8.2

Transformations of Logarithmic Functions

functions we have seen this course:

$$\left.\begin{array}{l} y=\sqrt{x} \\ y=\cos(x) \end{array}\right\} \text{they each have inputs}$$

Focus on...

- explaining the effects of the parameters a, b, h, and k in $y = a \log_c (b(x - h)) + k$ on the graph of $y = \log_c x$, where $c > 1$
- sketching the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_c x$, where $c > 1$, and stating the characteristics of the graph

$$\text{base}^{\text{exp}} = \text{ans} \qquad \log_{\text{base}}(\text{ans}) = \text{exp}$$

input

Allowable Bases and Arguments for Logarithms:

input of a function is called an argument

1. The base cannot be 0 or 1

Things we know:

0 raised to any power is 0 $(0^1=0)$

1 raised to any power is 1 $(1^5=1)$

$\log_a x$

a Base

$\log_1 5 = a$

convert to exponential

$1^a = 5$

no value of "a" exists to make this true

$\log_0 7 = b$

$0^b = 7$

not true.

2. Bases cannot be negative ($\log_{-2} 7$ is Bad...) why?

Consider $y = (-2)^x$

Complete the table of values:

x	y
0	1
0.5	
1	-2
1.5	
2	4

$(-2)^{\frac{1}{2}} = \sqrt{-2}$ Not possible

not a real #

$(-2)(-2)$

3. The argument of a logarithm must be positive ($\log_2(7)$)

A positive number, raised to *any* power, is always positive.

argument

$\log_2(-3) = a$

convert to exponential

$2^a = -3$

no value of "a" to make

2^a a negative.

To graph logarithmic functions, use transformations and mapping rules:

Transformational form: $y = \overset{R_x}{-}af(\overset{R_y}{-}b(x-h))+k$ ← Ch 1 ☺

$$y = \overset{R_x}{-}a\log_c(\overset{R_y}{-}b(x-h))+k$$

VS a HS $\frac{1}{b}$

VT k HT $+h$

R_x R_y

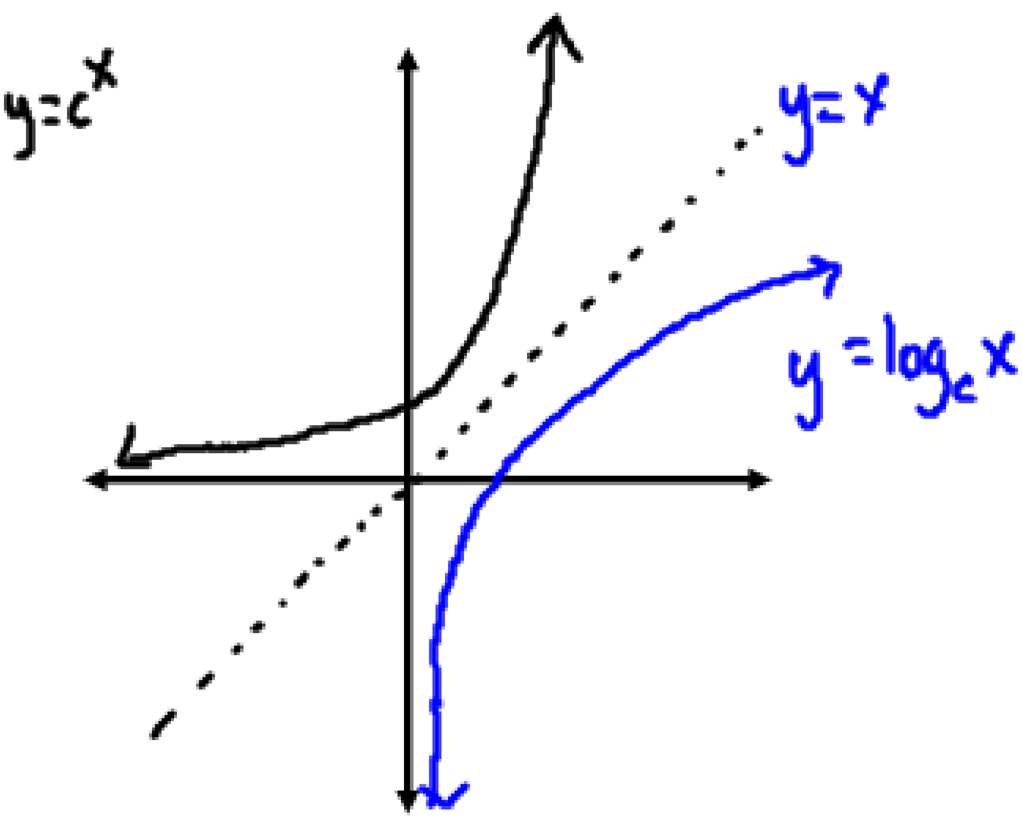
Mapping Rule

$$(x,y) \rightarrow \left(\underset{R_y}{-\tfrac{1}{b}}x+h,\ \underset{R_x}{-a}y+k\right)$$

Base Functions

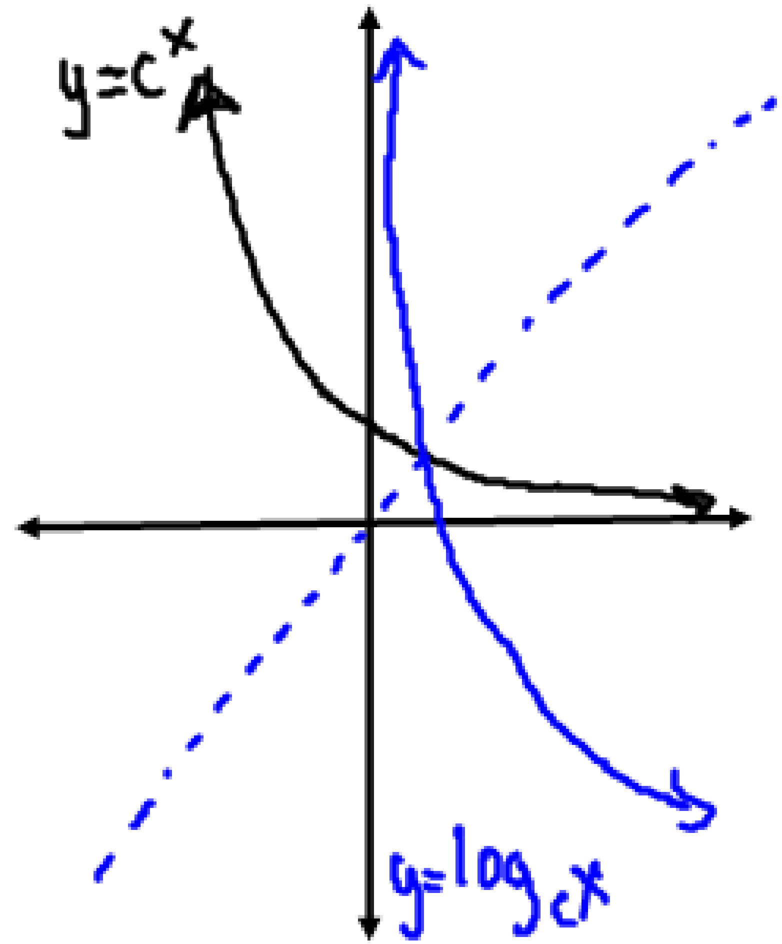
$y = \log_c x$

$c > 1$

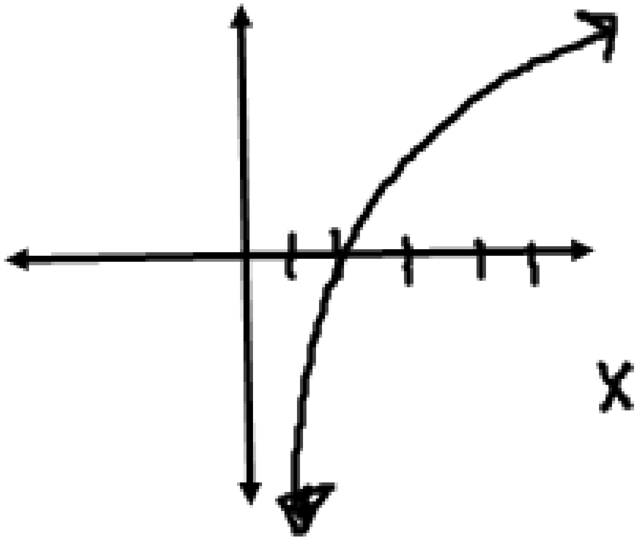


$y = \log_c x$

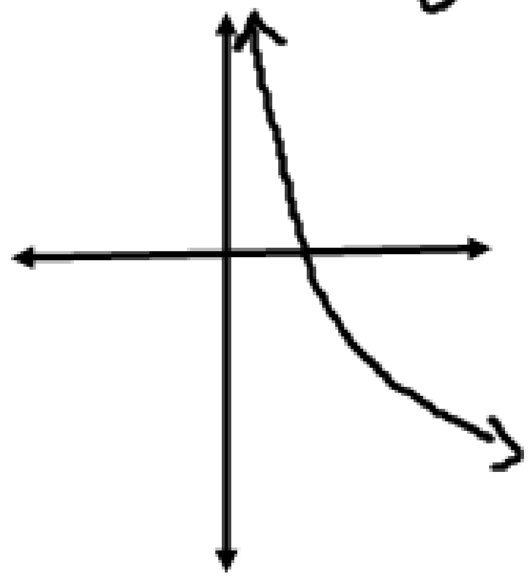
$0 < c < 1$



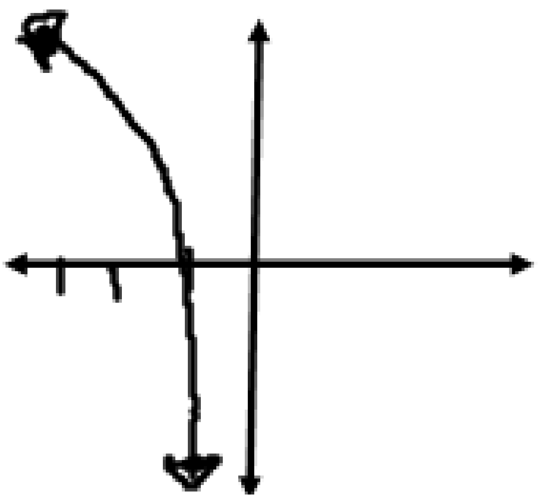
$y = \log_c x \quad , c > 1$



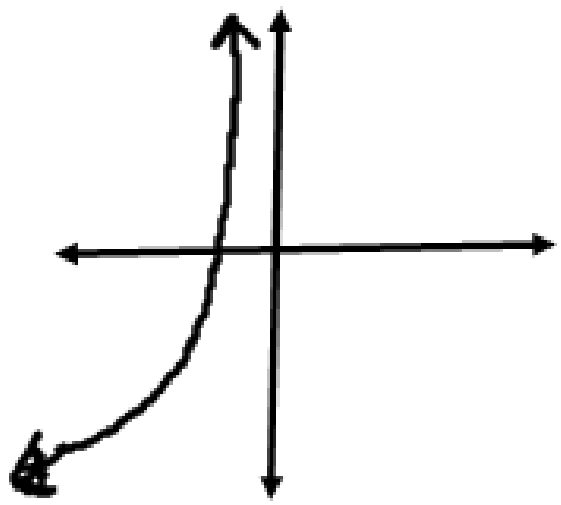
<u>R_x</u> $y = -\log_c(x)$



<u>R_y</u> $y = \log_c(-x)$

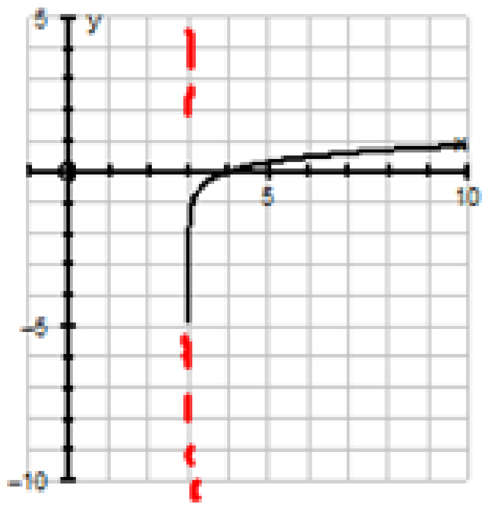
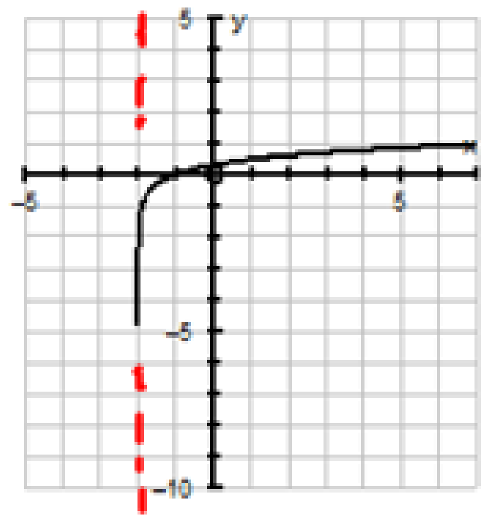
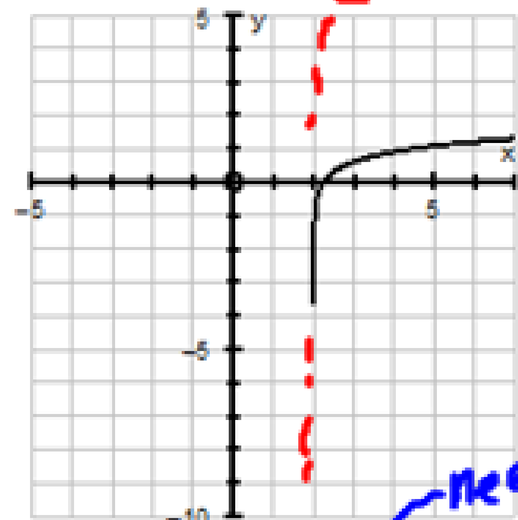


<u>R_x and R_y</u> $y = -\log_c(-x)$



When we graph log functions with *horizontal transformations* we see that the graphs move as shown:

Vertical Asymptote

3  $y = \log(x-3)$ HT +3	-2  $y = \log(x+2)$ HT -2	2  needs to be factored $y = \log(4x-8)$ $(4(x-2))$

HS $\frac{1}{4}$ HT+2

We can easily see the domain of each:

$\{x \mid x > 3, x \varepsilon R\}$	$\{x \mid x > -2, x \varepsilon R\}$	$\{x \mid x > 2, x \varepsilon R\}$

We could have found these domains without graphing them by recalling that $y = \log_b(x)$, x >0.

$y = \log(x-3)$

$x-3>0$

$x>3$

$D: \{x \mid x>3\}$

$y = \log(x+2)$

$x+2>0$

$x>-2$

$D: \{x \mid x>-2\}$

$y = \log(4x-8)$

$4x-8>0$

$\frac{4x}{4} > \frac{8}{4}$

$x>2$

$y = \log(-3x+12)$

$-3x+12>0$

$\frac{-3x}{-3} > \frac{-12}{-3}$

$x<4$

Example:

Use transformations to sketch the graph of the function $c>1$

$$y=-\frac{1}{2}\log_{2}(-2x-6)+1$$

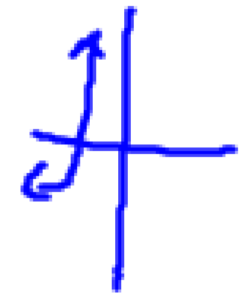
factor

$$y=-\frac{1}{2}\log_2(-2(x+3))+1$$

VS $\frac{1}{2}$ HS $\frac{1}{2}$

VT $+1$ HT -3

R_x Yes R_y Yes



State:

A) The equation of the asymptote

B) The domain and range

C) The x and y-intercepts (if they exist)

y-int: $(x=0)$

$$y=-\frac{1}{2}\log_2(-6)+1$$

No y-int :)

A) $x=-3$

B) $\{y \mid y\in R\}$

D: $-2x-6>0$

$$\frac{-2x}{-2}>\frac{6}{-2}$$

$$x<-3$$

$$\{x \mid x<-3\}$$

* to graph these we will only consider main features: Asymptotes, intercepts and general shape :)

x-int $(y=0)$

$0 = -\frac{1}{2}\log_2(-2x-6)+1$

$-1 = -\frac{1}{2}\log_2(-2x-6)$

$2 = \log_2(-2x-6)$

convert to exp!

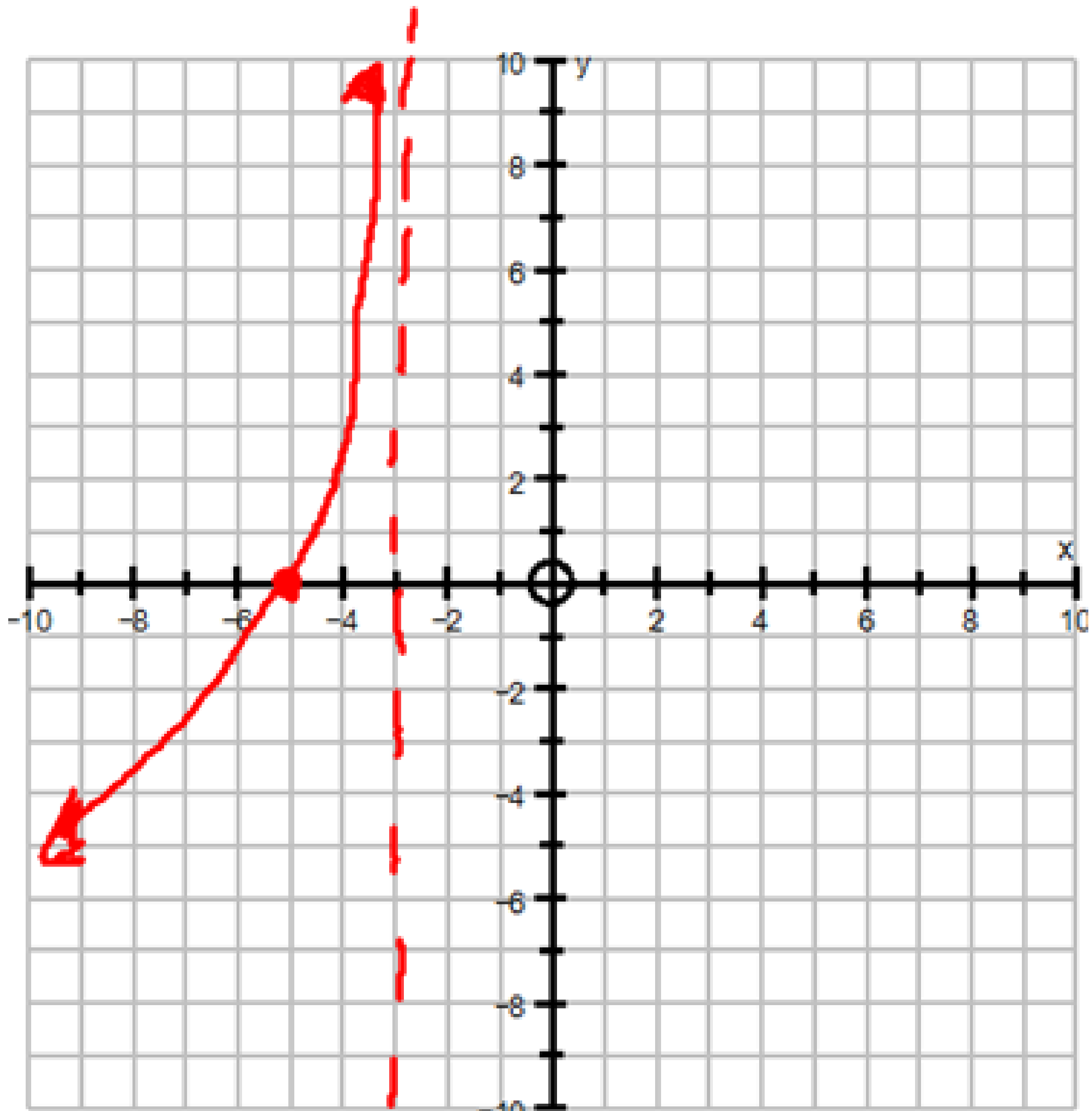
$2^2 = -2x-6$

$4 = -2x-6$

$10 = -2x$

$-5 = x$

$(-5,0)$



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last nights.....