Ch 8.2

Example:

Use transformations to sketch the graph of the function $c>1$

$$y=-\frac{1}{2}\log_2(-2x-6)+1$$

$$y=-\frac{1}{2}\log_2(-2(x+3))+1$$

VS $\frac{1}{2}$ HS $\frac{1}{2}$

VT 1 HT -3

R_x Y R_y Yes

State:

A The equation of the asymptote

B The domain and range

c The x and y-intercepts (if they exist)

VA: (HT) $x=-3$

B) R: $\{y \mid y \in R\}$

Domain

$-2x-6>0$

$\frac{-2x}{-2} > \frac{6}{-2}$

$x<-3$

$\{x \mid x<-3\}$

y-int ($x=0$)

$y=-\frac{1}{2}\log_2(-6)+1$

NO SOL'N

no y-int

R_x

R_y

x-int (y=0)

$0 = -\frac{1}{2}\log_2(-2x-6)+1$

$-1 = -\frac{1}{2}\log_2(-2x-6)$

$2 = \log_2(-2x-6)$

convert to exp.

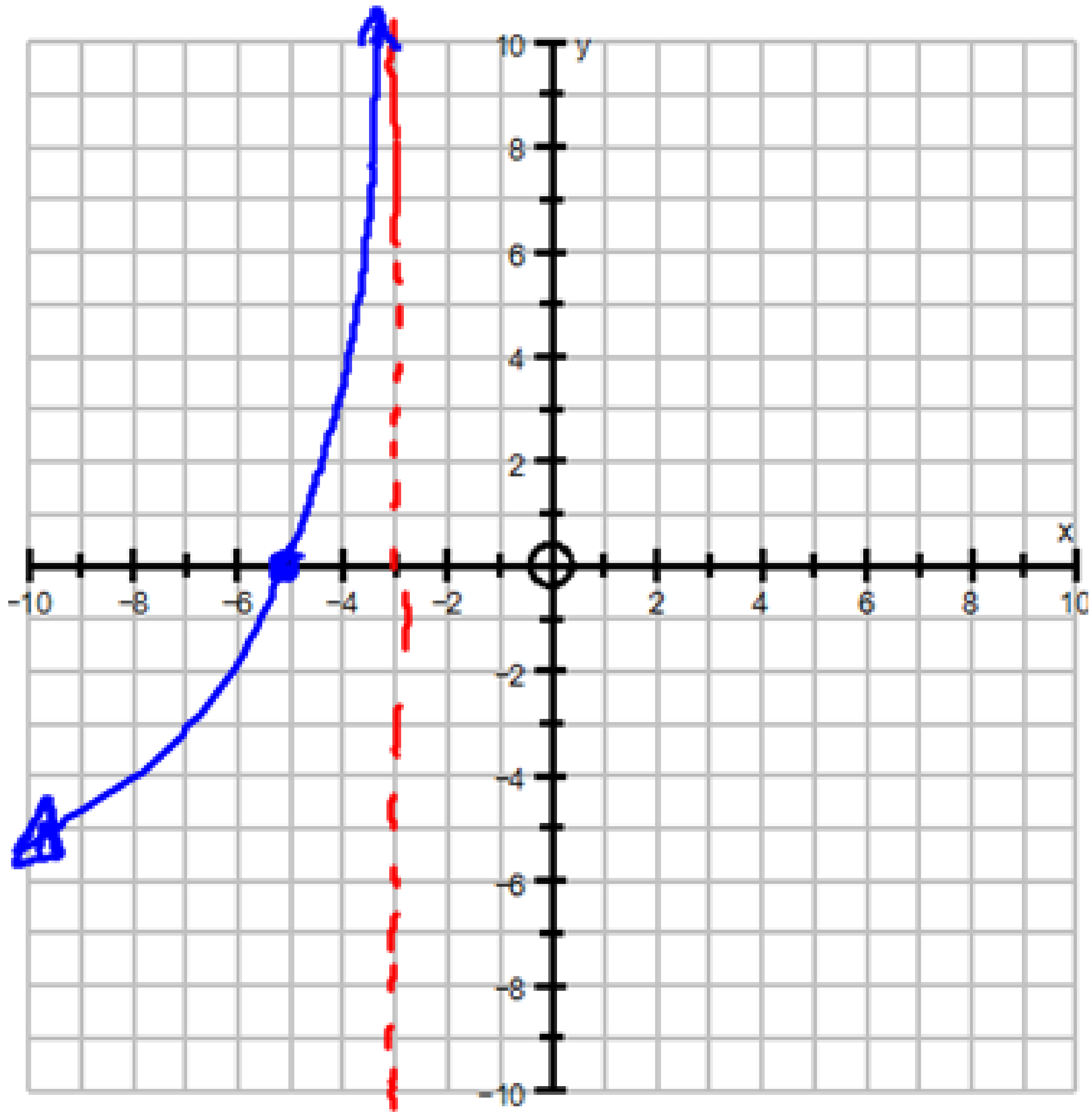
$2^2 = -2x-6$

$4 = -2x-6$

$10 = -2x$

$x = -5$

$(-5,0)$



HW pg390 # 4, 5, 8, 9

last nights.....

Laws of Logarithms

Product Law –

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

Quotient Law –

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

Power Law –

$$\log_a x^b = b \cdot \log_a x$$

The Change of Base Rule

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Evaluate: $\log_4 7$

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log(7)/log(4)
          1.403677461
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(a) Express $3^x = 7$ in log form.

$\log_3 7 = x$

(b) Solve the equation $3^x = 7$ by taking the log of both sides.

$\log_{10}(3^x) = \log_{10}(7)$ ← Power Law

$$\frac{x \cdot \log 3}{\log 3} = \frac{\log 7}{\log 3}$$

$$x = \frac{\log 7}{\log 3}$$

$$3^x = 7$$

$$\log_3(3^x) = \log_3(7)$$

$$x \cdot \log_3 3 = \log_3 7$$

$$x = \log_3 7$$

$$\log_7(3^x) = \log_7 7$$

$$x \cdot \log_7 3 = 1$$

$$x = \frac{1}{\log_7 3}$$

8.4

Logarithmic and Exponential Equations

Focus on...

- solving a logarithmic equation and verifying the solution
- explaining why a value obtained in solving a logarithmic equation may be extraneous
- solving an exponential equation in which the bases are not powers of one another
- solving a problem that involves exponential growth or decay
- solving a problem that involves the application of exponential equations to loans, mortgages, and investments
- solving a problem by modelling a situation with an exponential or logarithmic equation

Find the exact value for x:

$$3^{x-1} = 7$$

convert to logarithm

$$x-1 = \log_3 7$$

$$\boxed{x = \log_3 7 + 1}$$

take the log of both sides

$$\log(3^{x-1}) = \log(7)$$ ← Power Law

$$(x-1)\log 3 = \log 7$$

$$x\log 3 - \log 3 = \log 7$$

$$x\log 3 = \log 7 + \log 3$$ ← Product Law

$$x\log 3 = \log(7\cdot 3)$$

$$\frac{x\log 3}{\log 3} = \frac{\log 21}{\log 3}$$

$$\boxed{x = \frac{\log 21}{\log 3}}$$

$$3^{x-1} = 7$$ use exp laws first

$$(3^x)(3^{-1}) = 7$$

$$\frac{3^x}{3} = 7$$

$$3^x = 3 \cdot 7$$

$$3^x = 21$$ convert to logs

$$\boxed{\log_3 21 = x}$$

$$4^{2x} - 5(4^{x}) + 6 = 0$$

$(4^x)^2 - 5(4^x) + 6 = 0$

let $m = 4^x$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2 \quad m = 3$

$4^x = 2$

$(2^2)^x = 2^1$

$2^{2x} = 2^1$

$2x = 1$

$x = \frac{1}{2}$

$x = \log_4 2$ ← don't leave like this... evaluate

$4^x = 3$

$\boxed{x = \log_4 3}$

Solving Equations with Logs

- If the bases of the logarithms are all the same, use the laws of logarithms to simplify so there is one log on each side.
- If the bases of the logarithms are different, evaluate each and simplify to a single log.

$\log_3 x = \log_3 5$

Examples: Solve for x:

(A) $3\log_5 6 - 2\log_5 3 = \log_5 x$

$\log_5(6^3) - \log_5(3^2) = \log_5 x$

$\log_5\left(\frac{6^3}{3^2}\right) = \log_5 x$

$\log_5\left(\frac{216}{9}\right) = \log_5 x$

$x = \frac{216}{9}$

$x = 24$

$$\text{(B) } \log_3 27 - \log_x 125 = \log_4 16$$

- can't use log laws... not the same base.
- evaluate each log seperatly.

$$3 - \log_x 125 = 2$$

$$3 - 2 = \log_x 125$$

$$1 = \log_x 125$$

$$x^1 = 125$$

$$x = 125$$

(D) $\log_2(x-6) = 3 - \log_2(x-4)$

$\log_2(x-6) + \log_2(x-4) = 3$

$\log_2[(x-6)(x-4)] = 3$

convert to expo.

$2^3 = (x-6)(x-4)$

$8 = x^2 - 10x + 24$

$0 = x^2 - 10x + 16$

$0 = (x-8)(x-2)$

$x = 8$ $\quad$ $x = 2$

Check:

$x = 2$

$\log_2(x-6) \rightarrow \log_2(2-6)$

$\log_2(-4)$

NOT POSSIBLE

reject $x = 2$

$x = 8$ is the only answer

(E) $(\log_7 x)^2 - \log_7 x^2 - 3 = 0$

$(\log_7 x)^2 - 2(\log_7 x) - 3 = 0$

let $m = \log_7 x$

$m^2 - 2m - 3 = 0$

$(m-3)(m+1) = 0$

$m = 3 \quad m = -1$

$\log_7 x = 3 \quad \log_7 x = -1$

convert to exp

$7^3 = x \qquad 7^{-1} = x$

$x = 343 \qquad x = \frac{1}{7}$

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\# 1 - 6, 8