7.3

Solving Exponential Equations

Focus on...

- determining the solution of an exponential equation in which the bases are powers of one another
- solving problems that involve exponential growth or decay
- solving problems that involve the application of exponential equations to loans, mortgages, and investments

Laws of Exponents:

Multiplication: $a^m \times a^n = a^{m+n}$

Division: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power: $(a^m)^n = a^{mn}$

Power of a product: $(ab)^m = a^m b^m$

Different from $(a+b)^m$

$(a+b)(a+b)(a+b)\cdots\cdots$

m times

Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Rational Exponent $a^{\frac{1}{n}} = \sqrt[n]{a}$

$(a+b)^2$

$a^2+2ab+b^2$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

OR

$$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$$

$$a^{-m} = \frac{1}{a^m}$$

Review of Laws of Exponents: BEDMAS

1. Simplify each expression. Leave only positive exponents in your answer.

A) $\dfrac{(x^n)^2(x^{n+2})^2}{x^n}$

$\dfrac{(x^{2n})(x^{2n+4})}{x^n}$

$\dfrac{x^{4n+4}}{x^n} = x^{(4n+4)-(n)}$

$= x^{3n+4}$

C) $\left(\sqrt{(a^{\frac{2}{3}})}\right)^{-3}$

$\left((a^{\frac{2}{3}})^{\frac{1}{2}}\right)^{-3}$

$(a^{\frac{1}{3}})^{-3} = a^{-1}$

$= \dfrac{1}{a}$

D) $(2\sqrt[3]{c})\left(\frac{1}{\sqrt{c}}\right)$

$2(c)^{\frac{1}{3}}(c)^{-\frac{1}{2}}$

$2c^{\frac{1}{3}+\left(-\frac{1}{2}\right)}$

$2\cdot c^{-\frac{1}{6}} = 2\left(\frac{1}{c^{1/6}}\right)$

$= \frac{2}{\sqrt[6]{c}}$

$\frac{1}{3}+\left(-\frac{1}{2}\right)$

$\frac{2}{6}-\frac{3}{6}$

$\frac{2-3}{6} = -\frac{1}{6}$

2. Evaluate each expression:

B) $(-6)^0 = 1$

C) $-6^0 = -1$

Think subtract 6^0

D) $\frac{5^{-3}}{5^{-4}}$

$5^{-3-(-4)}$

$5^{-3+4} = 5^1$

$= 5$

3. Express each as a base of 2 and evaluate if possible.

A) $(4^2)^3$

$= ((2^2)^2)^3$

$= (2^4)^3$

$= 2^{12}$

$= 4096$

B) $(4^{m+3})(2)$

$((2^2)^{m+3})(2)$

$(2^{2m+6})(2^1)$

2^{2m+7}

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

$2^{11} = 2048$

$2^{12} = 4096$

$2^{13} = 8192$

$2^{14} = 16384$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

$3^7 = 2187$

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

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$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

$3^6 = 729$

4. Write each as a single ~~power~~ base.

A) $\dfrac{(8^{2n+1})(4^{2-n})}{(2^{2n})^3}$

$$= \frac{\left((2^3)^{2n+1}\right)\left((2^2)^{2-n}\right)}{2^{6n}}$$

$$= \frac{(2^{6n+3})(2^{4-2n})}{2^{6n}}$$

$$= \frac{2^{(6n+3)+(4-2n)}}{2^{6n}}$$

$$= \frac{2^{4n+7}}{2^{6n}}$$

$$= 2^{(4n+7)-(6n)}$$

$$= 2^{7-2n}$$ the same as 2^{-2n+7}

$$2^{-(2n-7)}$$

$$= \frac{1}{2^{2n-7}}$$

write as a single base

B) $\dfrac{(6^{-3})(2^{4})(9^{-2})}{(27^{-4})(4^{-2})}$

$= \dfrac{(6^{-3})(2^{4})((3^{2})^{-2})}{((3^{3})^{-4})((2^{2})^{-2})}$

$= \dfrac{(6^{-3})(2^{4})(3^{-4})}{(3^{-12})(2^{-4})}$

$\rightarrow (6^{-3})(2^{4-(-4)})(3^{-4-(-12)})$

$(6^{-3})(2^{8})(3^{8})$

$(6^{-3})(2\cdot 3)^{8}$ $\quad (ab)^{m} = a^{m}b^{m}$

$(6^{-3})(6^{8})$

6^{5}

BEDMAS NO NEG EXPONENTS

4. Simplify.

A) $\left(y^{-2}+x^{-1}\right)^{-2}$

$= \left(\frac{1}{y^2}+\frac{1}{x}\right)^{-2}$

$= \left(\frac{1}{y^2}\left(\frac{x}{x}\right)+\frac{1}{x}\left(\frac{y^2}{y^2}\right)\right)^{-2}$

$= \left(\frac{x}{xy^2}+\frac{y^2}{xy^2}\right)^{-2}$

$= \left(\frac{x+y^2}{xy^2}\right)^{-2}$

$= \frac{(x+y^2)^{-2}}{(xy^2)^{-2}}$

$= \frac{(xy^2)^2}{(x+y^2)^2}$

$= \frac{x^2y^4}{(x+y^2)(x+y^2)}$

$= \frac{x^2y^4}{x^2+2xy^2+y^4}$

We're done :)

Solve each of the following equations:

A) $7^{x} = 7^{10}$

$x = 10$

B) $5^{2x+3} = 5^{x-9}$

$2x+3 = x-9$

$2x - x = -9-3$

$x = -12$

C) $8^{2x} = 2^{4x+1}$

$(2^{3})^{2x} = 2^{4x+1}$

$2^{6x} = 2^{4x+1}$

$6x = 4x+1$

$6x - 4x = 1$

$2x = 1$

$x = \frac{1}{2}$

D) $\frac{3}{48} = \frac{48\left(\frac{1}{2}\right)^{\frac{x}{92}}}{48}$

$\frac{3}{48} = \left(\frac{1}{2}\right)^{\frac{x}{92}}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{x}{92}}$

$\frac{1}{2^4} = \left(\frac{1}{2}\right)^{\frac{x}{92}}$

$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{x}{92}}$

$4 = \frac{x}{92} \quad \therefore \boxed{x = 368}$

E) $81^{2x} = 27^{4x-5}$

$(3^4)^{2x} = (3^3)^{4x-5}$

$3^{8x} = 3^{12x-15}$

$8x = 12x - 15$

$15 = 12x - 8x$

$15 = 4x$

$\boxed{\frac{15}{4} = x}$

remember this question from ch 6?? pg321

19. Find the general solution for the equation $4(16^{\cos^2 x}) = 2^{6\cos x}$. Give your answer in radians.

$(2^2)(2^4)^{\cos^2 x} = 2^{6\cos x}$

$(2^2)(2^{4\cos^2 x}) = 2^{6\cos x}$

$2^{4\cos^2 x + 2} = 2^{6\cos x}$

$4\cos^2 x + 2 = 6\cos x$

$4\cos^2 x - 6\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = 1$

$\vdots$ $\qquad$ $\vdots$

H) $9^x = 3^x + 6$

$(3^2)^x = 3^x + 6$

$(3^x)^2 = 3^x + 6$

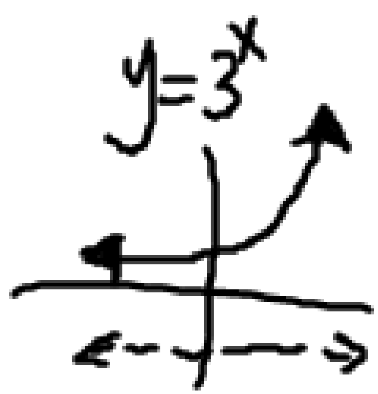
let $m = 3^x$

$m^2 = m + 6$

$m^2 - m - 6 = 0$

$(m-3)(m+2) = 0$

$m = 3 \qquad m = -2$



$(a^m)^n = (a^n)^m$

$3^x = 3^1$

$x = 1$

$3^x = -2$

NO SOL'N

There is no value of x that will make 3^x negative

#4

HW: pg 364 #1-5, C2, sheet (#6,7)