C) $2\sin 2x = 1$ $\quad 0 \le x \le 2\pi$

$\sin 2x = \frac{1}{2}$

we don't need to use an identity

$2x = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} + 2\pi k, k \in I$

$x = \begin{cases} \frac{\pi}{12} \\ \frac{5\pi}{12} \end{cases} + \pi k, k \in I$

$\frac{24\pi}{12}$

$\frac{12\pi}{12}$

interval:

$x = \begin{cases} \frac{\pi}{12}, \frac{13\pi}{12} \\ \frac{5\pi}{12}, \frac{17\pi}{12} \end{cases}$

Pg 321

Extend

17. Solve $4\sin^2 x = 3\tan^2 x - 1$ algebraically. Give the general solution expressed in radians.

- change $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$
- multiply each term by $\cos^2 x$
- eliminate $\cos^2 x$ by using identity $\sin^2\theta + \cos^2\theta = 1$, $\cos^2\theta = 1 - \sin^2\theta$
- expand, collect like terms

$$4\sin^2 x = 3\left(\frac{\sin^2 x}{\cos^2 x}\right) - 1$$

$$4\sin^2 x\cos^2 x = 3\sin^2 x - \cos^2 x$$

$$4\sin^2 x(1-\sin^2 x) = 3\sin^2 x - (1-\sin^2 x)$$

$$4\sin^2 x - 4\sin^4 x = 3\sin^2 x - 1 + \sin^2 x$$

$$0 = 4\sin^4 x - 1$$

$0 = 4\sin^4 x - 1$ ← Diff of squares

$0 = (2\sin^2 x + 1)(2\sin^2 x - 1)$

$2\sin^2 x + 1 = 0$

$\sin^2 x = -\frac{1}{2}$

NO SOL'N

$2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$

$$\boxed{x = \begin{cases} \frac{\pi}{4} \\ \frac{3\pi}{4} \end{cases} + 2\pi k,\ k \in I}$$

$\sin x = -\frac{\sqrt{2}}{2}$

$$\boxed{x = \begin{cases} \frac{5\pi}{4} \\ \frac{7\pi}{4} \end{cases} + 2\pi k,\ k \in I}$$

Pg 321

Extend

17. Solve $4\sin^2 x = 3\tan^2 x - 1$ algebraically. Give the general solution expressed in radians.

- change $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$
- look for NPV: $\cos x \neq 0$

$$x \neq \begin{cases} \frac{\pi}{2} + 2\pi k \\ \frac{3\pi}{2} \end{cases}$$

$$4\sin^2 x = 3\frac{\sin^2 x}{\cos^2 x} - 1$$

- mult each term by $\cos^2 x$

$$4\sin^2 x\cos^2 x = 3\sin^2 x - \cos^2 x$$

- use identity $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$4(1-\cos^2 x)\cos^2 x = 3(1-\cos^2 x) - \cos^2 x$$

$$4\cos^2 x - 4\cos^4 x = 3 - 3\cos^2 x - \cos^2 x$$

$$0 = 4\cos^4 x - 8\cos^2 x + 3$$

$0 = 4\cos^4 x - 8\cos^2 x + 3$

let $a = \cos^2 x$

$0 = 4a^2 - 8a + 3$

$\_\_ \times \_\_ = 12$

$\_\_ + \_\_ = -8$

$0 = \underbrace{4a^2 - 6a}\underbrace{- 2a + 3}$

$0 = 2a(2a-3) - 1(2a-3)$

$0 = (2a-3)(2a-1)$

$a = \frac{3}{2}$ $\quad a = \frac{1}{2}$

$\cos^2 x = \frac{3}{2}$ $\quad \cos^2 x = \frac{1}{2}$

$\cos^2 x = \frac{3}{2}$

$\cos^2 x = 1.5$

$\cos x = \pm\sqrt{1.5}$

Bigger than 1

NO SOL'N

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$$\cos x = \pm\frac{\sqrt{2}}{2}$$

$$\cos x = \frac{\sqrt{2}}{2} \qquad \cos x = -\frac{\sqrt{2}}{2}$$

$$\boxed{x = \begin{cases} \frac{\pi}{4} \\ \frac{7\pi}{4} \end{cases} + 2\pi k,\ k \in I} \qquad \boxed{x = \begin{cases} \frac{3\pi}{4} \\ \frac{5\pi}{4} \end{cases} + 2\pi k,\ k \in I}$$

14. Solve $\sin 2x = 2 \cos x \cos 2x$ algebraically. Give the general solution expressed in radians.

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$2\sin x\cos x = 2\cos x(2\cos^2 x - 1)$

$2\sin x\cos x = 4\cos^3 x - 2\cos x$

$0 = 4\cos^3 x - 2\cos x - 2\sin x\cos x$

$0 = 2\cos x(2\cos^2 x - 1 - \sin x)$

$0 = 2\cos x(2(1-\sin^2 x) - 1 - \sin x)$

$0 = 2\cos x(2 - 2\sin^2 x - 1 - \sin x)$

$0 = 2\cos x(-2\sin^2 x - \sin x + 1)$

$0 = 2\cos x(-2\sin^2 x - \sin x + 1)$

$\frac{2\cos x}{2} = \frac{0}{2}$

$-2\sin^2 x - \sin x + 1 = 0$ ← mult each term by -1

$\cos x = 0$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi k,\ k \in I$

$\sin x = \frac{1}{2}$

$\sin x = -1$

$x = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} + 2\pi k,\ k \in I$

$x = \left\{ \frac{3\pi}{2} + 2\pi k,\ k \in I \right.$

14. Solve $\sin 2x = 2\cos x \cos 2x$ algebraically. Give the general solution expressed in radians.

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$2\sin x\cos x = 2\cos x(\cos^2 x - \sin^2 x)$

$2\sin x\cos x = 2\cos^3 x - 2\cos x\sin^2 x$

$0 = 2\cos^3 x - 2\cos x\sin^2 x - 2\sin x\cos x$

$0 = 2\cos x[\cos^2 x - \sin^2 x - \sin x]$

$0 = 2\cos x[(1 - \sin^2 x) - \sin^2 x - \sin x]$

$0 = 2\cos x[1 - \sin x - 2\sin^2 x]$

$$0 = 2\cos x\left[1 - \sin x - 2\sin^2 x\right]$$

$$2\cos x = 0 \qquad 1 - \sin x - 2\sin^2 x = 0$$

$$\frac{2\cos x}{2} = \frac{0}{2}$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\cos x = 0$$

$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi k,\ k \in I$$

$$x = \begin{cases} \frac{\pi}{6} + 2\pi k,\ k \in I \\ \frac{5\pi}{6} \end{cases} \qquad x = \left\{ \frac{3\pi}{2} + 2\pi k,\ k \in I \right.$$

18. Solve $\dfrac{(1 - \sin^2 x) - 2\cos x}{\cos^2 x - \cos x - 2} = -\dfrac{1}{3}$ algebraically over the domain $-\pi \le x \le \pi$.

$$\frac{\cos^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \frac{-1}{3}$$

$$\frac{\cos x\,(\cancel{\cos x - 2})}{(\cancel{\cos x - 2})(\cos x + 1)} = \frac{-1}{3}$$

NPV:

$\cos x - 2 \ne 0$

$\cos x \ne 2$

X

$\cos x + 1 \ne 0$

$\cos x \ne -1$

$x \ne \{\pi + 2\pi k,\ k \in I$

$$\frac{\cos x}{\cos x + 1} = \frac{-1}{3}$$

$$3\cos x = -1(\cos x + 1)$$

$$3\cos x = -\cos x - 1$$

$$4\cos x = -1$$

$$\cos x = \frac{-1}{4}$$

solve with calculator

Stay tuned!- We will solve this question in chapter 7 :)

19. Find the general solution for the equation $4(16^{\cos^2 x}) = 2^{6\cos x}$. Give your answer in radians.

#1-5

Hw: pg321 #6, 14, 16, 18, C2