8.3

Laws of Logarithms

$\log_5 625 = 4$

Focus on...

- developing the laws of logarithms
- determining an equivalent form of a logarithmic expression using the laws of logarithms
- applying the laws of logarithms to logarithmic scales

$a^m \cdot a^n = a^{m+n}$

Laws of Logarithms

$\log_{base}(ans) = exp$

Product Law –

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

Proof:

let $x = a^m$ $y = a^n$

$$x \cdot y = a^m \cdot a^n$$

$$xy = a^{m+n}$$

- rewrite x and y in terms of m and n

$m = \log_a x$ $n = \log_a y$

- rewrite $xy = a^{m+n}$ in terms of m+n

$$m + n = \log_a(xy)$$

$$\log_a x + \log_a y = \log_a(xy)$$

Quotient Law – $\frac{a^m}{a^n} = a^{m-n}$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

let $x = a^m$ $\quad y = a^n$

ans $\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$ exp

- rewrite x and y in terms of m and n

$m = \log_a x \quad n = \log_a y$

- rewrite $\frac{x}{y} = a^{m-n}$ in terms of $m-n$

$m - n = \log_a\left(\frac{x}{y}\right)$

$\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$

Power Law –

$$\log_a x^b = b \cdot \log_a x$$

let $x = a^m$

$(x)^b = (a^m)^b$

$x^b = a^{mb}$ exponent

Ans base

- rewrite $x = a^m$ in terms of m

$m = \log_a x$

- rewrite $x^b = a^{mb}$ in terms of mb

$mb = \log_a (x^b)$

$b \cdot \log_a x = \log_a (x^b)$

Simplify by writing as a single logarithm

A) $\log_3 5 + \log_3 10$

Product Law

$\log_3(5 \cdot 10) = \log_3(50)$

B) $\log_2 28 - \log_2 7$

Quotient

$\log_2\left(\frac{28}{7}\right) = \log_2(4)$

$= 2$

C) $2\log 9 + \log 2$

- Base 10
- Power law 1st

$\log(9^2) + \log 2$

$= \log 81 + \log 2$ ← Product Law

$= \log(81 \cdot 2) = \log(162)$

D) $3\log 3 - 1$ ← fancy 1

$\log_{10} 10 = 1$

$3\log 3 - \log 10$

Power Rule

$\log(3^3) - \log 10$

$\log(27) - \log 10$

$\log\left(\frac{27}{10}\right)$

F) $2\log 5 + \frac{1}{2}\log 16$

$16^{\frac{1}{2}} = \sqrt{16}$

$\log(5^2) + \log(16^{1/2})$

$\log 25 + \log 4$

$\log(25 \cdot 4)$

$\log(100) = \log_{10}(100)$

$= 2$

Write as a sum or difference of logarithms. Simplify if possible.

(a) $\log_3 xy$ Product Rule

$\log_3 x + \log_3 y$

(b) $\log 20$

$20 = 2 \cdot 10$
$= 4 \cdot 5$
$= \frac{40}{2}$
$= \frac{100}{5}$
$= \sqrt{400}$

$\rightarrow \log 2 + \log 10$

$\rightarrow \log 40 - \log 2$

$\log(\sqrt{400})$

$\rightarrow \log(400^{\frac{1}{2}}) = \frac{1}{2}\log 400$

(c) $\log(ab^2c)$

$\log a + \log b^2 + \log c$

$\log a + 2\log b + \log c$

Simplify: (no calculator)

$$\frac{\log 8}{\log 32}$$

$$= \frac{\log_{10}(8)}{\log_{10}(32)} = \frac{\log_{10}(2^3)}{\log_{10}(2^5)}$$

$$= \frac{3 \cdot \log_{10} 2}{5 \cdot \log_{10} 2}$$

let $z = \log_{10} 2$

$$= \frac{3z}{5z}$$

$$= \frac{3}{5}$$

Write as a logarithmic equation (in base 10):

$$A = \frac{x^2 y^3}{z}$$

The same way we can take the square root of both sides, we can take the logarithm of both sides

$x^2 = 9$ ← function

$\sqrt{x^2} = \sqrt{9}$

Function → $\log(A) = \log\left(\frac{x^2 y^3}{z}\right)$

$= \log x^2 + \log y^3 - \log z$

$= 2\log x + 3\log y - \log z$

The Change of Base Rule

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$\log_2 8 = 3$

[redacted]

$$\log_2 8 = \frac{\log_{10} 8}{\log_{10} 2}$$

```
log(8)
          .903089987
log(2)
          .3010299957
log(8)/log(2)
                   3
```

HW pg 400
#1-3,5,8,9
Sheet (1st 3 parts)

Brackets are important!!

```
log(8/log(2
     1.424480215
```

HW: pg 400 #1-3,5,8,9
sheet (first three parts)