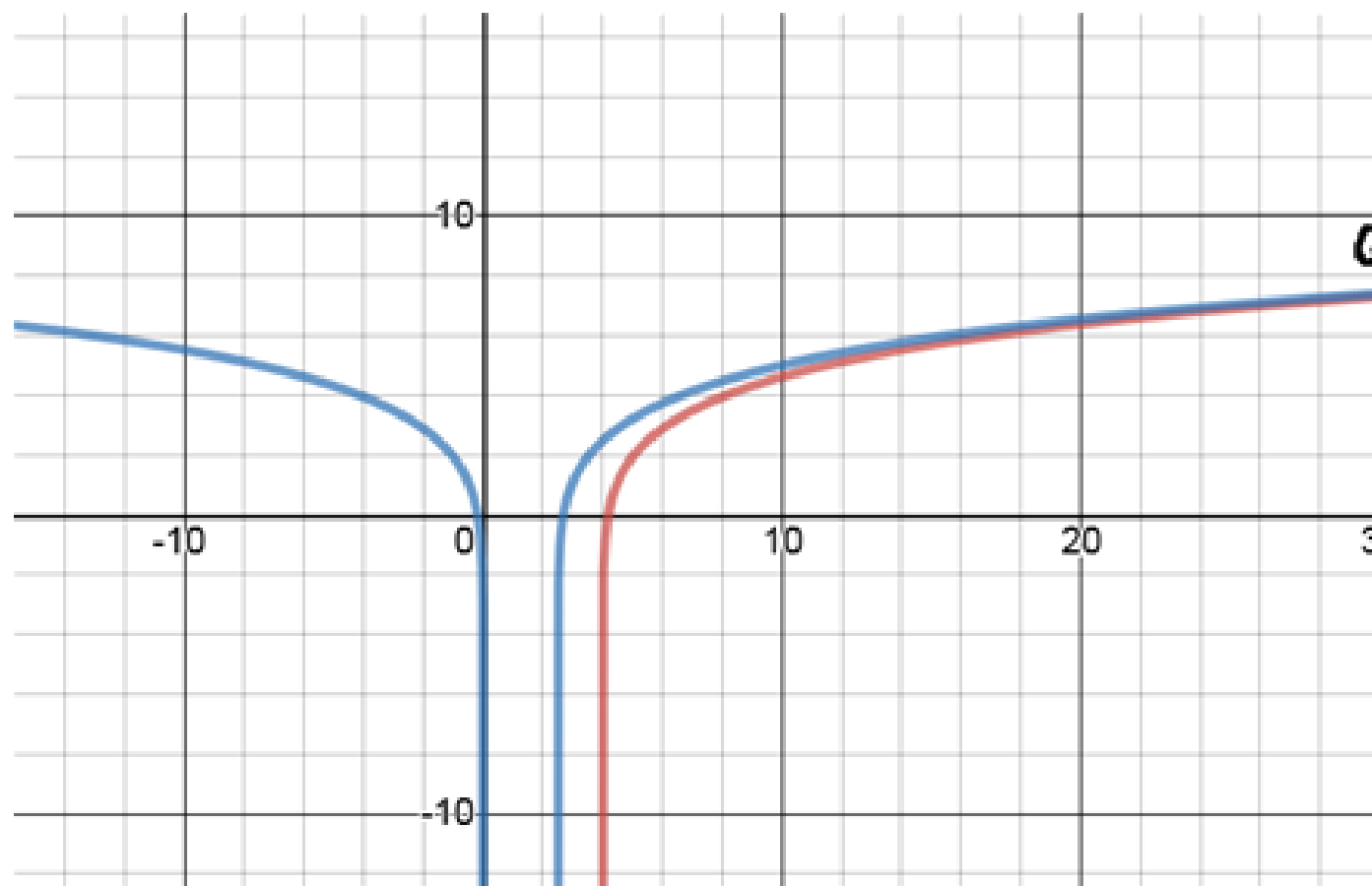
$$2 = x$$

check

$$\ln(2-4) = \ln(-2)$$

not
possible

No SOL'N to this
Question



these do not intersect, but they get super close to each other

$y = \ln(x - 4) + \ln(2x - 3)$

$y = \ln(2x^2 - 5x)$

\therefore NO point of intersection

Graph: $y = 2^x - 3$ (exact values only)

Increasing
 $c > 1$

x-int ($y=0$)

$$0 = 2^x - 3$$

$$2^x = 3$$

$$x = \log_2 3$$

$$2^1 = 2, 2^2 = 4$$

$$2 < 3 < 4$$

$$2^1 < 2^x < 2^2$$

$$1 < x < 2$$

$$(\log_2 3, 0)$$

y-int ($x=0$)

$$y = 2^0 - 3$$

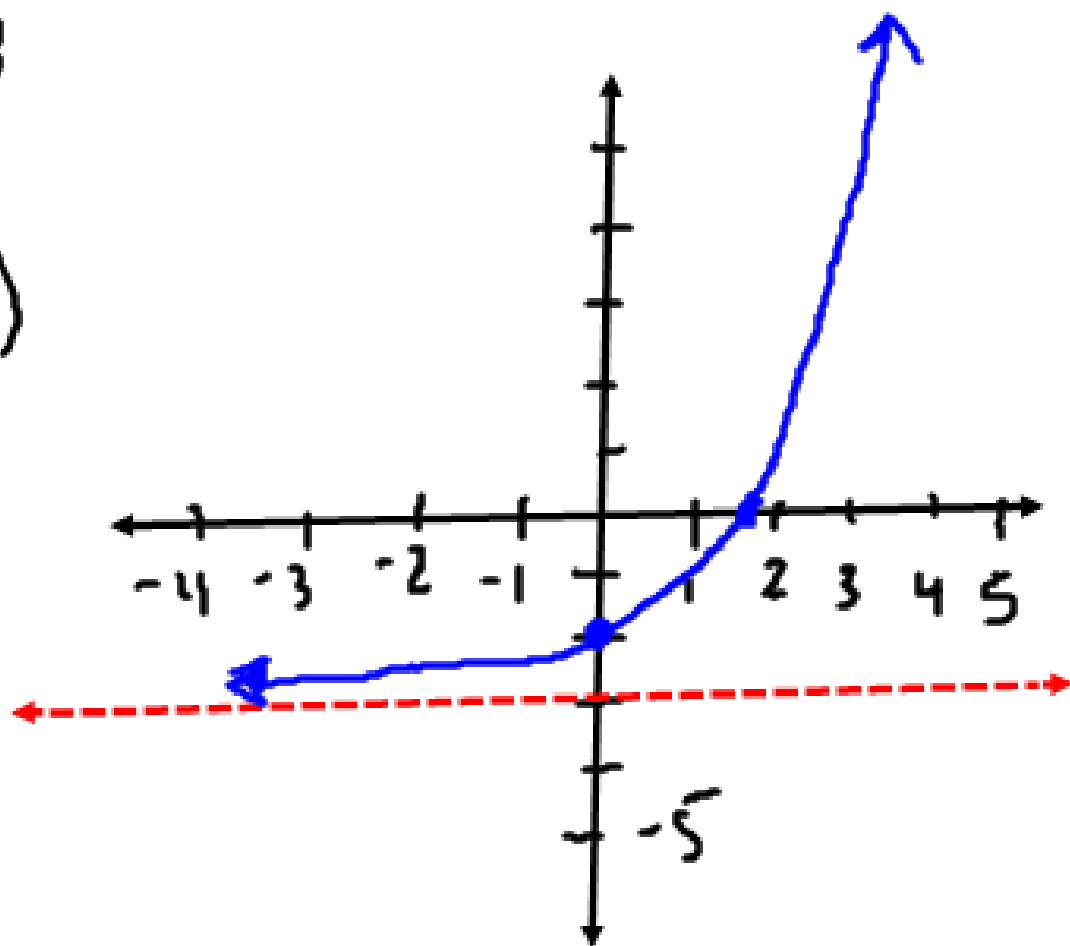
$$y = 1 - 3$$

$$y = -2$$

$$(0, -2)$$

HA (VT)

$$y = -3$$



Graph: $y = 2e^x - 1$ (exact values only)



increasing $c > 1$ $c = e$
 $= 2.718$

x-int ($y=0$)

$$0 = 2e^x - 1 \quad \left(\ln\left(\frac{1}{2}\right), 0\right)$$

$$1 = 2e^x$$

$$e^x = \frac{1}{2}$$

$$e^0 = 1, e^1 = 2.718$$

$$e^{-1} = \frac{1}{2.7}$$

$$x = \ln\left(\frac{1}{2}\right)$$

$$\approx -0.6931$$

y-int ($x=0$)

$$y = 2e^0 - 1$$

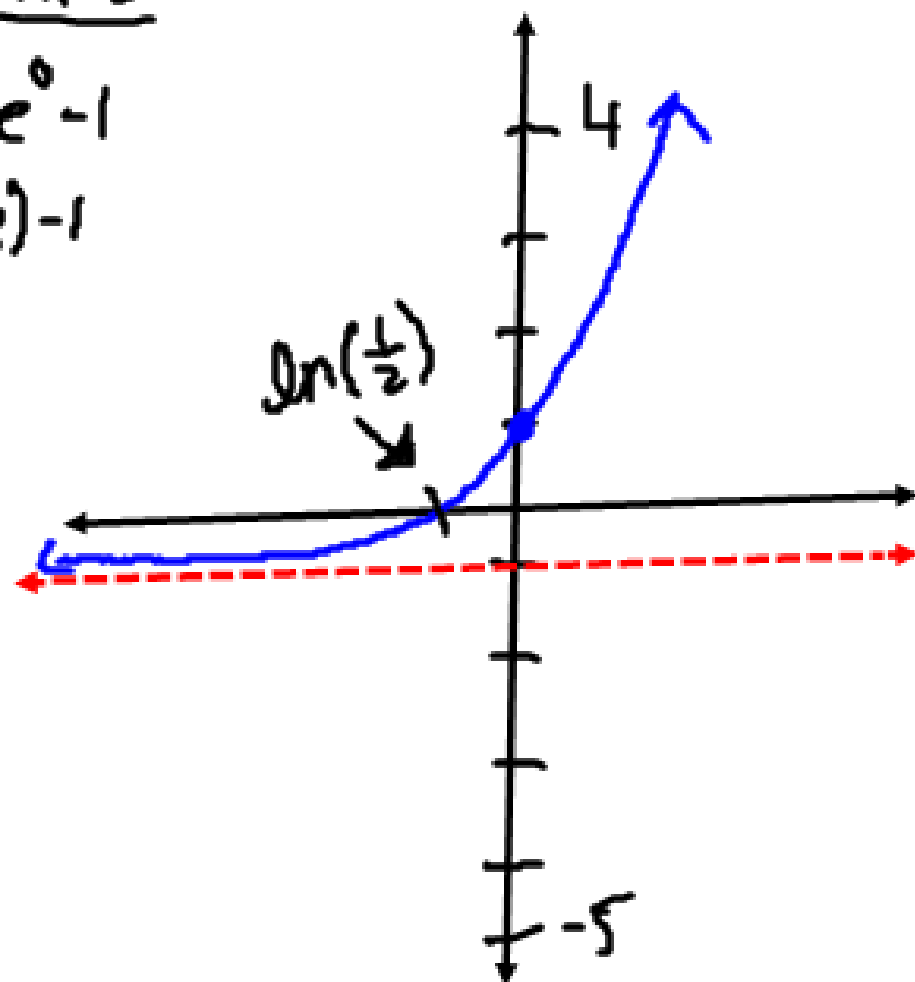
$$= 2(1) - 1$$

$$= 1$$

$(0, 1)$

HA (VT)

$$y = -1$$



Graph $y = \ln(2e - 2x) - 1$ (exact values only)

X-int (y=0)

$$0 = \ln(2e - 2x) - 1$$

$$1 = \ln(2e - 2x)$$

$$e^1 = 2e - 2x$$

$$2x = 2e - e$$

$$2x = e$$

$$x = \frac{1}{2}e$$

$$\left(\frac{1}{2}e, 0\right)$$

Y-int (x=0)

$$y = \ln(2e - 0) - 1$$

$$y = \ln 2e - 1$$

$$= \ln 2 + \ln e - 1$$

$$= \ln 2 + 1 - 1$$

$$y = \ln 2$$

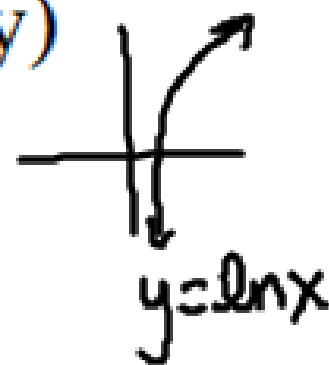
$$(0, \ln 2)$$

VA

$$2e - 2x > 0$$

$$\frac{-2x}{-2} > \frac{-2e}{-2}$$

$$x < e$$



$$\ln(2e - 2x)$$

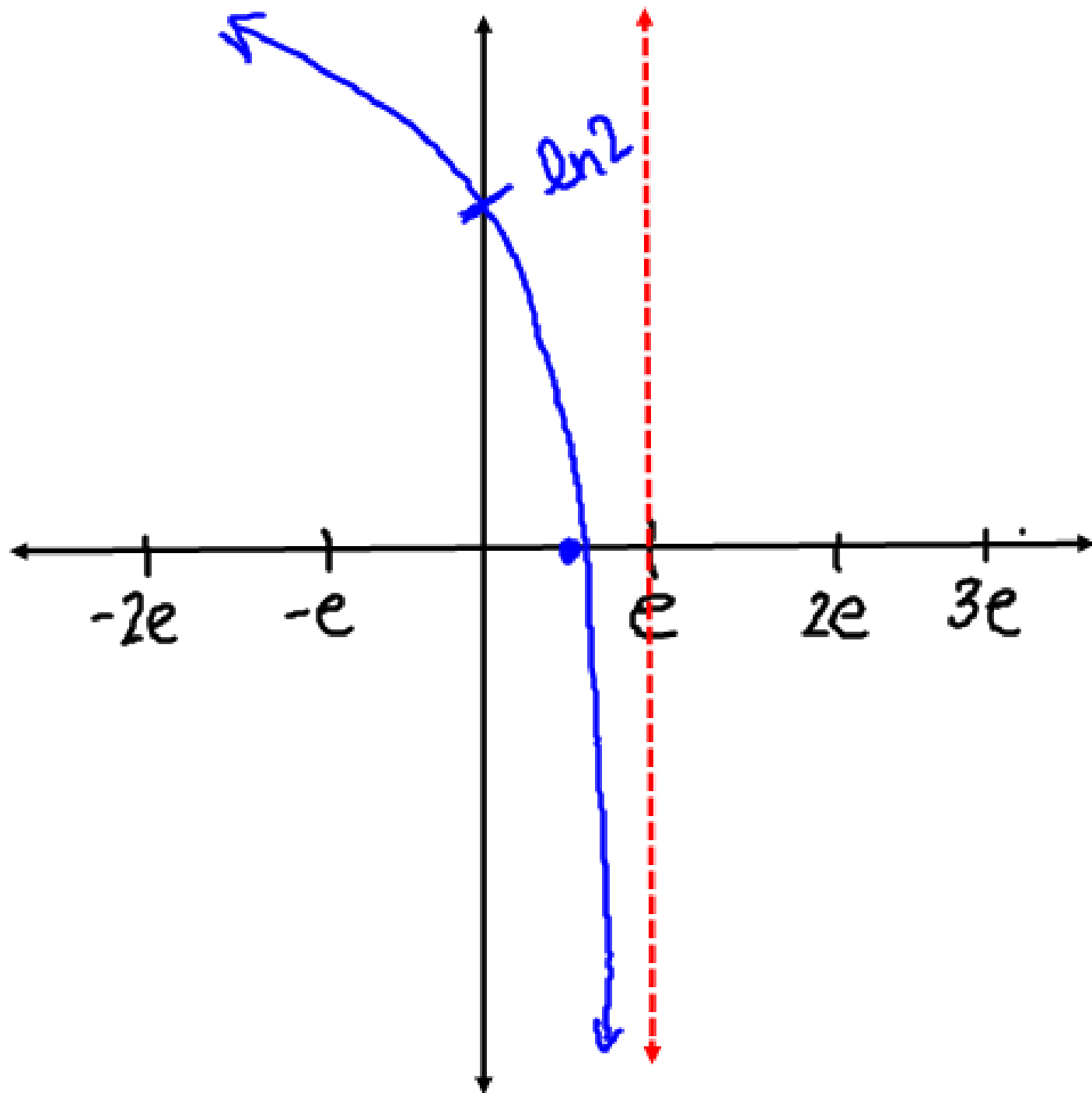
$$\ln(-2x + 2e)$$

$$\ln(-2(x - e))$$

Ry Yes

HS $\frac{1}{2}$

HT + e



8.4

Logarithmic and Exponential Equations

Focus on...

- solving a logarithmic equation and verifying the solution
- explaining why a value obtained in solving a logarithmic equation may be extraneous
- solving an exponential equation in which the bases are not powers of one another
- solving a problem that involves exponential growth or decay
- solving a problem that involves the application of exponential equations to loans, mortgages, and investments
- solving a problem by modelling a situation with an exponential or logarithmic equation

1. A certain material decays according to the equation
 $A = A_0(0.981)^t$, where t = time (years) and A = amount of
material (grams). How long will it take to decay to 75% of its
original amount?

hundredth

Original Amount: A_0
 $t = ?$ when $A = 0.75A_0$

$$0.75 \cancel{A_0} = \cancel{A_0} (0.981)^t$$

$$0.75 = (0.981)^t$$

$$t = \log_{0.981} (0.75)$$

$$t = \frac{\log(0.75)}{\log(0.981)}$$

$$t = 14.9968608$$

$$t = 15.00 \text{ years}$$

$$t = \frac{\ln 0.75}{\ln(0.981)}$$

$$t = 14.9968608$$

2. During an experiment, it was recorded that there were initially 250 bacteria present in a culture. After 1 hour, there were 1600 bacteria present. What was the doubling period?

60 min

$$y = a(c)^{bx}$$

time	0	?		
# bacteria	250	500	1000	2000

$$y = 1600$$

$$a = 250$$

$$c = 2$$

$$x = 60 \text{ min}$$

$$b = ?$$

$$1600 = 250(2)^{60b}$$

$$6.4 = 2^{60b}$$

$$60b = \log_2 6.4$$

$$60b = 2.67807$$

$$b = 0.04463$$

$$y = 250(2)^{0.04463x}$$

$$HS: \frac{1}{b}$$

$$\frac{1}{0.04463}$$

$$= 22.40417$$

→ it will take 22.4 minutes to double in size.

$$y = 250(2)^{\frac{x}{22.4}}$$

$$y \quad x = 60 \text{ min}$$
$$y = 1600$$

$$y = 250(2)^{\frac{60}{22.4}}$$
$$= 1600.554085$$

3. Determine how long it would take, (to the nearest month), for a principal sum of \$1000 to produce an amount of \$2000 if the annual interest rate is 6% with interest compounded monthly?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$\begin{aligned} A &= 2000 \\ P &= 1000 \\ r &= 0.06 \\ n &= 12 \\ t &= ? \end{aligned}$$

$$2000 = 1000 \left(1 + \frac{0.06}{12}\right)^{12t}$$

$$2 = (1.005)^{12t}$$

$$12t = \log_{1.005}(2)$$

$$12t = \frac{\ln 2}{\ln 1.005}$$

$$12t = \underbrace{138.9757216}_{\text{\# months}}$$

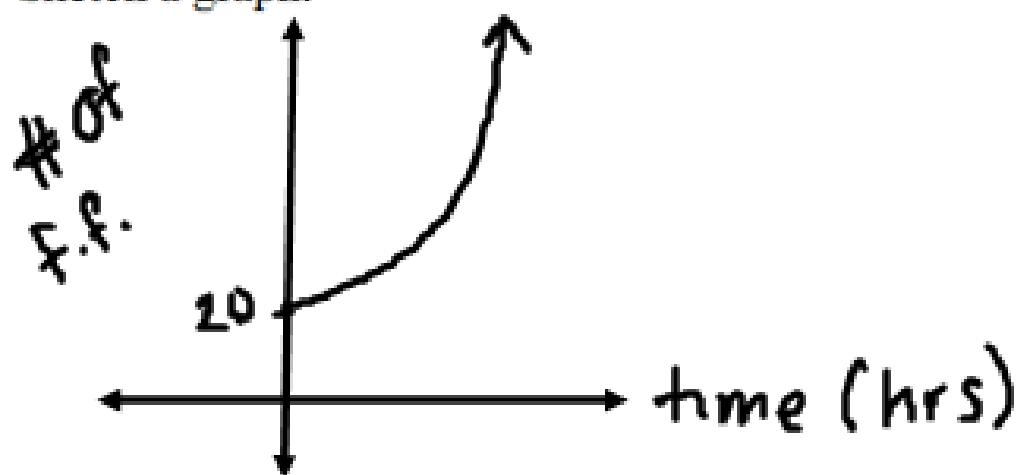
$$t = 11.58131018 \text{ years}$$

→ It will take 139 months

4. $F(t) = 20e^{0.03t}$ models the number of fruit flies in an experimental population at time t .

Increasing function, y -int ($t=0$)

Sketch a graph.



After how many hours would there be 121 fruit flies?

$$\frac{121}{20} = \frac{20e^{0.03t}}{20}$$

$$6.05 = e^{0.03t}$$

$$\rightarrow 0.03t = \ln 6.05$$

$$t = \frac{\ln 6.05}{0.03}$$

$$t = 60.0019424$$

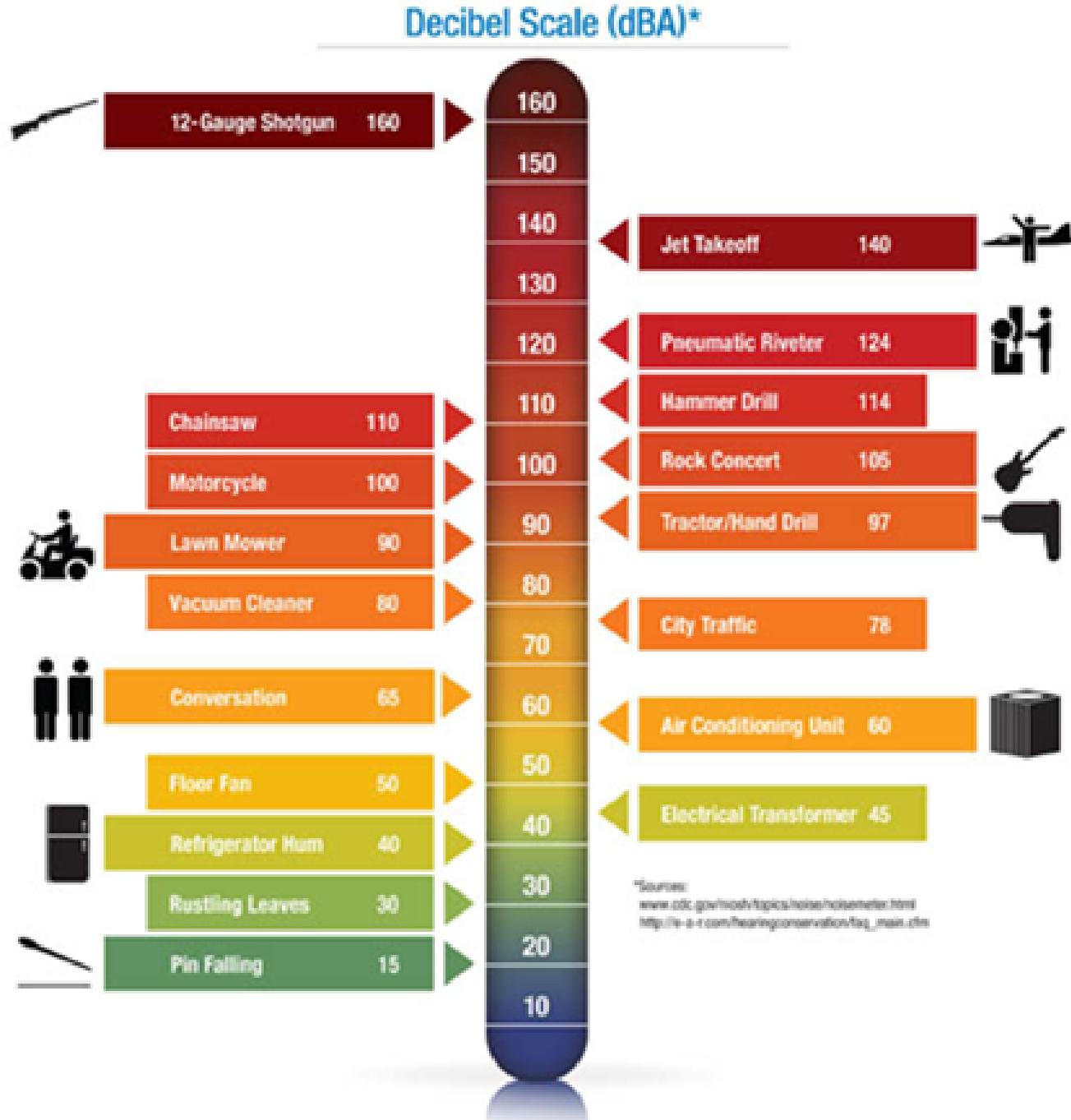
\rightarrow it will take 60 hrs.

Logarithmic Scales – used to describe many natural phenomena.

Number (N)	10^0	10^1	10^2	10^3	10^4	10^5
Exponent ($\log_{10}N$)	0	1	2	3	4	5

In this table the numbers increase by a factor of 10, but the exponents increase by 1.

1. Sound



Decibel levels:

The difference in sound levels, in decibels, can be found using the equation:

$$\beta_2 - \beta_1 = 10 \log \left(\frac{I_2}{I_1} \right)$$

where $\beta_2 - \beta_1$ is the difference in sound levels, in decibels, and $\frac{I_2}{I_1}$ is the ratio of their sound intensities, where I is measured in watts per square metre (W/m^2).

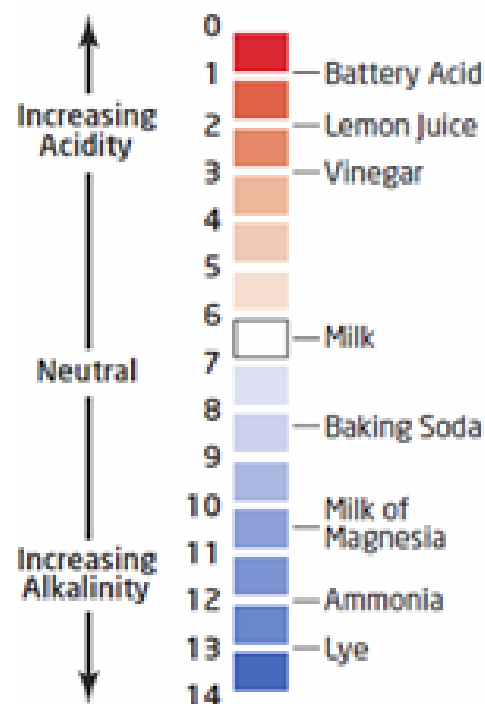
Examples:

(a) How many times as intense as a whisper (30 dB) is the sound of a normal conversation (60 dB)?

(b) The sound level in normal city traffic is approximately 85 dB. The sound level while riding a snowmobile is about 32 times as intense. What is the sound level while riding a snowmobile, in decibels?

Your Turn

The pH scale is used to measure the acidity or alkalinity of a solution. The pH of a solution is defined as $\text{pH} = -\log [\text{H}^+]$, where $[\text{H}^+]$ is the hydrogen ion concentration in moles per litre (mol/L). A neutral solution, such as pure water, has a pH of 7. Solutions with a pH of less than 7 are acidic and solutions with a pH of greater than 7 are basic or alkaline. The closer the pH is to 0, the more acidic the solution is.



- a) A common ingredient in cola drinks is phosphoric acid, the same ingredient found in many rust removers. A cola drink has a pH of 2.5. Milk has a pH of 6.6. How many times as acidic as milk is a cola drink?

- b) An apple is 5 times as acidic as a pear. If a pear has a pH of 3.8, then what is the pH of an apple?