Review of chapter 7 :)

Solve for x:

$$3^x + 3^x = 18$$

let $m = 3^x$

$m + m = 18$

$2m = 18$

$m = 9$

$3^x = 9$

$3^x = 3^2$

$x = 2.$

$$3^{x+1} + 3^{x-1} = \frac{10}{81}$$

$$a^m \cdot a^n = a^{m+n}$$

$$(3^x)(3^1) + (3^x)(3^{-1}) = \frac{10}{81}$$

let $m = 3^x$

$$3m + \frac{1}{3}m = \frac{10}{81}$$

$$\frac{9m}{3} + \frac{m}{3} = \frac{10}{81}$$

$$\frac{10m}{3} = \frac{10}{81}$$

$$\frac{\cancel{10}m}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{10}}\right) = \frac{\cancel{10}}{81}\left(\frac{3}{\cancel{10}}\right)$$

$$m = \frac{3}{81}$$

$$m = \frac{1}{27}$$

$$3^x = \frac{1}{27}$$

$$3^x = 3^{-3}$$

$$x = -3$$

$$2^{2x+1} - 9(2^x) + 4 = 0$$

Solve for x

$$(a^m)^n = (a^n)^m$$
$$= a^{mn}$$

$$(2^{2x})(2^1) - 9(2^x) + 4 = 0$$

$$(2^x)^2(2) - 9(2^x) + 4 = 0$$

$$a = 2^x$$

$$2a^2 - 9a + 4 = 0 \qquad \_\times\_ = 8$$

$$2a^2 - 8a - 1a + 4 = 0 \qquad \_+\_ = -9$$

$$2a(a-4) - 1(a-4) = 0$$

$$(a-4)(2a-1) = 0$$

$$a = 4$$
$$2^x = 4$$
$$2^x = 2^2$$
$$x = 2$$

$$a = \frac{1}{2}$$
$$2^x = \frac{1}{2}$$
$$2^x = 2^{-1}$$
$$x = -1$$

The weight W_t of a radioactive uranium-235 sample remaining after t years is given by the formula $W_t = W_0 \times 2^{-0.0002t}$ grams, $t \geqslant 0$. Find:

a the original weight

b the percentage weight loss after 1000 years

c the time required until $\frac{1}{512}$ of the sample remains.

$t=0$

$W_t = W_0(2)^{-0.0002(0)}$

$= W_0(1)$

B) $W_{1000} = W_0 2^{-0.0002(1000)}$

$= W_0(0.87055)$

$= 0.87W_0$

I have 87% of my original mass

% loss $= 100\% W_0 - 87\% W_0$

$= 13\% W_0$

What is t when $W_t = \frac{1}{512} W_0$

$$W_t = W_0 2^{-0.0002t}$$

$$\frac{1}{512} \cancel{W_0} = \cancel{W_0} 2^{-0.0002t}$$

$$\frac{1}{512} = 2^{-0.0002t}$$

$$2^{-9} = 2^{-0.0002t}$$

$$-9 = -0.0002t \qquad t = 45000$$

8.1

Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

Inverse Functions

Recall: An inverse function is a function that "undoes" what the original function did.

Example:

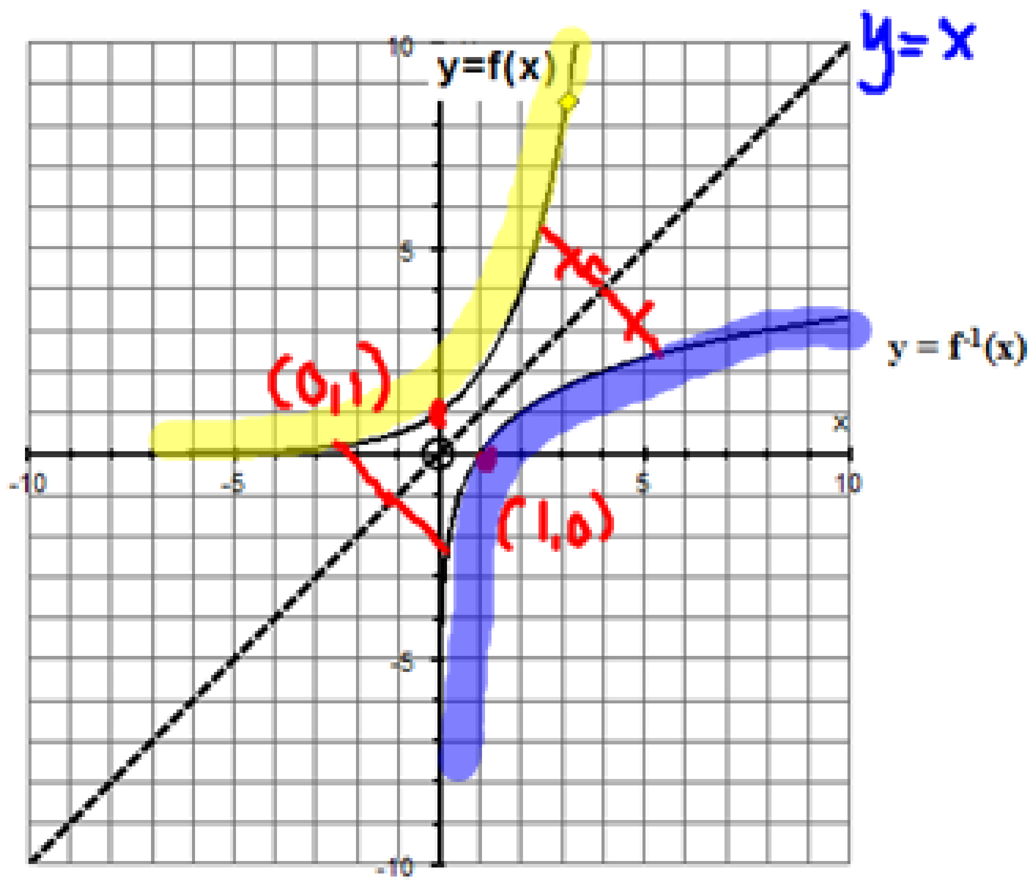
The inverse of the function $y = x^2$ is $y = \sqrt{x}$

- Graphically, the x and y values of points are switched

function	inverse
(x, y)	(y, x)
Domain	Range
Range	Domain

Reflection in $y = x$ axis

- The graph of a function and its inverse are mirror images of each other about the line $y = x$



function

HA: $y=0$

D: $\{x \mid x \in R\}$

R: $\{y \mid y>0\}$

inverse $f^{-1}(x)$

VA: $x=0$

D: $\{x \mid x>0\}$

R: $\{y \mid y \in R\}$

Example: Find the inverse of:

(a) $y = 6x - 12$

$x = 6y - 12$

$\frac{6y}{6} = \frac{x}{6} + \frac{12}{6}$

$y = \frac{1}{6}x + 2$

$f^{-1}(x) = \frac{1}{6}x + 2$

(b) $y = 3x^2 + 2$

$x = 3y^2 + 2$

$\frac{3y^2}{3} = \frac{x-2}{3}$

$y^2 = \frac{x-2}{3}$

$\sqrt{y^2} = \sqrt{\frac{x-2}{3}}$

$y = \pm\sqrt{\frac{x-2}{3}}$

(c) $y = 2^x$

$x = 2^y$

How do I solve for y?!?

Logarithm – used to describe the inverse function of an exponential function.

Inverse Function

Logarithmic Function

$x = 10^y$ $\xleftrightarrow{\text{equivalent}}$ $y = \log_{10} x$

$\text{answer} = \text{base}^{\text{exponent}}$

$\text{exp} = \log_{\text{base}}(\text{ans})$

Examples:

$\text{exp} = \log_{\text{base}}(\text{ans})$

1. Write each in logarithmic form:

ans

exp

(a) $16 = 2^4$

Base

$4 = \log_2 16$

Say "4 is log base 2 of 16"

(b) $\frac{1}{9} = 3^{-2}$

$-2 = \log_3\left(\frac{1}{9}\right)$

Examples: $\quad \exp = \log_{base}(\text{ans})$

1. Write each in ~~logarithm~~ exponential ic form:

(a) $\log_3 81 = 4$

$81 = 3^4$

(b) $\log 100 = 2$

When the base is missing we assume it is 10.

$\log_{10} 100 = 2$

$10^2 = 100$

3. Evaluate:

(a) $\log_2 32 = x$

Think: 2 raised to what exponent is 32

convert to exponential

$2^x = 32$

$2^x = 2^5$

$x = 5$

$\log_2 32 = 5$

(b) $\log_3 \frac{1}{9} = y$

$3^y = \frac{1}{9}$

$3^y = 3^{-2}$

$y = -2$

$\log_3\left(\frac{1}{9}\right) = -2$

4. Solve each of the following equations:

(a) $\log_3 m = 4$

Convert to an exponential

$3^4 = m$

$m = 81$

(b) $\log_8 2 = y$

$8^y = 2$

$(2^3)^y = 2$

$2^{3y} = 2^1$

$3y = 1$

$y = \frac{1}{3}$

Graph $y = 3^x - 2$ and its inverse.

For each graph:

State the domain and range

State the intercepts

Any asymptotes

$y = 3^x - 2$

VT -2

HA: $y = -2$

$\{x \mid x \in R\}$

$\{y \mid y > -2\}$

$\underline{y\text{-int}}\ (x=0)$

$y = 3^0 - 2 \quad y = -1$

$(0,-1)$

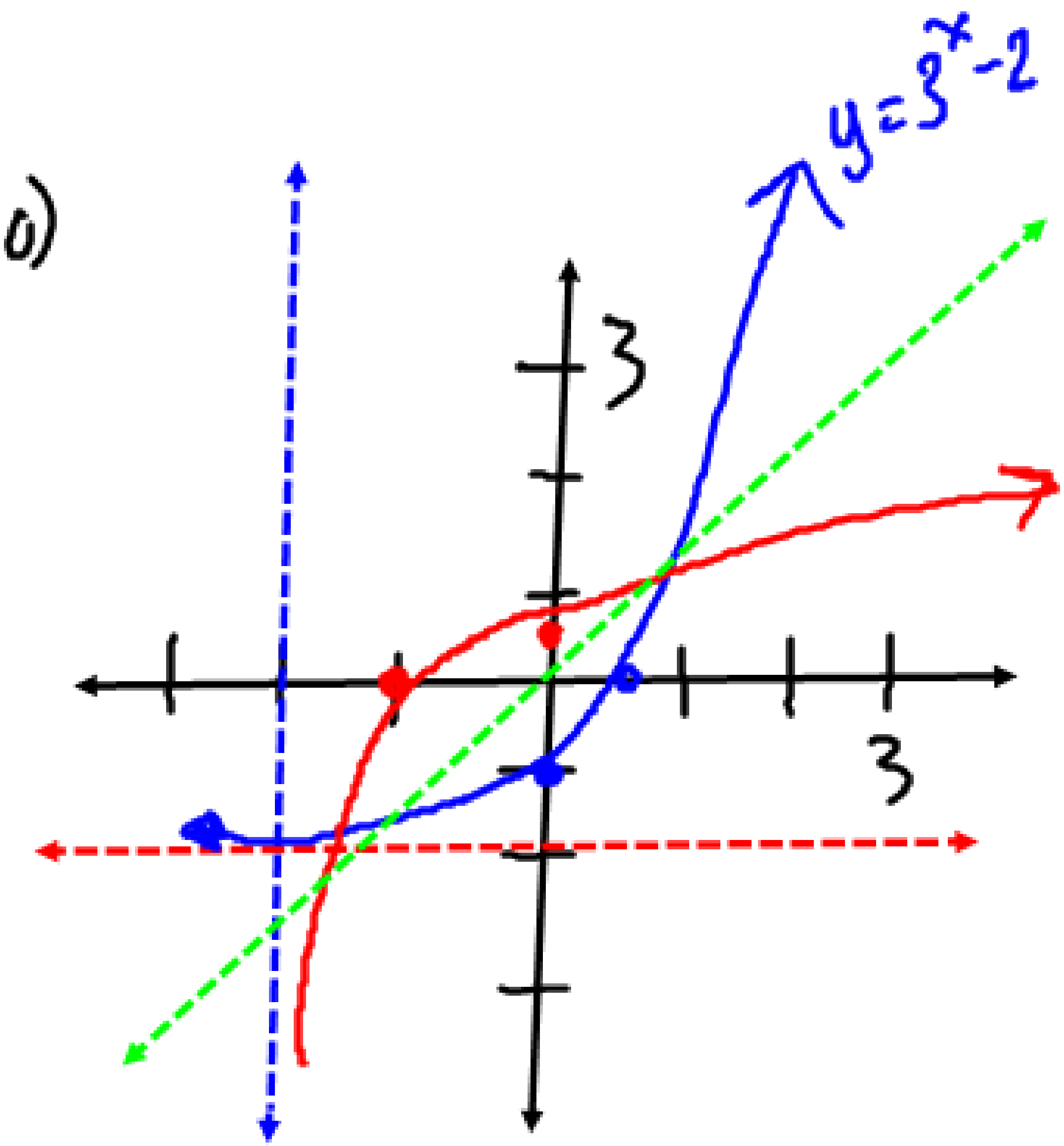
$\underline{x\text{-int}}\ (y=0)$

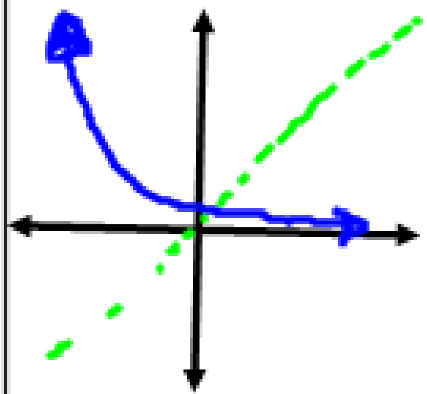
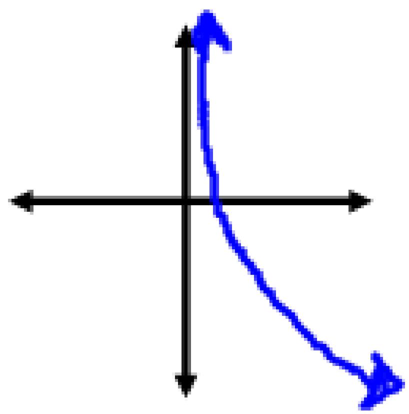
$0 = 3^x - 2$

$2 = 3^x \rightsquigarrow$

$x = \log_3 2$

$0 < x < 1$



	Equation	Sketch	Inc/Dec	Domain & Range	Intercepts
)	$y = 3^{-x}$		Dec	$\{x \mid x \in R\}$ $\{y \mid y > 0\}$	x-int: NONE y-int: (0,1)
)	$x = 3^{-y}$ $-y = \log_3 x$ $y = -\log_3 x$		Dec	$\{x \mid x > 0\}$ $\{y \mid y \in R\}$	x-int: (1,0) y-int: NONE

HW: pg 380
2-7,9,12,13
sheet